Hypothesis Testing

Sessions 8

Research Hypothesis

Statistical Hypothesis for the following Research Hypothesis:

- Older workers are more loyal to a company.
  - H_0: Loyalty of Old Workers = Loyalty of Young Workers
  - H_A: Loyalty of Old Workers $>$ Loyalty of New Workers
- Bigger companies spend a higher percentage of their annual budget on advertising than do smaller companies.
- Using a Six Sigma quality approach to manufacturing results in greater productivity.
- Scrap metal price is a good leading indicator of the Industrial Production Index.

One Tailed and Two Tailed Tests

Suppose a company has held an 18% share of the market. However, because of an increased marketing effort, company officials believe the company's market share is now greater than 18%, and the officials would like to prove it.

- H_0: p = 0.18
- H_A: p > 0.18

Suppose flour packaged by a manufacturer is sold by weight, and a particular size of package is supposed to average 250 gms. Suppose the manufacturer wants to test to determine whether their packaging process is out of control as determined by the weight of the flour packages.

- H_0: $\mu = 250$ gms
- H_A: $\mu \neq 250$ gms

Statistical Hypothesis

- A statistical hypothesis is a formal hypothesis structure set up with a null and an alternative hypothesis to scientifically test research hypotheses
- the **null hypothesis** states that the “null” condition exists; that is, there is nothing new happening, the old theory is still true, the old standard is correct, and the system is in control.
- The **alternative hypothesis**, on the other hand, states that the new theory is true, there are new standards, the system is out of control, and/or something is happening.
- A **substantive result** occurs when the outcome of a statistical study produces statistically significant results that are also important to the decision maker.

Accept or Reject

- Do we accept or reject the null hypothesis?
- Two types of errors possible:
  - Not accept a null hypothesis when it should be accepted
  - Accept a null hypothesis when it should be rejected

State of Nature

Action		**Null True**	**Null False**
	Fail to Reject Null	Correct Decision	Type II Error (β)
	Reject Null	Type I Error (α)	Correct Decision

Accept or Reject

- A prisoner is to be punished
  - Null: Not Guilty
  - Type I Error: Prisoner not guilty but is punished (False Positive)
  - Type II Error: Prisoner guilty but not punished
- Legal Principle: Even if several guilty have to be set free, one innocent should not be punished
- Which error is more serious – Type I or Type II?
- Which error is being minimized?

False Positive in Medical Test: Result indicates that a given condition exists when in reality it does not.

State of Nature

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Accept or Reject

Suppose flour packaged by a manufacturer is sold by weight, and a particular size of package is supposed to average 250 gms. Suppose the manufacturer wants to test to determine whether their packaging process is out of control as determined by the weight of the flour packages.

H_0: μ = 250 gms

H_A: $\mu \neq$ 250 gms

- What would be Type I and Type II errors?
- Type I error: Actual filling is 250 gms but reject null (The process is faulty → Replacement / Repair costs incurred even when not necessary)
- Suppose we write the hypothesis as

H_0: $\mu \neq$ 250 gms

H_A: μ = 250 gms

- What would be Type I and Type II errors?
- Type I error: Filling is faulty but reject null (The process is correct → No repairs → Defective product continues to go in the market)

Accept or Reject

- Type I errors can be easily controlled
- So, set-up hypothesis such that minimizing Type I errors will minimize loss due to error
- As the true value goes farther away from the hypothesized H_0, the probability of committing Type II error reduces
- If alpha is reduced, beta is increased, and vice versa.
- **Power**, which is equal to $1 - \beta$, is *the probability of a statistical test rejecting the null hypothesis when the null hypothesis is false*.

