

Lean Manufacturing Management & Analytics

Job Scheduling Decisions in Manufacturing (04 May, 2025)



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Lean manufacturing management is defined as to produce a **right product** and to deliver it at **right location** in **right time** in **right quantity** with **right price** having **right quality**.

It deals with high service level.

Some of prominent names in Manufacturing Management

| Date | Area of Contribution | Contributor |
|------|--|-------------------------------------|
| 1776 | Specialization of labour in manufacturing | Adam Smith |
| 1799 | Interchangeable parts, cost accounting | Eli Whitney |
| 1832 | Classification of skill based workforce | Charles Babbage |
| 1900 | Ergonomics (Work study, Time and motion study) | G.B. Gilberth |
| 1901 | Scheduling and sequencing | Henry L. Gantt |
| 1915 | Inventory model (EOQ-Economic Order Quantity) | F.W. Harris |
| 1927 | Human resource management | Elton Mayo |
| 1931 | Statistical control charts | W.A. Shewart |
| 1935 | Statistical sampling plan, OC Curve | H.F. Dodge |
| 1940 | Operations Research | P.M. Blacker |
| 1947 | Linear Programming | G.B. Dantzig |
| 1950 | Non-linear and stochastic programming, Data Envelopment Analysis | A. Charnes, W. W. Cooper |
| 1970 | Integrating operations into strategy, Scheduling and control, Materials Requirement Planning (MRP) | W. Skinner J. Orlicky and G. Wright |
| 1980 | Quality principle and productivity | W.E. Deming, J. Juran |

A good sequence provides *less waiting time, decreased delivery delays, and better due date performance.*

There are costs associated with waiting and delays. There are other costs associated, such as, set up cost and in-process inventory costs.

The objective function can be to minimize total production time.

Scheduling

A production schedule is the time table that specifies the times at which the jobs in a production department will be processed on various machines.

The schedule gives the starting and ending times of each job on the machines on which the job has to be processed.

Example

Suppose there are three jobs (A, B, and C) in a shop floor that are to be processed on four types of machines (M_1 , M_2 , M_3 , and M_4).

These three jobs consist of 4, 3, and 4 operations respectively; and there are four machines-one machine of each operations.

The operations for job A are A_1 , A_2 , A_3 , and A_4 . The operations of job B are B_1 , B_2 , and B_3 . Similarly, the four operations of job C are C_1 , C_2 , C_3 , and C_4 .

Each job is characterized by its routing that specifies the information about the number of operations to be performed, the sequence of these operations, and the machines required for processing these operations.

The times required for processing these operations are also required for developing a production schedule.

The table gives the machine required for each operation of each job. For example, the first operation of job A, A₁, is processed on machine M₁; second operation, A₂, is processed on machine M₃ and so on.

The operations of all jobs have to follow their processing sequences. For example operation A₃ of job A can not be processed before operation A₂.

The processing time for each operation is also given in the table.

| Job | Operation Number | Machine Number | Processing Time (Days) |
|-----|------------------|----------------|------------------------|
| A | A1 | M1 | 5 |
| | A2 | M3 | 3 |
| | A3 | M4 | 7 |
| | A4 | M2 | 4 |
| B | B1 | M2 | 2 |
| | B2 | M3 | 6 |
| | B3 | M4 | 8 |
| C | C1 | M1 | 4 |
| | C2 | M2 | 6 |
| | C3 | M3 | 8 |
| | C4 | M4 | 2 |

What should be an objective?

The objective is to schedule these jobs so as to minimize the time to complete all jobs.

