Certificate Program in Machine Learning & Artificial Intelligence: Batch-3

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Session Outline

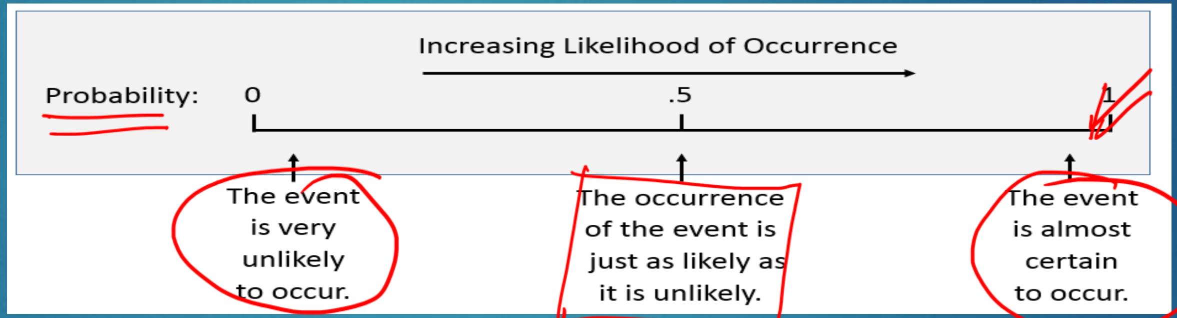
- Probability
  - Why Probability?
  - What is Probability? - Definition and Meaning
  - Properties of Probability
  - Conditional Probability
  - Bayes' Theorem

Business Use Cases of Probability

- Managers often base their decisions on an analysis of uncertainties such as the following
- What are the *chances* that the sales will decrease if we increase prices?
- What is the *likelihood* a new assembly method will increase productivity?
- What are the *odds* that a new investment will be profitable?

Probability & Random Experiments

- What is probability?
  - Numerical measure of the likelihood that an event will occur.



Probability: A Few Terms

{H, T} {1, 2, 3, 4, 5, 6}

- Experiment, Sample space, Single-step and multi-step experiment
- Combinations and Permutations
  - Number of experimental outcomes when the experiment involves selecting n objects from a set of N objects – Combinations

    {H,1} {H,2} ... {H,6}

    {T,1} ... {T,6}
  - Number of experimental outcomes when the experiment involves selecting n objects from a set of N objects and the order of selection is important – Permutations

Assigning Probabilities

$\rightarrow \{1, 2, \ldots, 1000000\}$ $\{H, T\}$ $\{(6,4), (\ ), \ldots\}$

$P(1)$ $P(2)$ $\hookrightarrow E_1 \ldots E_2 \ldots$

- The probability assigned to each experimental outcome must be between 0 and 1, inclusively.

$$\underline{0 \leq P(E_i) \leq 1 \text{ for all } i}$$

where E_i is the i th experimental outcome and $\underline{P(E_i) \text{ is its}}$ probability

- The sum of the probabilities for all experimental outcomes must equal 1.

$$\underline{P(E_1) + P(E_2) + \ldots + P(E_n) = 1}$$

where n is the number of experimental outcomes

Assigning Probabilities

- Classical Method
  - Assigning probabilities based on the assumption of equally likely outcomes; E.g., Rolling a die
- Relative Frequency Method
  - Assigning probabilities based on experimental or historical data; E.g., No. of deliveries on a particular day in a week
- Subjective Method
  - Assigning probability based on judgment.

The best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimate

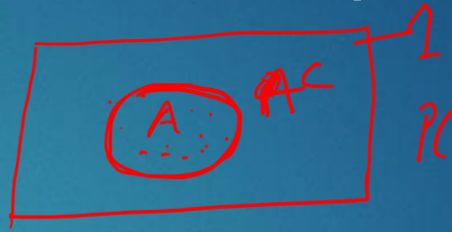
Events & Their Probabilities

$\{(H,1)\ (H,2)\ \dots\ (H,6)\ \ (T,1)\ \dots\ (T,6)\}$ → experimental outcome; sample space

- Event
  - A collection of sample points
  - Two upstream suppliers' case – together being able to deliver in less than 10 days
- Probability of an event
  - Sum of the probabilities of the sample points in the event

Some Probability Relationships

- Complement of an Event, A



$P(A) = 1 - P(A^c)$

  - Event consisting of all sample points that are not in A
  - Computing Probability using the complement

- Addition Law – Union of Two Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

- Mutually Exclusive events
  - If the events have no sample points in common
  - P(A intersection B) = 0

Conditional Probability

{(H,1) . . . (H,6)
(T,1) . . . . (T,6)}

A – custom
B → – demo

A&B

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of A given B has already occurred denoted by $P(A \mid B)$.

buy

demo

P(A|

date

relative freq

- A conditional probability is computed as follows:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Examples? Business Use Cases?

