



# Hypothesis Testing

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Sessions 13-14



# Research Hypothesis

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- Older workers are more loyal to a company.
- Bigger companies spend a higher percentage of their annual budget on advertising than do smaller companies.
- Using a Six Sigma quality approach to manufacturing results in greater productivity.
- Scrap metal price is a good leading indicator of the Industrial Production Index.

# Statistical Hypothesis

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- A statistical hypothesis is a formal hypothesis structure set up with a null and an alternative hypothesis to scientifically test research hypotheses
- the **null hypothesis** states that the “null” condition exists; that is, there is nothing new happening, the old theory is still true, the old standard is correct, and the system is in control.
- The **alternative hypothesis**, on the other hand, states that the new theory is true, there are new standards, the system is out of control, and/or something is happening.
- A **substantive result** occurs when the outcome of a statistical study produces statistically significant results that are also important to the decision maker.



# Research Hypothesis

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## Statistical Hypothesis for the following Research Hypothesis:

- Older workers are more loyal to a company.
  - $H_0$ : Loyalty of Old Workers = Loyalty of Young Workers
  - $H_A$ : Loyalty of Old Workers  $>$  Loyalty of New Workers
- Bigger companies spend a higher percentage of their annual budget on advertising than do smaller companies.
- Using a Six Sigma quality approach to manufacturing results in greater productivity.
- Scrap metal price is a good leading indicator of the Industrial Production Index.

# One Tailed and Two Tailed Tests

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Suppose a company has held an 18% share of the market. However, because of an increased marketing effort, company officials believe the company's market share is now greater than 18%, and the officials would like to prove it.

- $H_0: p = 0.18$
- $H_A: p > 0.18$

Suppose flour packaged by a manufacturer is sold by weight, and a particular size of package is supposed to average 250 gms. Suppose the manufacturer wants to test to determine whether their packaging process is out of control as determined by the weight of the flour packages.

- $H_0: \mu = 250 \text{ gms}$
- $H_A: \mu \neq 250 \text{ gms}$



# Accept or Reject

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- Do we accept or reject the null hypothesis?
- Two types of errors possible:
  - Not accept a null hypothesis when it should be accepted
  - Accept a null hypothesis when it should be rejected

		State of Nature	
		Null True	Null False
Action	Fail to Reject Null	Correct Decision	Type II Error $(\beta)$
	Reject Null	Type I Error $(\alpha)$	Correct Decision



# Accept or Reject

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- A prisoner is to be punished
  - Null: Not Guilty
  - Type I Error: Prisoner not guilty but is punished
  - Type II Error: Prisoner guilty but not punished
- Legal Principle: Even if several guilty have to be set free, one innocent should not be punished
- Which error is more serious – Type I or Type II?
- Which error is being minimized?

		State of Nature	
		Null True	Null False
Action	Fail to Reject Null	Correct Decision	Type II Error ( $\beta$ )
	Reject Null	Type I Error ( $\alpha$ )	Correct Decision



# Accept or Reject

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Suppose flour packaged by a manufacturer is sold by weight, and a particular size of package is supposed to average 250 gms. Suppose the manufacturer wants to test to determine whether their packaging process is out of control as determined by the weight of the flour packages.

$$H_0: \mu = 250 \text{ gms}$$

$$H_A: \mu \neq 250 \text{ gms}$$

- What would be Type I and Type II errors?
- Type I error: Actual filling is 250 gms but reject null (The process is faulty → Replacement / Repair costs incurred even when not necessary)

- Suppose we write the hypothesis as

$$H_0: \mu \neq 250 \text{ gms}$$

$$H_A: \mu = 250 \text{ gms}$$

- What would be Type I and Type II errors?
- Type I error: Filling is faulty but reject null (The process is correct → No repairs → Defective product continues to go in the market)



# Accept or Reject

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- Type I errors can be easily controlled
- So, set-up hypothesis such that minimizing Type I errors will minimize loss due to error
- As the true value goes farther away from the hypothesized  $H_0$ , the probability of committing Type II error reduces
- If alpha is reduced, beta is increased, and vice versa.
- **Power**, which is equal to  $1 - \beta$ , is *the probability of a statistical test rejecting the null hypothesis when the null hypothesis is false.*

# $H_0$ and $H_1$

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- The  $H_0$  represents the current belief in a situation.
- The  $H_1$  represents a research claim.
- If you reject the  $H_0$ , you have statistical proof that the  $H_1$  is correct.
- If you do not reject the  $H_0$ , you have failed to prove the  $H_1$ . The failure to prove the  $H_1$ , however, does not mean that you have proven the  $H_0$ .
- The  $H_0$  always refers to a specified value of the population parameter (such as  $\mu$ ), not a sample statistic (such as  $\bar{X}$ ).
- The statement of the  $H_0$  always contains an equal sign regarding the specified value of the population parameter (e.g.,  $H_0 : \mu = 368$  grams).
- The statement of the  $H_1$  never contains an equal sign regarding the specified value of the population parameter (e.g.,  $H_1 : \mu \neq 368$  grams).