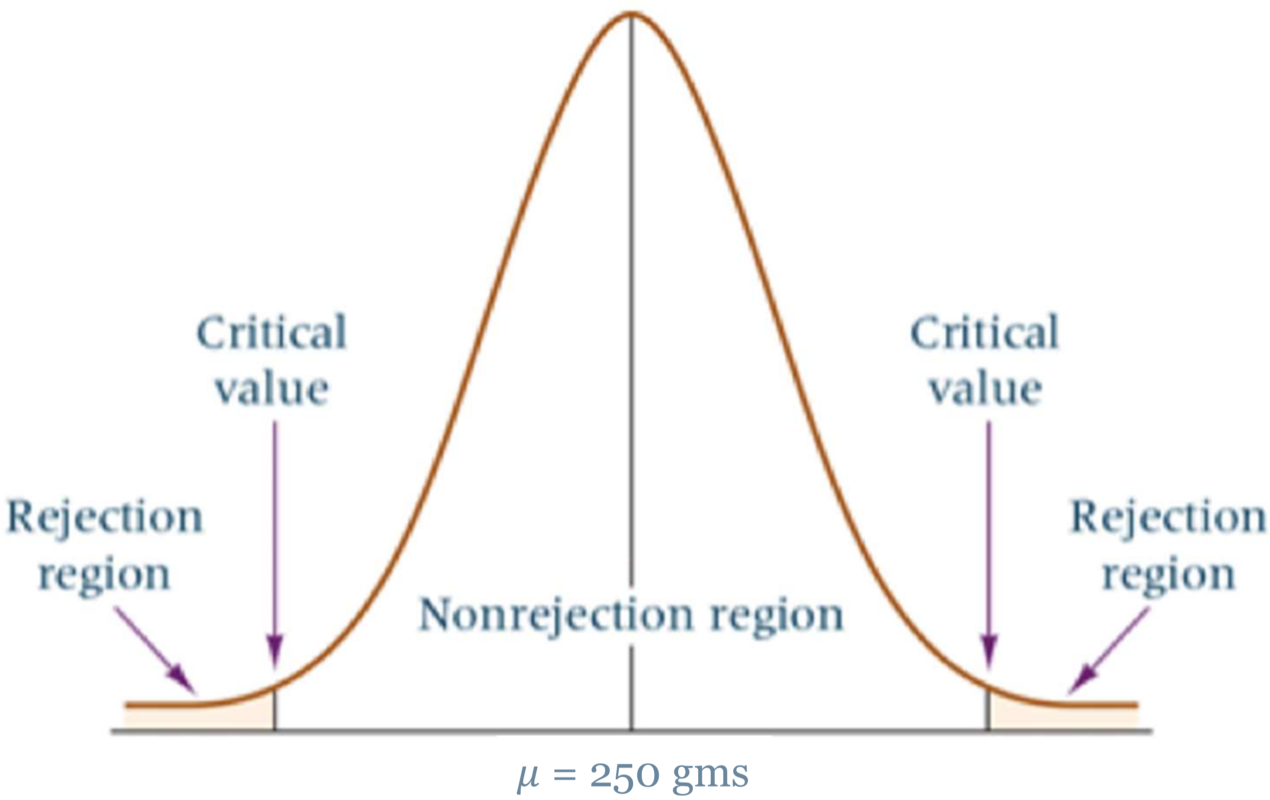
H_0 and H_1

- The H_0 represents the current belief in a situation.
- The H_1 represents a research claim.
- If you reject the H_0, you have statistical proof that the H_1 is correct.
- If you do not reject the H_0, you have failed to prove the H_1. The failure to prove the H_1, however, does not mean that you have proven the H_0.
- The H_0 always refers to a specified value of the population parameter (such as μ), not a sample statistic (such as $\bar{X}$).
- The statement of the H_0 always contains an equal sign regarding the specified value of the population parameter (e.g., $H_0 : \mu = 368$ grams).
- The statement of the H_1 never contains an equal sign regarding the specified value of the population parameter (e.g., $H_1 : \mu \neq 368$ grams).

Type I or Type II Error

- An analyst is testing to determine if 0.31 of all families own more than one car. His null hypothesis is that the population proportion is 0.31. He randomly samples 600 families and obtains a sample proportion of 0.33 that own more than one car. Based on these sample data, his decision is to fail to reject the null hypothesis. The actual population proportion is .31.
- Suppose it is generally known that the average price per square foot for a home in a particular locality is Rs. 15 per sq.ft. A business analyst believes that due to the economy, the average may now be less than that. To test her belief, she takes a random sample of 45 homes in this community, resulting in a sample mean of Rs. 13 per sq.ft. The analyst's decision based on this sample information is to fail to reject the null hypothesis. The actual average price per square foot is now Rs. 12.
- Suppose a utility analyst knows from past experience that the average water bill for a 2000-square-foot home in a large city is Rs. 250 per month. The utility analyst wants to test to determine if this figure is still true today. Her null hypothesis is that the population mean is Rs.250. To test this, she randomly samples 63 homes, resulting in a sample mean of Rs.290. From this, she decides to reject the null hypothesis. The actual average is Rs.270.

Acceptance Region



To minimize probability of Type I error, minimize the rejection region

→ Set α to low value: 0.1, 0.05, 0.01

Hypothesis Testing

A survey of CA firms across the country found that the average net income for sole-proprietor CA firms is Rs. 98,500. Because this survey is over two decades old, an accounting analyst wants to test this figure by taking a random sample of 112 sole-proprietor firms to determine whether the net income figure has changed or not. It is assumed that the population standard deviation of net incomes for sole-proprietor CA firms is Rs. 14,530. The sample mean is Rs. 102,220.

Step 1: Write the null and alternate hypothesis

H_0: μ = 98500; H_1: μ = 98500

Step 2: Determine the test statistic

- Test of mean
- σ known
- Sample > 30
- Use normal distribution (z)

Step 3: Decide on α – acceptable probability for Type I error – assume 0.05

Hypothesis Testing

Step 4: Is the test one-tailed or two-tailed? Accordingly determine the critical value – Value of standard test statistic for appropriate α (α for one tailed; $\alpha/2$ for two-tailed)

Step 5: For given data calculate z

$Z = (102220 - 98500)/14530 = 2.71$

Step 6: Statistical decision

- If $Z_{calc} > Z_{critical}$ → Reject H_0
- If $Z_{calc} < Z_{critical}$ → Fail to reject H_0

Step 7: Conclusion

- Average net income of sole-proprietary CA firms is not equal to Rs. 98,500.
- We cannot conclude that the income is Rs. 102,220.

Using the p-value method

- Find the probability associated with calculated z statistic
- For z = 2.71 → $P(z \lesseqgtr 2.71)$ = 0.0034
- If this is less than $\alpha/2$ = 0.025 → reject H_0

Critical Value Method

Construct a 95% interval around μ

If $\bar{X}$ is outside the interval, reject H_0

For μ = 98500 and σ = 14530

$$\bar{X} = 98500 \pm 14530/112 = 98500 \pm 2691$$

$$95809 \leq \bar{X} \leq 101191$$

Since $\bar{X}$ = 102220 is outside the interval – reject H_0

Other Cases

- If σ is not known – Use t-statistic and t-distribution

$$t_{\alpha/2,(n-1)} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

- Hypothesis testing for proportion

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

- Hypothesis testing of Variance

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma^2}$$

- In all cases the p-value method and critical value method can be used with appropriate modifications