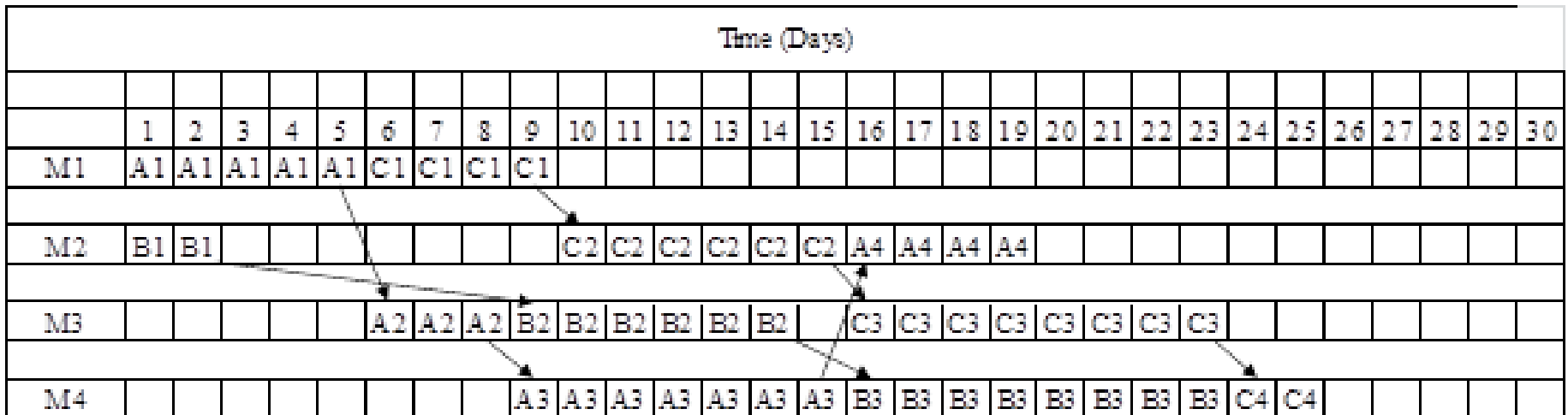
This time is called *make-span* or the *schedule time*.

Make-span is referred in manufacturing management.

Let's take a random solution

One of the schedules for this example is presented below in the form of a Gantt Chart.

The Gantt chart, for each machine, shows the start and finish times of all operations scheduled on that machine.



Can we have better schedules?

Several alternative schedules can be generated for this example. The schedules differ in the order in which the jobs are processed on the four machines.

Three of these schedules are:

- The first schedule orders jobs as: A first, then B and then C (A-B-C).
- The second schedule orders jobs as: B first, then A, and then C (B-A-C).
- The third schedule orders jobs as: C first, then A, and then B (C-A-B).

The Gantt charts for these schedules are shown in next slide.

The values of make-span for these three schedules are 25, 27 and 30 days respectively. Schedule A-B-C is the best of these three schedules.

Is sequence A-B-C the best solution (**global optimal**)? Can we find a better sequence than this? So, the scheduling techniques attempt to answer it.

It should be noted that there are different effectiveness measures of a schedule in different situations. Minimizing make-span is the widely considered as it **optimize production time (product lead time i.e. make span)**.

Assumptions in operations scheduling

Once a job is started on a machine, its processing can not be interrupted, that is, **preemption is not allowed**.

The machines are continuously available and will not break down during the operations (*This assumption is rather unrealistic but we make this assumption to avoid complexity in discussing scheduling concepts*).

A machine is not kept idle if a job is available for operation.

Also, each machine can process only one job at a time.

Classification of Scheduling Problems

The scheduling problems can be classified based on the following criteria:

- Sequence of machines
- Number of machines
- Processing times
- Job arrival time
- Objective function

- Sequence of Machines

The sequencing problems, based on the sequence of machines, are classified as:

➤ Flow Shops

➤ Job Shops

Flow Shop

In a flow-shop , processing of all jobs require machines in the same order.

The following table gives an example of a flow-shop in which three jobs, A, B, and C are processed on four machines, M₁, M₂, M₃, and M₄.

The sequences of machines to process these jobs are same (M₁-M₃-M₄-M₂).

| Example of a Flow Shop | | | | | | | | |
|------------------------|---------------|---------------|---------------|---------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Job | Operation # 1 | Operation # 2 | Operation # 3 | Operation # 4 | Machine for Operation # 1 | Machine for Operation # 2 | Machine for Operation # 3 | Machine for Operation # 4 |
| A | A1 | A2 | A3 | A4 | M1 | M3 | M4 | M2 |
| B | B1 | B2 | B3 | B4 | M1 | M3 | M4 | M2 |
| C | C1 | C2 | C3 | C4 | M1 | M3 | M4 | M2 |



Job Shop

In a job shop the sequence of machines will be mixed, that is, the jobs may require machines in different sequences.

