Continuous Probability Distributions

Sessions 6

Applying Probability Distributions

- A survey conducted by International Society for Training and Development reported that on overage an employee receives 32 hours of training per year. Suppose the number of hours of training is uniformly distributed across all employees varying from 0 to 64 hours. What percentage of employees receive 50 hours or more of training?
- As a result of another survey, it was estimated that, on average, it costs $3,270 to hire an employee. Suppose such costs are normally distributed with a standard deviation of $400. Based on these figures, what is the probability that a randomly selected hire costs more than $4000? What percentage of employees is hired for less than $3000?

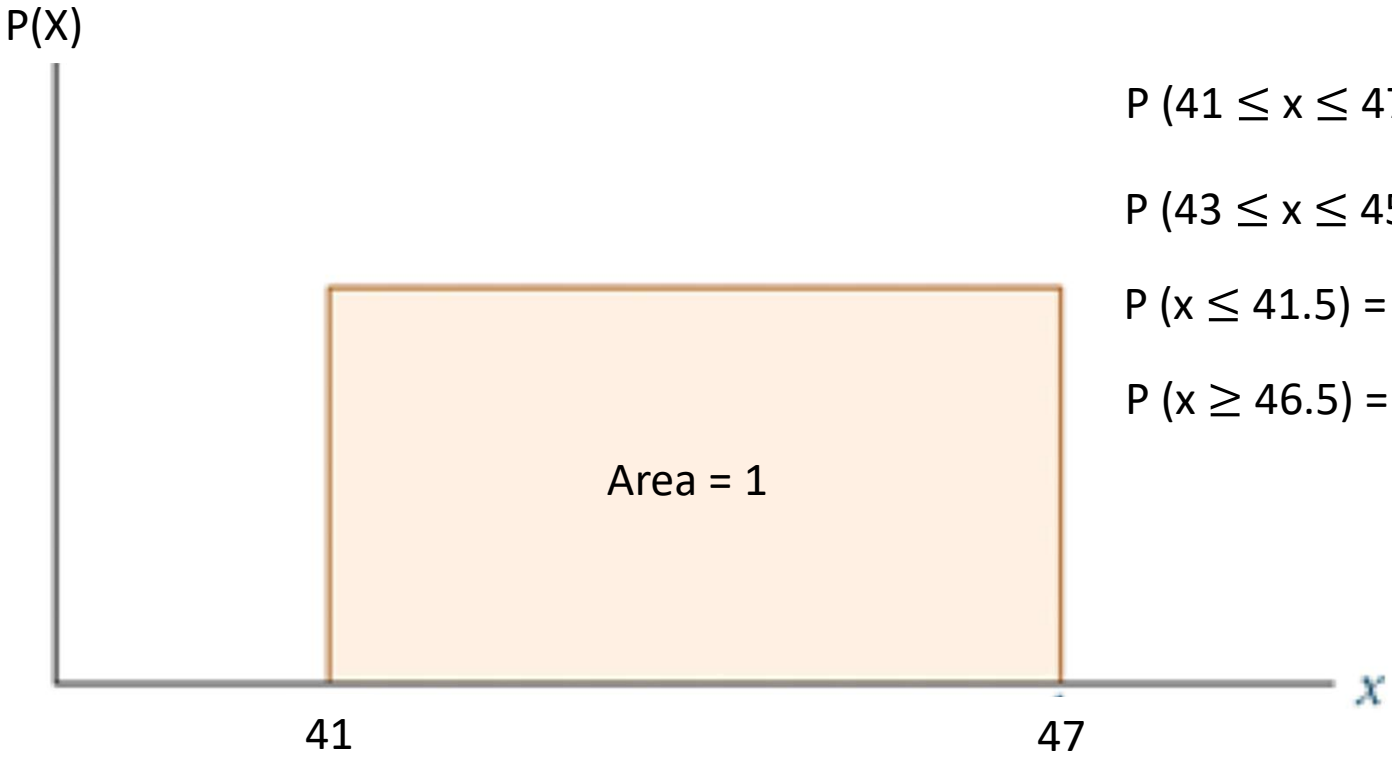
Definitions

Continuous random variables *take on values at every point over a given interval*.