Independent events

A B demo

buy

$A \quad A^c$

$\boxed{A \quad A^c}$

- Can two events with non-zero probability be both mutually exclusive and independent? →
- Mutually exclusive events are always dependent events
- A → S1 Non-mutually exclusive events may or may not be
B → S2 independent

A 16 17 ... 35 B → Vizag-rain

independent

- Independent events:

$$P(A \mid B) = P(A) \text{ or } P(B \mid A) = P(B)$$

dependent

$$\overbrace{P(A \mid B) = P(A)}$$

buy email

Questions

$$\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

- Question 1
  - P(A) = 0.30; p(B) = 0.40; A and B are mutually independent;
  - p(A Intersection B) = ? $P(A \cap B) = P(A) \times P(B)$
  - P(A | B)= ? $P(A|B) = \frac{P(A) \times P(B)}{P(B)} = P(A) = 0.30$

$P(A) = 0.30$

- Question 2
  - P(A) = 0.30; p(B) = 0.40; A and B are mutually exclusive;
  - p(A Intersection B) = ? 0
  - P(A | B)= ? 0

demo → bought

A & B are dependent

$P(A) = 0.30$
$P(B) = 0.40$

$\frac{P(A \cap B)}{P(B)}$ $\frac{0.3}{0.4}$

Bayes' Theorem

- Probability Analysis begins with 'prior probabilities'
- Then, additional information is obtained – from a sample, product test, report
- Using this information, 'posterior probabilities' are calculated – using Bayes' Theorem – A means to revise prior probabilities

P(A) Prior Probabilities → 100 New Information → Application of Bayes' Theorem → Posterior Probabilities P(A/B)

P(A|B)

Bayes' Theorem: Applications

$E_1 \cup E_2 \cup E_3 \cup \ldots$ $\{A_1, A_2 \ldots\ldots A_{10}\}$

$S_1 \rightarrow E_1$ an item S_1

$S_2 \rightarrow E_2$ S_2

$S_3 \rightarrow E_3$ S_3

E_4

- Use case
  - Impact of marketing approaches

A_1 buy, A_2 Nbuy, A_3 postpone

$P(A_1|B)$

$A_1, A_2, A_3 \ldots A_{10}$

posterior

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

- Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and collectively exhaustive

Bayes' Theorem: Applications

$P(A_1|B) = ?$

$$P(A_1|G) = \frac{P(G|A_1) \times P(A_1)}{P(G/A_1) \times P(A_1) + P(G/A_2) \times P(A_2)}$$

- Question 1
  - Two suppliers (A1 & A2)
  - 65% supplies from A1 & 35% from A2
  - P(G|A1) = 0.98 & p(G|A2) = 0.95
  - Question is: A bad part is detected, what is the probability that it came from A1 and what is the probability that it came from A2?
- Solution
  - If a part is chosen at random, p(A1) = 0.65; p(A2) = 0.35
  - P(A1|B) = p(A1)*p(B|A1)/(p(A1)*p(B|A1) + p(A2)*p(B|A2))

$P(A_1) = 0.65$; $P(A_2) = 0.35$

[mutu exc
colle. exhaust

$\rightarrow P(B|A_2) = 0.05$

$P(B|A_1) = 0.02$

$$P(A_1|B) = ? = \frac{P(B|A_1) \cdot P(A_1)}{P(B/A_1) \cdot P(A_1) + P(B/A_2) \cdot P(A_2)}$$

$P(A_2|B) = ?$

$\hookrightarrow \dfrac{P(B/A_2) \cdot P(A_2)}{}$

A Supply Chain Problem

- **Scenario: Identifying Defective Products from Multiple Manufacturing Plants**
- A company, *TechWear*, produces wearable fitness trackers at three different manufacturing plants: **Plant A**, **Plant B**, and **Plant C**. Recently, there has been an uptick in defective products, and *TechWear* wants to identify the most likely source of these defects.
- The company has the following information:
  - 50% of the fitness trackers are produced at **Plant A**.
  - 30% of the fitness trackers are produced at **Plant B**.
  - 20% of the fitness trackers are produced at **Plant C**.
  - The probability of a defective fitness tracker from **Plant A** is 2%.
  - The probability of a defective fitness tracker from **Plant B** is 4%.
  - The probability of a defective fitness tracker from **Plant C** is 6%.
- A fitness tracker has been identified as defective, and the company wants to determine the probability that it was produced at **Plant C**.

A Customer Service Problem

- **Scenario: Customer Service Issue in a Telecom Company**
- A telecom company, *ConnectCo*, provides customer support through three different channels: **Phone Support**, **Live Chat**, and **Email Support**. Recently, the company has been receiving customer complaints about poor service experiences, and it wants to identify which channel is most likely responsible for the complaints.
- The company has the following data:
  - 50% of customer interactions happen via **Phone Support**. A1 $P(A_1) = 0.5$
  - 30% of customer interactions happen via **Live Chat**. A2 0.3
  - 20% of customer interactions happen via **Email Support**. A3 0.2
  - The probability of a complaint arising from **Phone Support** is 4%.
  - The probability of a complaint arising from **Live Chat** is 6%.
  - The probability of a complaint arising from **Email Support** is 10%. $P(A_3|C) = \frac{\quad}{\_ + \_ + \_}$
- A customer complaint has been received, and the company wants to determine the probability that this complaint came from **Email Support**.

$P(A_1|C) =$

$P(A_2|C)$