| Example of a Job Shop | | | | | | | | |
|-----------------------|---------------|---------------|---------------|---------------|---------------------------|---------------------------|---------------------------|---------------------------|
| Job | Operation # 1 | Operation # 2 | Operation # 3 | Operation # 4 | Machine for Operation # 1 | Machine for Operation # 2 | Machine for Operation # 3 | Machine for Operation # 4 |
| A | A1 | A2 | A3 | A4 | M1 | M3 | M4 | M2 |
| B | B1 | B2 | B3 | | M2 | M3 | M4 | |
| C | C1 | C2 | C3 | C4 | M1 | M2 | M3 | M4 |

- Number of Machines

Based on the number of machines, the scheduling problems are classified as:

1. Single machine problems  Referred as Sequencing problem
2. Two-machine problems
3. Multiple (≥ 3 machine) problems  managed by mathematical modelling (or prescriptive analytics)

- Processing Times

➤ *Deterministic*: If processing times of all jobs are known and constant the scheduling problem is called a deterministic problem.

➤ *Probabilistic*: The scheduling problem is called probabilistic (or stochastic) if the processing times are not fixed; i.e., the processing times must be represented by a probability distribution.



Subject of
research

- Job Arrival Times

Based on this criterion, scheduling problems are classified as static and dynamic problems.

- *Static*: In the case of static problems the number of jobs is fixed and will not change until the current set of jobs has been processed.
- *Dynamic*: In the case of dynamic problems, new jobs enter the system and become part of the current set of unprocessed jobs. The arrival rate of jobs is given in the case of dynamic problems.



Subject of
research

• Objective Functions

Operations scheduling problems has several objectives.

- Minimize make-span (*mostly considered*)
- Minimize average flow time (or job completion time)
- Average number of jobs in the system
- Minimize average tardiness
- Minimize maximum tardiness
- Minimize number of tardy jobs

1. Single Machine Scheduling (or Sequencing problem)

There is one machine on which several jobs have to be processed.

The order in which these jobs will be processed needs to be specified. This schedule will not be changed until all jobs have been processed. This is the “static” version of the problem.

Few sequencing rules

There are several rules that can be used to find the order of processing. We will study the following three rules.

- First Come First Served (FCFS)
- Shortest Processing Time (SPT) or Shortest Operation Time.
- Earliest (shortest) Due Date (EDD)

Objective Functions

There are several objective functions that can be minimized in a single machine problem. We will study the following objective functions.

- ❖ Minimize average completion (flow) time.
- ❖ Minimize average number of jobs in the system.
- ❖ Minimize average tardiness.
- ❖ Minimize maximum tardiness
- ❖ Minimize number of tardy jobs.

Note: A job is tardy if it is not completed by its due date.

Example

Let's consider an Example.

There are five jobs A, B, C, D, and E. A is the first job that arrived in the production department. B,C, D, and E followed A in this order.

The processing times and due dates of all jobs are also given.

The order in which these jobs have to be processed needs to be specified.

| | Days | |
|-----|------|----------|
| Job | Time | Due Date |
| A | 17 | 45 |
| B | 12 | 35 |
| C | 22 | 27 |
| D | 18 | 54 |
| E | 26 | 47 |

Let's apply FCFS rule

A is the first job to be processed and will be completed at time 17. Its due date is 45. So job A is not late (tardy); tardiness is zero.

Job B starts after job A, and is completed at time 29 (17+12). This is also not tardy.

In this way the completion time and tardiness of all jobs are completed.

| First Come First Served | | | | |
|-------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| A | 17 | 45 | 17 | 0 |
| B | 12 | 35 | 29 | 0 |
| C | 22 | 27 | 51 | 24 |
| D | 18 | 54 | 69 | 15 |
| E | 26 | 47 | 95 | 48 |

$$\text{Tardiness} = \text{Completion Time} - \text{Due Date}$$

Make it zero if you get a negative value.

FCFS: Calculation of Objective Functions

Average Completion Time: Add completion times of all jobs and divide by the number of jobs ($261/5$). It is 52.5.

Average Number of Jobs in the System: This is obtained by dividing the total of completion times of all jobs by the completion time of the last job ($261/95$). It is 2.75.

Average Tardiness : This is obtained by adding the tardiness of all jobs and dividing it by the number of jobs ($87/5$). It is 17.4.

Maximum Tardiness: This is the maximum of all tardiness values, It is 48.

Number of Tardy Jobs: Count the number of jobs that are tardy. Three jobs (C, D, and E) are tardy for this problem. It is 3.