- Examples:
  - Sampling the volume of liquid nitrogen in a storage tank
  - Measuring the time between customer arrivals at a retail outlet
  - Measuring the length of newly designed automobiles
  - Measuring the weight of grain in a grain elevator at different points of time
- Continuous Probability Distributions
  - Uniform distribution
  - Normal distribution
  - Exponential distribution
  - *t* distribution
  - Chi-square distribution
  - *F* distribution

Uniform Distribution

Suppose a production line is set up to manufacture bolts in lots of five per minute during a shift. When the lots are weighed, variation among the weights is detected, with lot weights ranging from 41 to 47 grams in a uniform distribution



$P(41 \leq x \leq 47) = 1$

$P(43 \leq x \leq 45) = ?$

$P(x \leq 41.5) = ?$

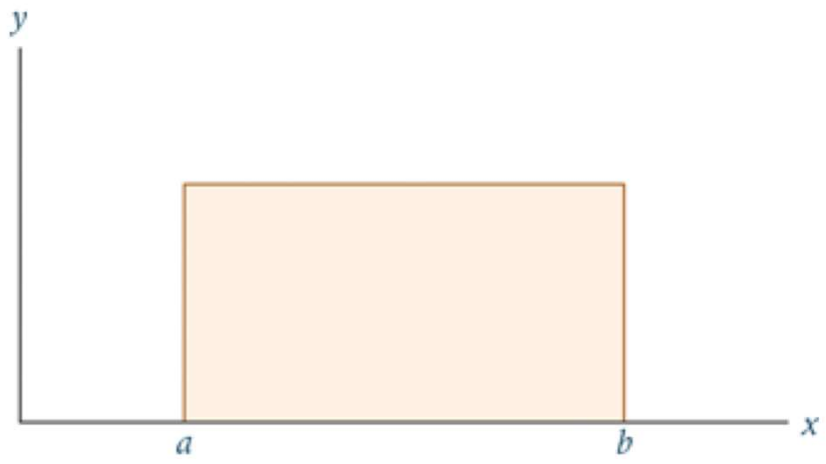
$P(x \geq 46.5) = ?$

Uniform Distribution

Height of the distribution: $\frac{1}{b-a}$

Mean: $\mu = \frac{a+b}{2}$

Standard Deviation: $\sigma = \frac{b-a}{\sqrt{12}}$



Probability: $P(x) = \frac{x_2 - x_1}{b-a} \quad where, \quad a \leq x_1 \leq x_2 \leq b$

Applications

- Suppose the amount of time it takes to assemble a plastic module ranges from 27 to 39 seconds and that assembly times are uniformly distributed. Describe the distribution. What is the probability that a given assembly will take between 30 and 35 seconds? Fewer than 30 seconds?
- According to the National Association of Insurance Commissioners, the average annual cost for automobile insurance in the United States in a recent year was $691. Suppose automobile insurance costs are uniformly distributed in the United States with a range from $200 to $1,182. What is the standard deviation of this uniform distribution? What is the height of the distribution? What is the probability that a person's annual cost for automobile insurance in the United States is between $410 and $825?

Application

How long does it take to download a two-hour movie from the iTunes store? According to Apple's technical support site, support.apple.com/HT201587, downloading such a movie using a 5 Mbit/s broadband connection should take 18 to 24 minutes. Assume that the download times are uniformly distributed between 18 and 24 minutes. If you download a two-hour movie, what is the probability that the download time will be:

a) Less than 19 minutes?