# Type I or Type II Error

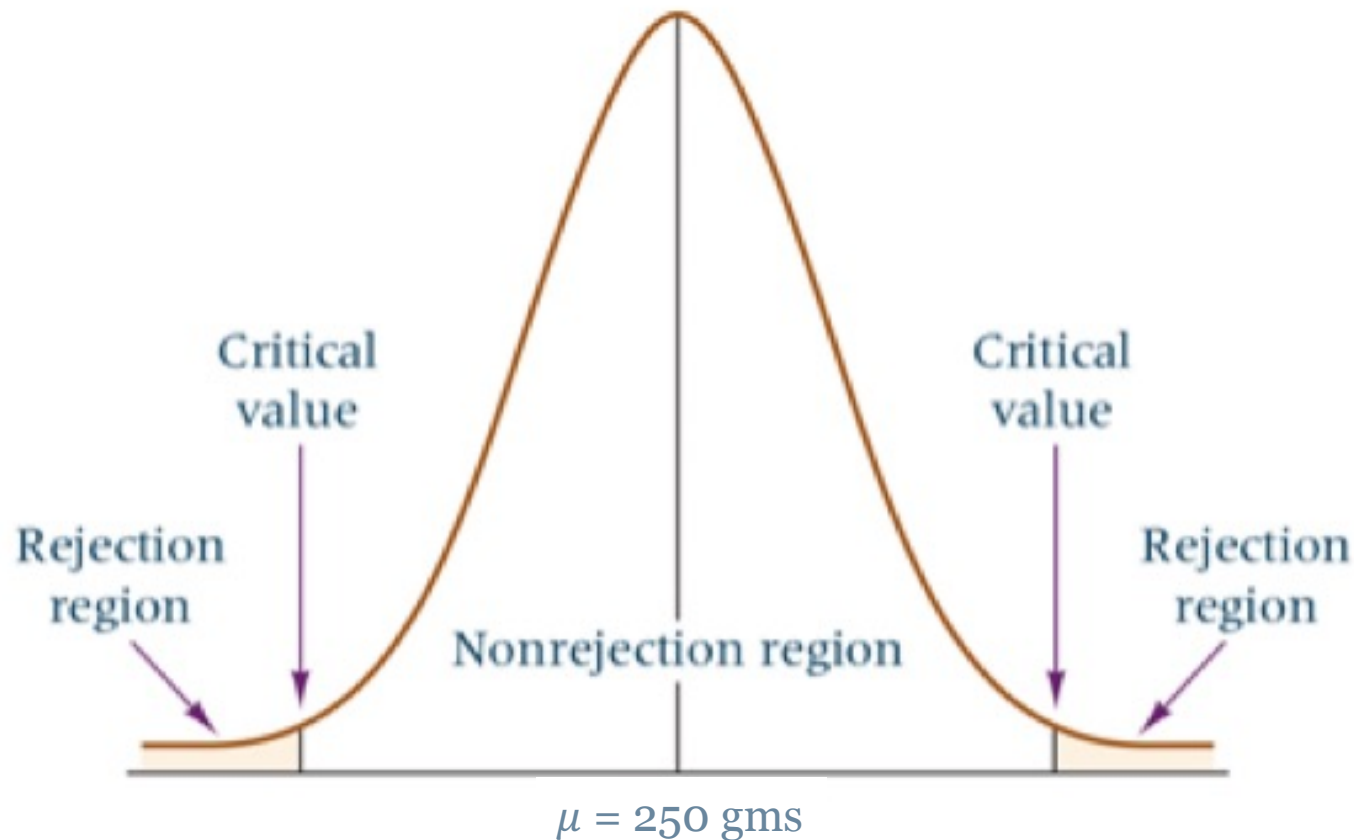
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- An analyst is testing to determine if 0.31 of all families own more than one car. His null hypothesis is that the population proportion is 0.31. He randomly samples 600 families and obtains a sample proportion of 0.33 that own more than one car. Based on these sample data, his decision is to fail to reject the null hypothesis. The actual population proportion is .31.
- Suppose it is generally known that the average price per square foot for a home in a particular locality is Rs. 15 per sq.ft. A business analyst believes that due to the economy, the average may now be less than that. To test her belief, she takes a random sample of 45 homes in this community, resulting in a sample mean of Rs. 13 per sq.ft. The analyst's decision based on this sample information is to fail to reject the null hypothesis. The actual average price per square foot is now Rs. 12.
- Suppose a utility analyst knows from past experience that the average water bill for a 2000-square-foot home in a large city is Rs. 250 per month. The utility analyst wants to test to determine if this figure is still true today. Her null hypothesis is that the population mean is Rs.250. To test this, she randomly samples 63 homes, resulting in a sample mean of Rs.290. From this, she decides to reject the null hypothesis. The actual average is Rs.270.