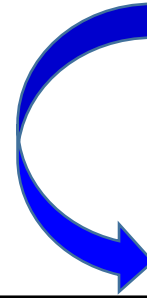
| First Come First Served | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| A | 17 | 45 | 17 | 0 |
| B | 12 | 35 | 29 | 0 |
| C | 22 | 27 | 51 | 24 |
| D | 18 | 54 | 69 | 15 |
| E | 26 | 47 | 95 | 48 |
| | | Total | 261 | 87 |
| Average Completion Time | | | | 52.2 |
| Average Number of Jobs in System | | | | 2.75 |
| Average Tardiness | | | | 17.4 |
| Maximum Tardiness | | | | 48 |
| Number of Tardy Jobs | | | | 3 |

Let's apply SPT rule

The jobs are processed in the increasing order of their processing times.

The job with the minimum processing time (B) is processed first. B is followed by A, D, C, and E.

The calculations of the objective functions follow the same procedure as for the FCFS rule.



| Job | Time | Due Date | Completion Time | Tardiness |
|-----|------|----------|-----------------|-----------|
| A | 17 | 45 | 17 | 0 |
| B | 12 | 35 | 29 | 0 |
| C | 22 | 27 | 51 | 24 |
| D | 18 | 54 | 69 | 15 |
| E | 26 | 47 | 95 | 48 |

| Shortest Processing Time | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| B | 12 | 35 | 12 | 0 |
| A | 17 | 45 | 29 | 0 |
| D | 18 | 54 | 47 | 0 |
| C | 22 | 27 | 69 | 42 |
| E | 26 | 47 | 95 | 48 |
| | | Total | 252 | 90 |
| Average Completion Time | | | | 50.4 |
| Average Number of Jobs in System | | | | 2.65 |
| Average Tardiness | | | | 18 |
| Maximum Tardiness | | | | 48 |
| Number of Tardy Jobs | | | | 2 |

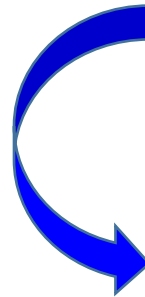
Let's apply EDD

The jobs are processed in the increasing order of their due dates.

The job with the minimum due date (C) is processed first; and is followed by B, A, E, and D.

The calculations of the objective functions follow the same procedure as for the FCFS rule

| Job | Time | Due Date | Completion Time | Tardiness |
|-----|------|----------|-----------------|-----------|
| A | 17 | 45 | 17 | 0 |
| B | 12 | 35 | 29 | 0 |
| C | 22 | 27 | 51 | 24 |
| D | 18 | 54 | 69 | 15 |
| E | 26 | 47 | 95 | 48 |



| Earliest Due Date | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| C | 22 | 27 | 22 | 0 |
| B | 12 | 35 | 34 | 0 |
| A | 17 | 45 | 51 | 6 |
| E | 26 | 47 | 77 | 30 |
| D | 18 | 54 | 95 | 41 |
| | | Total | 279 | 77 |
| Average Completion Time | | | | 55.8 |
| Average Number of Jobs in System | | | | 2.94 |
| Average Tardiness | | | | 15.4 |
| Maximum Tardiness | | | | 41 |
| Number of Tardy Jobs | | | | 3 |

| First Come First Served | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| A | 17 | 45 | 17 | 0 |
| B | 12 | 35 | 29 | 0 |
| C | 22 | 27 | 51 | 24 |
| D | 18 | 54 | 69 | 15 |
| E | 26 | 47 | 95 | 48 |
| | | Total | 261 | 87 |
| Average Completion Time | | | | 52.2 |
| Average Number of Jobs in System | | | | 2.75 |
| Average Tardiness | | | | 17.4 |
| Maximum Tardiness | | | | 48 |
| Number of Tardy Jobs | | | | 3 |

| Earliest Due Date | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| C | 22 | 27 | 22 | 0 |
| B | 12 | 35 | 34 | 0 |
| A | 17 | 45 | 51 | 6 |
| E | 26 | 47 | 77 | 30 |
| D | 18 | 54 | 95 | 41 |
| | | Total | 279 | 77 |
| Average Completion Time | | | | 55.8 |
| Average Number of Jobs in System | | | | 2.94 |
| Average Tardiness | | | | 15.4 |
| Maximum Tardiness | | | | 41 |
| Number of Tardy Jobs | | | | 3 |

| Shortest Processing Time | | | | |
|----------------------------------|------|----------|-----------------|-----------|
| Job | Time | Due Date | Completion Time | Tardiness |
| B | 12 | 35 | 12 | 0 |
| A | 17 | 45 | 29 | 0 |
| D | 18 | 54 | 47 | 0 |
| C | 22 | 27 | 69 | 42 |
| E | 26 | 47 | 95 | 48 |
| | | Total | 252 | 90 |
| Average Completion Time | | | | 50.4 |
| Average Number of Jobs in System | | | | 2.65 |
| Average Tardiness | | | | 18 |
| Maximum Tardiness | | | | 48 |
| Number of Tardy Jobs | | | | 2 |

Which rule is the best?