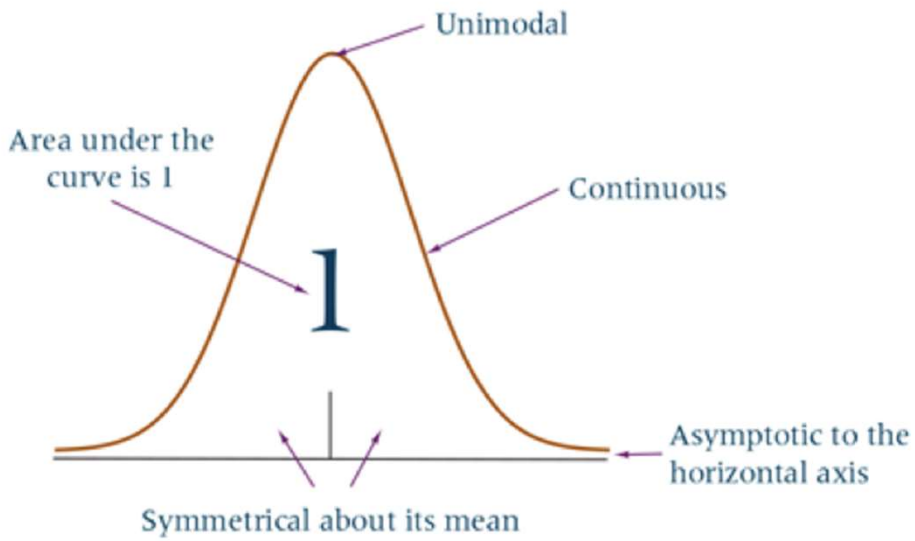
b) More than 23 minutes?

c) Between 20 and 22 minutes?

d) What are the mean and standard deviation of the download times?

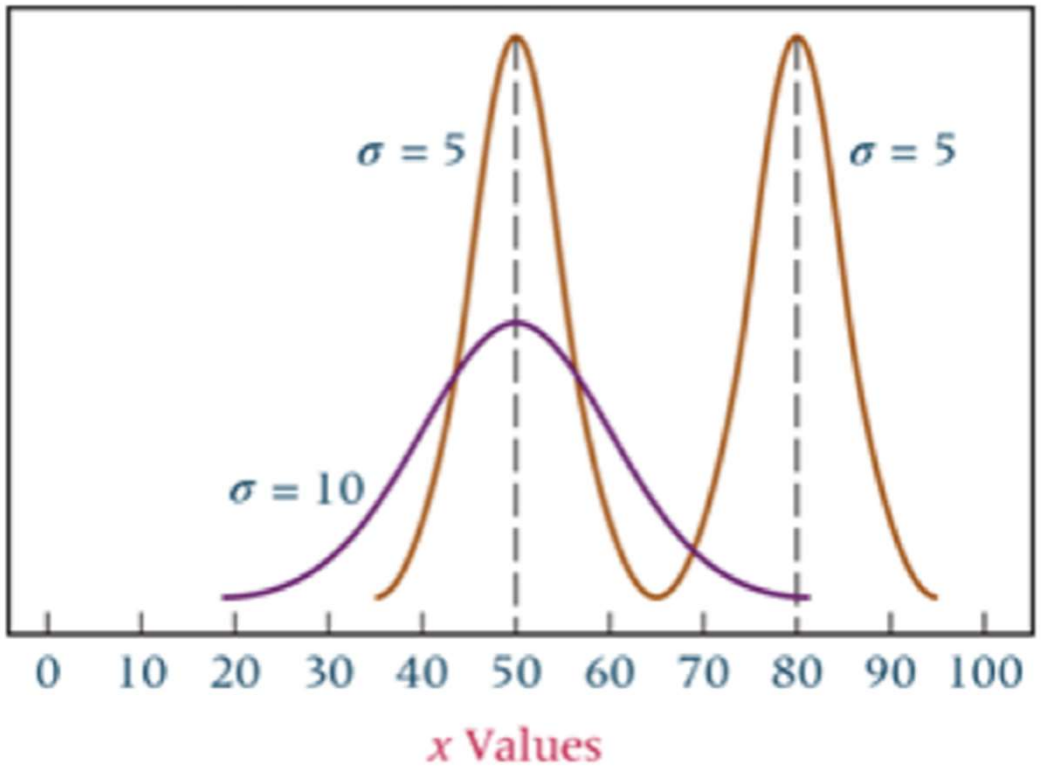
Normal Distribution

- It is a continuous distribution.
- It is a symmetrical distribution about its mean.
- It is asymptotic to the horizontal axis.
- It is unimodal.
- It is a family of curves.
- Area under the curve is 1