# Acceptance Region

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To minimize probability of Type I error, minimize the rejection region

→ Set  $\alpha$  to low value: 0.1, 0.05, 0.01



# Hypothesis Testing

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A survey of CA firms across the country found that the average net income for sole-proprietor CA firms is Rs. 98,500. Because this survey is over two decades old, an accounting analyst wants to test this figure by taking a random sample of 112 sole-proprietor firms to determine whether the net income figure has changed or not. It is assumed that the population standard deviation of net incomes for sole-proprietor CA firms is Rs. 14,530. The sample mean is Rs. 102,220.

**Step 1:** Write the null and alternate hypothesis

$$H_0: \mu = 98500; H_1: \mu \neq 98500$$

**Step 2:** Determine the test statistic

- Test of mean
- $\sigma$  known
- Sample  $> 30$
- Use normal distribution (z)

**Step 3:** Decide on  $\alpha$  – acceptable probability for Type I error – assume 0.05



# Hypothesis Testing

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**Step 4:** Is the test one-tailed or two-tailed? Accordingly determine the critical value – Value of standard test statistic for appropriate  $\alpha$  ( $\alpha$  for one tailed;  $\alpha/2$  for two-tailed)

**Step 5:** For given data calculate z

$$Z = (102220 - 98500)/14530 = 2.71$$

**Step 6:** Statistical decision

- If  $Z_{\text{calc}} > Z_{\text{critical}} \rightarrow$  Reject  $H_0$
- If  $Z_{\text{calc}} < Z_{\text{critical}} \rightarrow$  Fail to reject  $H_0$

**Step 7:** Conclusion

- Average net income of sole-proprietary CA firms is not equal to Rs. 98,500.
- We cannot conclude that the income is Rs. 102,220.



# Using the p-value method

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- Find the probability associated with calculated  $z$  statistic
- For  $z = 2.71 \rightarrow P(z \leq 2.71) = 0.0034$
- If this is less than  $\alpha/2 = 0.025 \rightarrow$  reject  $H_0$



# Critical Value Method

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Construct a 95% interval around  $\mu$

If  $\bar{X}$  is outside the interval, reject  $H_0$

For  $\mu = 98500$  and  $\sigma = 14530$

$$\bar{X} = 98500 \pm 14530/\sqrt{112} = 98500 \pm 2691$$

$$95809 \leq \bar{X} \leq 101191$$

Since  $\bar{X} = 102220$  is outside the interval – reject  $H_0$



# Other Cases

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- If  $\sigma$  is not known – Use t-statistic and t-distribution

$$t_{\alpha/2, (n-1)} = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

- Hypothesis testing for proportion

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

- Hypothesis testing of Variance

$$\chi_{n-1}^2 = \frac{(n-1)s^2}{\sigma^2}$$

- In all cases the p-value method and critical value method can be used with appropriate modifications



# Type II Error

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- A Type II error can be committed only when the analyst fails to reject the null hypothesis and the null hypothesis is false.
- If the null hypothesis is false, what is the true value for the population mean?
- For each of the possible values of the population mean, the analyst can compute the probability of committing a Type II error.
- Often, when the null hypothesis is false, the value of the alternative mean is unknown, so the analyst will compute the probability of committing Type II errors for several possible values.
- How can the probability of committing a Type II error be computed for a specific alternative value of the mean?



# Type II Errors

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Suppose an analyst is conducting a statistical test on the following hypotheses regarding 12 gms tube of a cosmetic cream.

$$H_0: \mu = 12 \text{ gms}; H_A: \mu < 12 \text{ gms}$$

Suppose that, in testing the preceding hypotheses, a sample of 60 tubes of cosmetic yields a sample mean of 11.985 gms. Assume that the population standard deviation is 0.10 gms.

From  $\alpha = .05$  and a one-tailed test, the table  $z_{0.05}$  value is  $-1.645$ . The observed  $z$  value from sample data is

$$z = \frac{11.985 - 12.00}{0.10/\sqrt{60}} = -1.16$$

From this observed value of  $z$ , the analyst determines not to reject the null hypothesis.

