Executive Certificate Program in Machine Learning & Artificial Intelligence

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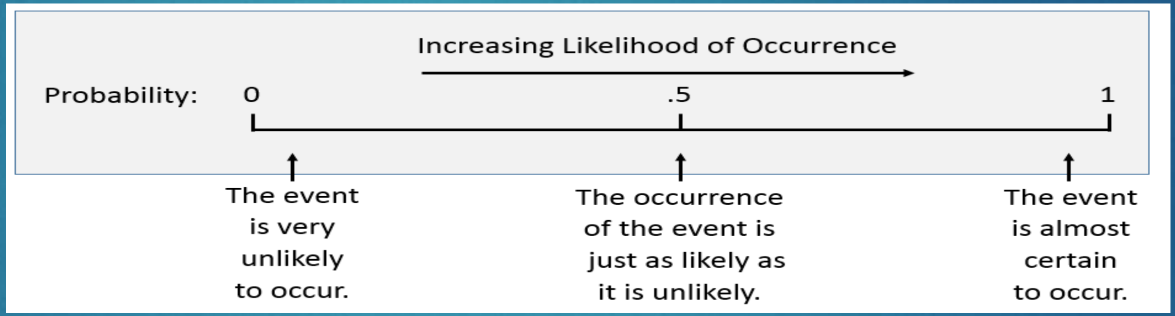
Module 1: Fundamentals of Statistical Learning

- Probability
  - Probability (Basic & Conditional), Bayes' Theorem
  - Probability distributions
- Hypothesis Testing
  - Point & Interval Estimation
  - Hypothesis Testing

Probability & Random Expriments

- What is probability?
  - Numerical measure of the likelihood that an event will occur.



- Statistical experiments versus Physical sciences experiments

Business Use Cases

- Managers often base their decisions on an analysis of uncertainties such as the following
  - What are the *chances* that the sales will decrease if we increase prices?
  - What is the *likelihood* a new assembly method will increase productivity?
  - What are the *odds* that a new investment will be profitable?

Assigning Probabilities

Basic Requirements for Assigning Probabilities

1. The probability assigned to each experimental outcome must be between 0 and 1, inclusively.

$$0 \leq P(E_i) \leq 1 \text{ for all } i$$

   where E_i is the i th experimental outcome and $P(E_i)$ is its probability

2. The sum of the probabilities for all experimental outcomes must equal 1.

$$P(E_1) + P(E_2) + \ldots + P(E_n) = 1$$

   where n is the number of experimental outcomes.

Assigning Probabilities

- Classical Method
  - Assigning probabilities based on the assumption of equally likely outcomes; E.g., Rolling a die
- Relative Frequency Method
  - Assigning probabilities based on experimental or historical data; E.g., No. of deliveries on a particular day in a week
- Subjective Method
  - Assigning probability based on judgment.

The best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimate.

Conditional Probability

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of *A* given B has already occurred denoted by $P(A \mid B)$.
- A conditional probability is computed as follows: $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
- Independent events: $$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$
- Examples? Business Use Cases?

Mutual Exclusive & Independent Events – Important to Note

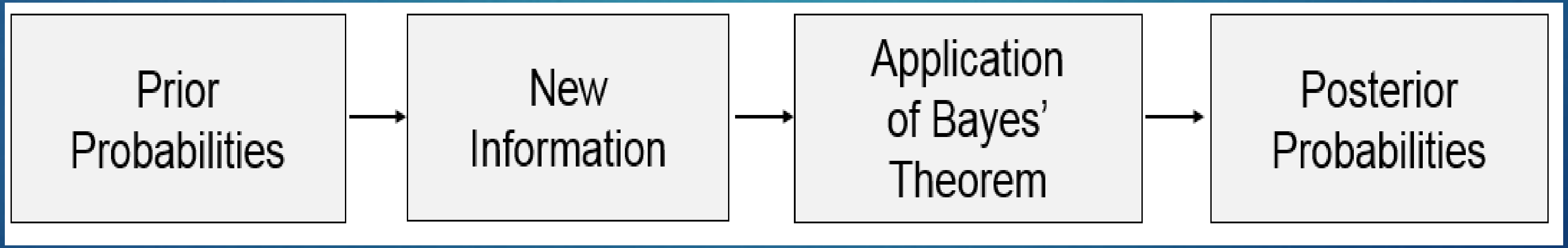
- Can two events with non-zero probability be both mutual exclusive and independent?
- Mutually exclusive events are always dependent events
- Non mutually exclusive events may or may not be independent

Questions

- Question 1
  - P(A) = 0.30; p(B) = 0.40; A and B are mutually exclusive;
  - p(A Intersection B) = ?
  - P(A | B)= ?