Normal Distribution

A Normal Distribution is characterized by two parameters: $\mu \; and \; \sigma$



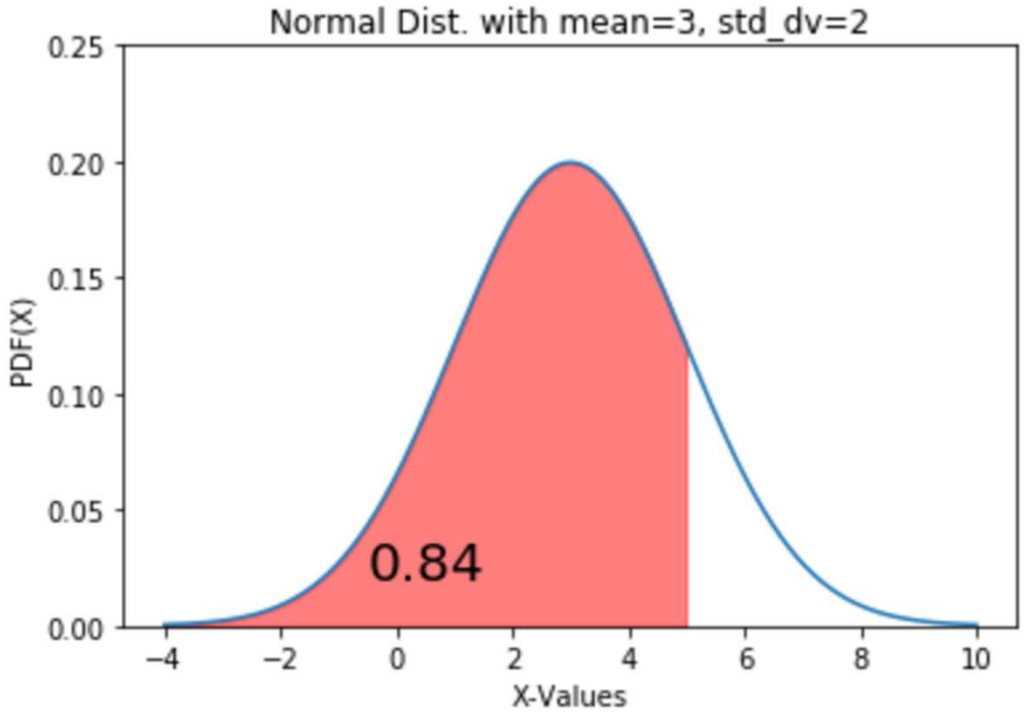
Normal Distribution

Calculating P ($X \leq x$) when X is normally distributed:

$\mu = 3$

$\sigma = 2$

P ($X \leq 5$) = ?



$$CDF = \phi(X) = P(X \leq x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2} .dx$$

Standard Normal Variate

- Z is normally distributed with $\mu = 0$ and $\sigma = 1$
- Area under the normal distribution curve between $\mu = 0$ and any point (Z = z) is given in a standard normal distribution table
- Any random series (X) can be converted to a standard normal variate $Z = (X - \mu)/\sigma$

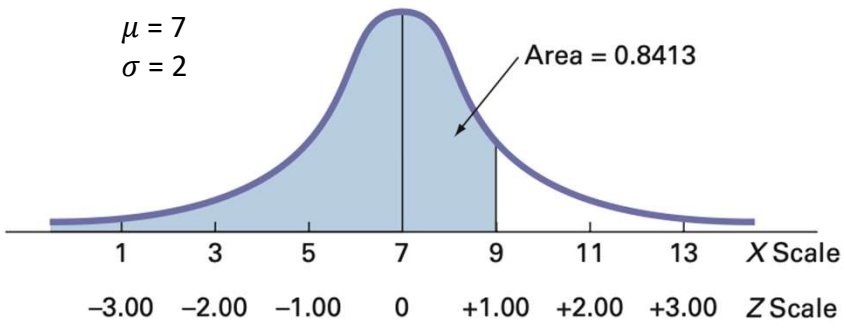
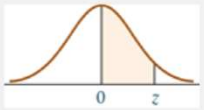