By not rejecting the null hypothesis, the analyst either makes a correct decision or commits a Type II error: *Failing to reject null when it is false*

What is the probability of committing a Type II error in this problem if the population mean actually is 11.99?



# Probability of Type II Errors

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- Calculate the critical value of  $\bar{X}$  for  $\mu = 12.00$

$$z_c = \frac{\bar{X}_c - 12.00}{0.10/\sqrt{60}}$$

$$\bar{X}_c = 11.979$$

- If  $\bar{X} > 11.979$  – accept null – possibility of Type II error
- If the true  $\mu$  is 11.99, what is the probability that we fail to reject the  $H_0$  of  $\mu = 12$ ?
- This is given by the area to the right of 11.979 in the distribution of  $\mu = 11.99$

$$z_c = \frac{11.979 - 11.99}{0.10/\sqrt{60}} = -0.85$$

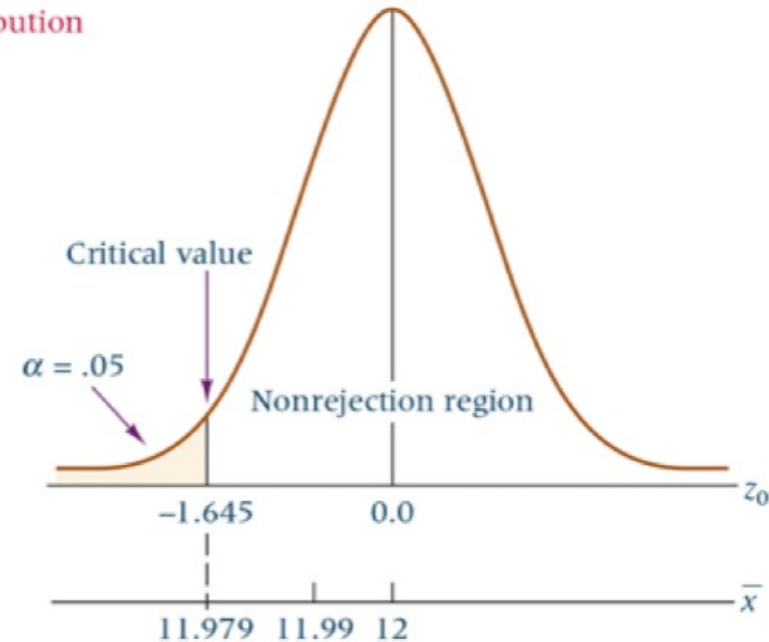
- The p-value associated with  $z = -0.85$  is 0.8023
- If alternative mean is 11.99, the probability of committing Type II error is 0.8023



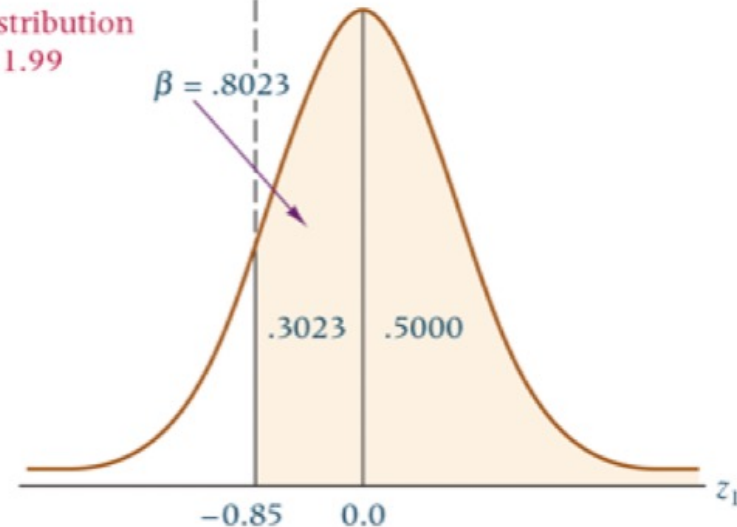
# Probability of Type II Error

- Probability of Type II error varies with value of true alternate mean
- If the true alternate mean is close to hypothesized mean -  $\beta$  is large
- As the true alternate mean moves away from hypothesized mean  $\beta$  becomes smaller
- If true alternate mean is close to hypothesized mean – probability of Type II error increases – but loss on account of Type II error is low

(a) Sampling distribution under  $H_0: \mu = 12$



(b) Sampling distribution under  $H_a: \mu = 11.99$



# Probability of Type II Error

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