Bayes' Theorem

- Probability Analysis begins with 'prior probabilities'
- Then, additional information is obtained – from a sample, product test, report
- Using this information, 'posterior probabilities' are calculated – using Bayes' Theorem – A means to revise prior probabilities



Bayes' Theorem: Applications

- Use case
  - Impact of marketing approaches

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

- Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and collectively exhaustive

Bayes' Theorem: Applications

- Question 1
  - Two suppliers (A1 & A2)
  - 65% supplies from A1 & 35% from A2
  - P(G|A1) = 0.98 & p(G|A2) = 0.95
  - Question is: A bad part is detected, what is the probability that it came from A1 and what is the probability that it came from A2?
- Solution
  - If a part is chosen at random, p(A1) = 0.65; p(A2) = 0.35
  - P(A1|B) = p(A1)*p(B|A1)/(p(A1)*p(B|A1) + p(A2)*p(B|A2))

Random Variables & Probability Distribution

- Random variables and probability distributions are models for populations of data
- Random Variable – numerical description of the outcome of an experiment
- Discrete & Continuous Random variables
- Discrete Random variables
  - X = the number of people arriving in a bank branch in a day
- Continuous Random variables
  - X = time between customer arrivals in a minute, at a bank
  - X = percentage of project complete after six months

Probability Distributions - Discrete

- Probability Distribution
  - How are probabilities distributed over the values of the random variable
  - Prob Distribution of number obtained on one roll of a die

Number obtained	x	1	2	3	4	5	6
Probability of x	f(x)	1/6	1/6	1/6	1/6	1/6	1/6

  - Prob distribution of the no of automobiles sold during a day at a dealer

x	0	1	2	3	4	5
f(x)	0.18	0.39	0.24	0.14	0.04	0.01

Probability Distributions - Discrete

- Discrete Uniform Probability Distribution
- Discrete Uniform Probability Function $f(x) = 1/n$

  where: n = the number of values the random variable may assume

  The values of the random variable are equally likely.
- Binomial Probability Distribution
  - ‘n’ identical independent trials, two possible outcomes in each trial
  - The probability of both events does not change from trial to trial
  - Trials are independent
  - Example: Showing an advertisement to 100 families, each either purchases a policy or not; Families are selected randomly (independent trials)

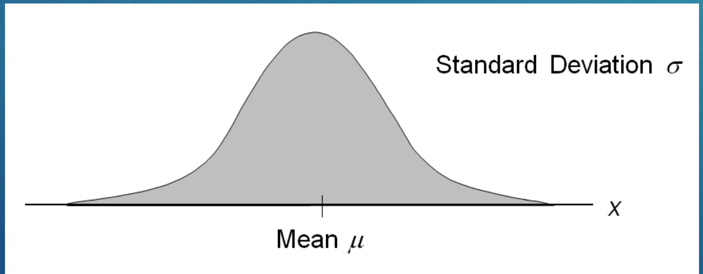
Probability Distributions - Continuous

- Continuous Uniform Probability Distribution
  - Probability is proportional to the interval's length
  - a, b: smallest, largest value the variable can assume
  - E.g., Flight time of an airplane traveling from Delhi to Mumbai
  - E.g., Battery life of iPad Mini is uniformly distributed between 8.5 and 12 hours – what's the probability that the battery life will be more than 11 hours?

$$f(x) = 1/(b-a) \quad \text{for } a \leq x \leq b$$
$$= 0 \quad \text{elsewhere}$$

- Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$$



Estimation: Point & Interval

- Sampling: Simple random sampling (with replacement, without replacement), stratified sampling, systematic, Convenience
- Point estimation – use sample data to estimate population parameter
  - sample mean, sample standard deviation etc.
- Interval estimation – includes margin of error
  - Point estimate +- Margin of error
  - Confidence interval
    - To have a higher confidence, width of confidence interval will be larger

Hypothesis Testing

- Performed to determine if a statement about the value of a population parameter should or should not be rejected
  - Null Hypothesis: H_0: tentative assumption about a population parameter
  - Alternate Hypothesis: H_1: Opposite of Null Hypothesis
- Developing Null & Alternate Hypothesis is very crucial
- Type 1 Error
  - Rejecting Null Hypothesis when it is true
  - The probability of Type 1 error – Level of significance
- Type II Error
  - Accepting Null Hypothesis when it is false

Type 1 & Type 11 Errors

	Population Condition	
Conclusion	H_0 True	H_0 False
Accept H_0	Correct Conclusion	Type II Error
Reject H_0	Type 1 Error	Correct Conclusion

The Importance of p-value: One-tailed Hypothesis Testing

- P-value: Support provided by the sample for the Null hypothesis
- If p-value is less than or equal to the level of significance (α Type 1 error), the value of the test statistic is in the rejection region
  - Reject H_0, if the p-value <= α
- Guidelines for interpreting p-values
  - Less than 0.01: Overwhelming evidence to conclude H_1 is true.
  - Between 0.01 & 0.05: Strong evidence to conclude H_1 is true.
  - Between .05 and .10: Weak evidence to conclude H_1 is true.
  - Greater than .10: Insufficient evidence to conclude H_1 is true

Example of Linear Regression (for p-value)

- We first check the distributions of the data collected (both dependent and independent variables)
- We check assumptions for setting a linear regression model
- On running linear regression, you get a model – coefficients for independent variable
- Then you need to check the significance of those coefficients – and hence we need the p-value
- Compare the p-value with the level of significance you have decided
  - A low p-value indicates that coefficient is likely not equal to zero (i.e., Reject H0)
  - A high p-value means we cannot conclude that the independent variable affects the dependent variable