TABLE 6.2

z Distribution



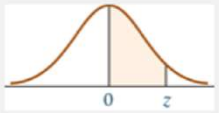
	Second Decimal Place in z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830

Application

According to National Environmental Protection Agency (EPA) data, on average there is 4.43 kgs of waste generated per person in the country per day. Suppose waste generated per person per day in the country is normally distributed with a standard deviation of 1.32 kgs. If a person is randomly selected, what is the probability that the person generates more than 6.00 kgs of waste per day?

TABLE 6.2

z Distribution



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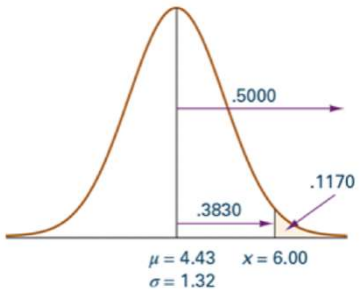
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177

$P(X \geq 6) = ?$

$Z = (6 - 4.43) / 1.32 = 1.19$

$P(X \geq 6) = 0.5 - 0.3830 = 0.117$



Application

Using this same waste-generation example, if a person is randomly selected, what is the probability that the person generates between 3.60 and 5.00 kgs of waste per day?

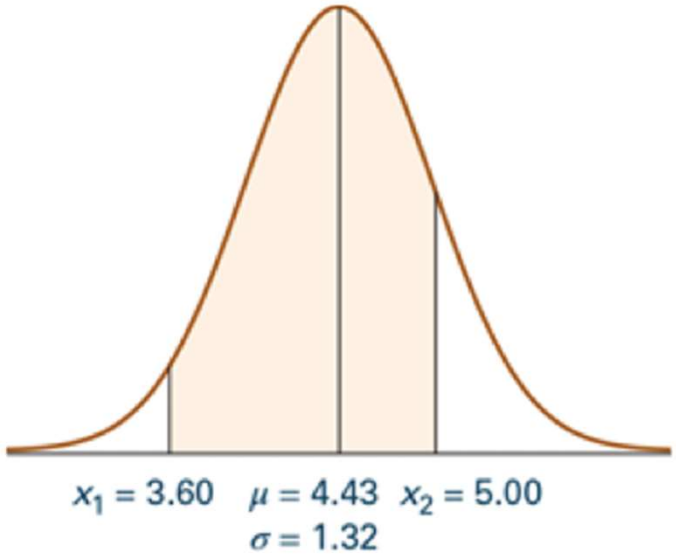
Step 1: Convert 3.6 and 5.0 in z scale

Step 2: Find the probabilities associated with these z values

Step 3: Calculate probability of P ($3.6 \leq x \leq 5.0$)

Excel Formula:

Prob =NORM.DIST(X, μ, σ, c = TRUE / FALSE)#



Note that excel calculates cumulative probability from -∞ to X

Application

A particular 10-inch (diameter) clay pipe weighs, on average, 44 kgs, and pipe weights are normally distributed in the population. If 74.22% of the pipe weights are more than 40 kgs, what is the value of the standard deviation?

$Z = (40 - 44) / \sigma$

What value of Z gives a probability of 0.7422?

$-0.65 = (40 - 44) / \sigma$

$\sigma = 6.154$

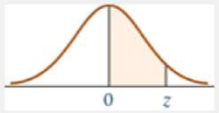
Excel Formula:

Z =NORM.INV(Prob, μ, σ)#

Note that excel calculates cumulative probability from -∞ to X

TABLE 6.2

z Distribution



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