Case Study

Thermochemical processes at Modi Engineering Workshop at Anand, Gujarat carries out job work for heat treatment and electro plating work. The heat treatment is being done on electric arc furnace. Recently, the company has installed a salt bath type treatment unit. In salt bath heat treatment process, in a vessel certain salts and chemicals are heated till they melt and get converted into liquid form. The components to be heat treated are kept in the tub and are heated by the molten salt and chemicals. The Modi Engineering Workshop has received materials for five orders for heat treatment on December 1. The job time required and the delivery date is as given below:

Earlier the company was following sequencing rule of FCFS (First Come First Serve). But now instead of following the old sequencing rule, Modi Engineering Workshop's manager wanted to evaluate performance of various sequencing rules. Following four sequencing rules were identified:

1. FCFS (First Come First Serve)
2. SPT (Shortest Processing Time)
3. Critical Ratio
4. Earliest Due Date

| JOB CODE | JOB | DURATION | DUE DATE |
|----------|-------------------|-----------|-------------|
| A | Lever | 5 (Days) | December 10 |
| B | Brake shaft | 10 (Days) | December 15 |
| C | High Tensile Stud | 2 (Days) | December 5 |
| D | Tie Rod | 8 (Days) | December 12 |
| E | Sleeve | 6 (Days) | December 8 |

Question:

Evaluate the above sequencing rules and suggest the best sequence for the jobs for different criteria.

2. Two Machines Problem

Consider a problem with five jobs (A, B, C, D, and E); and two machines as M₁ and M₂.

All five jobs consist of two operations each. The first operation of each job is processed on machine M₁; and the second operation is processed on machine M₂.

Data for a 5-Job 2-Machine Flow Shop Problem

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) |
|-----|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|
| A | A1 | A2 | M1 | M2 | 8 | 3 |
| B | B1 | B2 | M1 | M2 | 5 | 7 |
| C | C1 | C2 | M1 | M2 | 6 | 9 |
| D | D1 | D2 | M1 | M2 | 7 | 1 |
| E | E1 | E2 | M1 | M2 | 4 | 6 |

The scheduling objective is to find an optimal sequence that gives minimum make span for these five jobs on two machines.

Once we know an **optimal sequence**, the time to complete all jobs can be determined i.e. make span.

For example, A-B-C-D-E is a sequence order that tells us that A is the first job to be processed; B is the second job and so on. E is the last job to be processed. Is it optimal?

Another sequence could be B-C-A-E-D. Is it optimal?

What is the optimal sequence?

Number of Sequences

For this 5-job problem there are 120 (5!) different sequences. Similarly, for a six-job problem, the number of sequences will be 720 (6!).

In general, for a “ n ” job problem there are $n!$ (n -factorial) sequences.

Our goal is to find the best sequence that minimizes make-span.

Gantt Chart Sequence A-B-C-D-E

The Gantt chart for the sequence A-B-C-D-E is given below.

The value of **make-span** (time to complete all jobs) is 36 days.

Our objective is to identify the sequence that minimizes the value of make-span.

| Gantt Chart for Sequence A-B-C-D-E | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Time (Days) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| M1 | A1 | A1 | A1 | A1 | A1 | A1 | A1 | A1 | B1 | B1 | B1 | B1 | B1 | C1 | C1 | C1 | C1 | C1 | C1 | D1 | D1 | D1 | D1 | D1 | D1 | D1 | E1 | E1 | E1 | E1 | | | | | | |
| M2 | | | | | | | | | A2 | A2 | A2 | | | B2 | B2 | B2 | B2 | B2 | B2 | B2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | D2 | E2 | E2 | E2 | E2 | E2 | E2 |

Johnson's Rule to Minimize (optimal) Make-span

We use following four steps to find the **optimal sequence**.

