

Module 1-S3: Fundamentals of Statistical Learning

POSTGRADUATE CERTIFICATE PROGRAM IN DIGITAL
TRANSFORMATION STRATEGY & LEADERSHIP

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Random Variable

$x = 0, 1, 2, 3, \dots$
 $\left\{ \frac{1}{4}, 1, \dots \right\}$



- ▶ A numerical measure of the outcome of an experiment
- ▶ May be discrete or continuous $\rightarrow X \in (1, 2)$
- ▶ Examples?
 - ▶ No. of ~~days~~ that a supplier takes to supply raw material
 - ▶ No. of ~~defects~~ in a batch
 - ▶ Time between two customer arrivals —
- ▶ In supply chain, most models are created with an assumption of continuous demand

Probability Distributions - Discrete

- ▶ Discrete Probability Function
 - ▶ Probability Distributions - Discrete
- ▶ Discrete Uniform Probability Distribution
- ▶ Discrete Uniform Probability Function

$$f(x) = 1/n$$

$\frac{1}{2}$ $\frac{1}{2}$

where: n = the ~~number of values the~~
random variable may assume

The values of the random variable are equally likely. ✓

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 $\frac{1}{100}$, $\frac{1}{100}$, ..., $\frac{1}{100}$

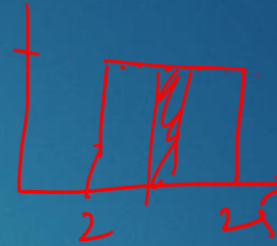
Probability Distributions - Discrete

- ▶ Binomial Probability Distribution

- ▶ 'n' identical independent trials, two possible outcomes in each trial
- ▶ The probability of both events does not change from trial to trial
- ▶ Trials are independent
- ▶ Example:
 - ▶ Showing an advertisement to 100 families, each either purchases a policy or not; Families are selected randomly (independent trials); Probability that at least 20 families buy the policy
 - ▶ Past experience – probability that any customer will buy from the store is 0.30. What is the probability that two out of the next three customers will buy



Probability Distributions - Continuous



▶ Continuous Uniform Probability Distribution

- ▶ Probability is proportional to the interval's length
- ▶ a, b : smallest, largest value the variable can assume
- ▶ E.g., Flight time of an airplane traveling from Delhi to Mumbai ✓
- ▶ E.g., Battery life of iPad Mini is uniformly distributed between 8.5 and 12 hours – what's the probability that the battery life will be more than 11 hours? ✓

$$f(x) = \begin{cases} 1/(b-a) & \text{for } a \leq x \leq b \\ 0 & \text{elsewhere} \end{cases}$$

▶ Area as a measure of probability

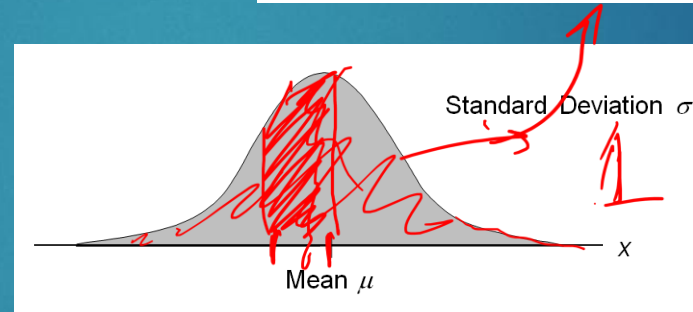
- ▶ Probability that x takes value between x_1 and x_2 = area under the curve $f(x)$

Probability Distributions – Continuous - Normal

- ▶ Normal Probability Density Function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

- ▶ Normal Distribution Curve:



- ▶ Total area under the curve = 1

Normal Probability Distribution

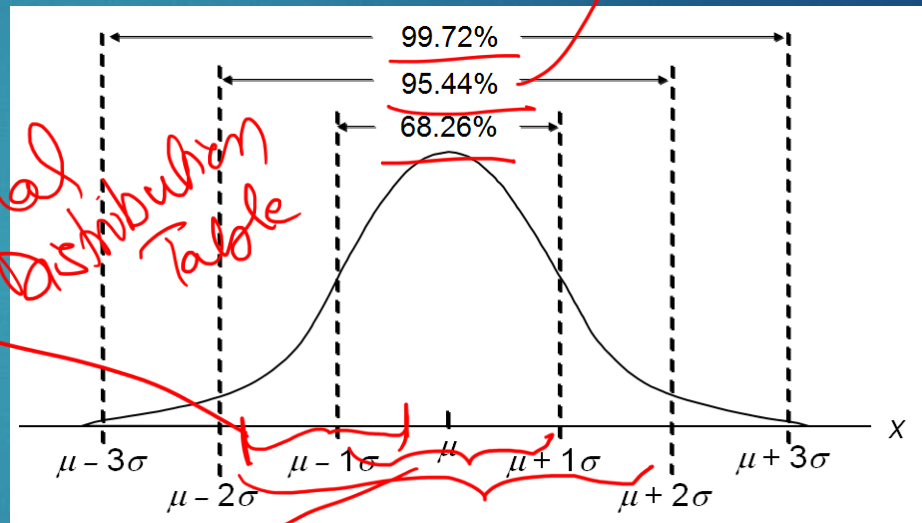
$X \rightarrow Z \sim N(0,1)$

$P(2.5 < X < 85) = 95.44$

$N(10, 2)$

Empirical Rule

- 68.26% of values of a normal random variable are within ± 1 standard deviation of its mean.
- 95.44% of values of a normal random variable are within ± 2 standard deviations of its mean.
- 99.72% of values of a normal random variable are within ± 3 standard deviations of its mean.



Normal Distribution Table

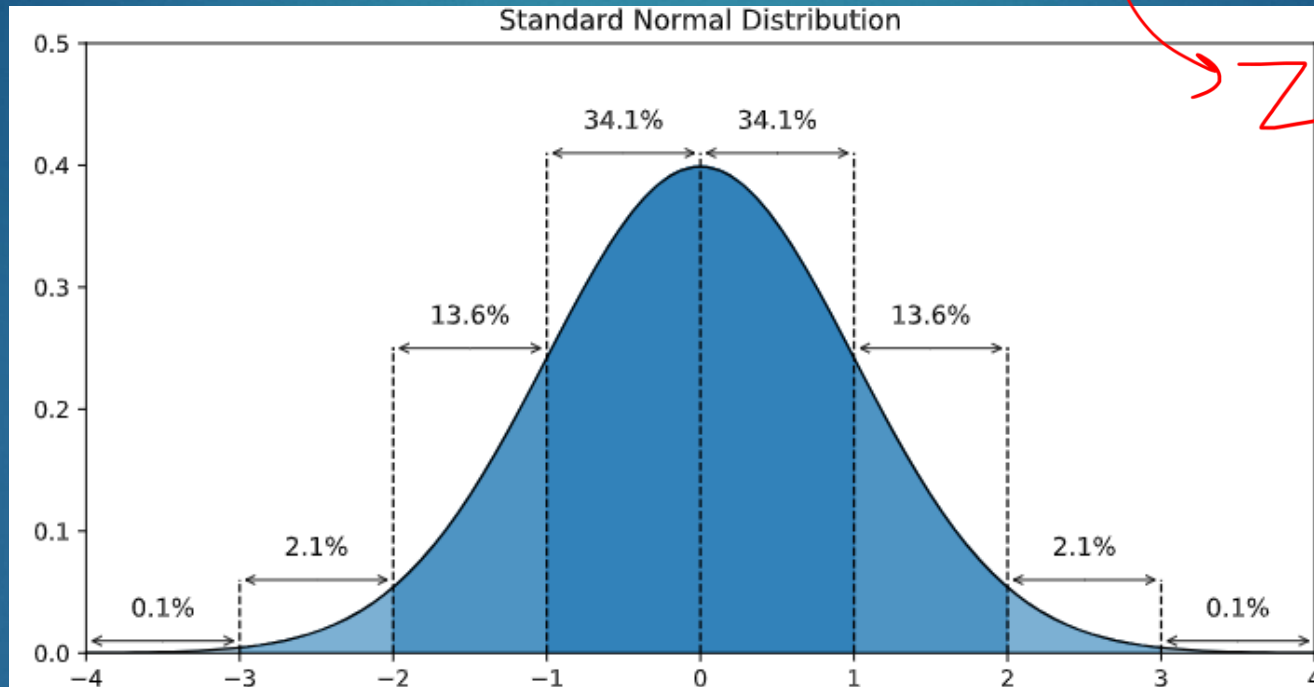
$P(2 < X < 5)$
 Normal Dist

$X \sim N(5.5, 1.5)$

$P(4 < X < 7) = 0.6826$

Standard Normal Distribution

20, 2.2



$X \sim N(\mu, \sigma^2)$

$Z \sim N(0, 1)$

$Z = \frac{X - \mu}{\sigma}$

$\left. \begin{array}{l} 5 \\ 7 \\ 10 \\ 12 \\ 14 \\ 15 \end{array} \right\} \begin{array}{l} (5-10)/2 = -2.5 \\ (7-10)/2 \\ (10-10)/2 \end{array}$

5, 2