Step 1: Find the minimum processing time on both machines.

Step 2: Identify the job & machine for minimum time identified at Step 1.

Step 3: Scheduling Rule

(a) If the machine identified in **Step 2** is the first machine i.e. M_1 then the job will be scheduled in the **first available position**.

(b) If the machine identified in **Step 2** is the second machine i.e. M_2 then the job will be scheduled in the **last available position**.

Step 4: Remove the job whose position is fixed in **Step 3**; and go to **Step 1**.

Continue this process until all jobs have been scheduled.

Iteration 1

Step 1: The minimum time is 1.

Step 2: The job is D and the machine is M₂.

Step 3: The machine identified at Step 2 is machine M₂, job D will be assigned to **last available position** i.e. position 5 as given below.

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) |
|-----|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|
| A | A1 | A2 | M1 | M2 | 8 | 3 |
| B | B1 | B2 | M1 | M2 | 5 | 7 |
| C | C1 | C2 | M1 | M2 | 6 | 9 |
| D | D1 | D2 | M1 | M2 | 7 | 1 |
| E | E1 | E2 | M1 | M2 | 4 | 6 |

| | | | | |
|-------------------|-------------------|-------------------|-------------------|----------|
| Position 1 | Position 2 | Position 3 | Position 4 | D |
|-------------------|-------------------|-------------------|-------------------|----------|

Step 4: Delete job D .

Iteration 2

Step 1: The next minimum time is 3.

Step 2: The job is A and the machine is M2.

Step 3: The job A will be assigned to **last available position** i.e. position 4 as given below.

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | |
|--------------|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|-----------|
| A | A1 | A2 | M1 | M2 | 8 | 3 | |
| B | B1 | B2 | M1 | M2 | 5 | 7 | |
| C | C1 | C2 | M1 | M2 | 6 | 9 | |
| D | D1 | D2 | M1 | M2 | 7 | 4 | Scheduled |
| E | E1 | E2 | M1 | M2 | 4 | 6 | |

| | | | | |
|-------------------|-------------------|-------------------|----------|----------|
| Position 1 | Position 2 | Position 3 | A | D |
|-------------------|-------------------|-------------------|----------|----------|

Step 4: Delete job A.

Iteration 3

Step 1: The minimum time is 4.

Step 2: The job is E and the machine is M₁.

Step 3: The job E will be assigned to the **first available schedule position** i.e. position 1 as given below.

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | |
|--------------|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|-----------|
| A | A1 | A2 | M1 | M2 | 8 | 3 | Scheduled |
| B | B1 | B2 | M1 | M2 | 5 | 7 | |
| C | C1 | C2 | M1 | M2 | 6 | 9 | |
| D | D1 | D2 | M1 | M2 | 7 | 1 | Scheduled |
| E | E1 | E2 | M1 | M2 | 4 | 6 | |

| | | | | |
|----------|-------------------|-------------------|----------|----------|
| E | Position 2 | Position 3 | A | D |
|----------|-------------------|-------------------|----------|----------|

Step 4: Delete job E.

Iteration 4

Step 1: The minimum time is 5.

Step 2: The job is B and the machine is M₁.

Step 3: The job B will be assigned to the **first available position** i.e. position 2 as given below.

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | |
|-----|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|-----------|
| A | A1 | A2 | M1 | M2 | 8 | 3 | Scheduled |
| B | B1 | B2 | M1 | M2 | 5 | 7 | |
| C | C1 | C2 | M1 | M2 | 6 | 9 | |
| D | D1 | D2 | M1 | M2 | 7 | 1 | Scheduled |
| E | E1 | E2 | M1 | M2 | 4 | 6 | Scheduled |

| | | | | |
|---|---|------------|---|---|
| E | B | Position 3 | A | D |
|---|---|------------|---|---|

Step 4: Delete job B.

Iteration 5

The only unscheduled job at this stage is C and; it will be assigned to the remaining unassigned position 3.

The final sequence is given below.

The value of make-span for this sequence will be determined by drawing the Gantt chart.

| Job | Operation # 1 | Operation # 2 | Machine for Operation # 1 | Machine for Operation # 2 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | |
|-----|---------------|---------------|---------------------------|---------------------------|-------------------------------|-------------------------------|-----------|
| A | A1 | A2 | M1 | M2 | 8 | 3 | Scheduled |
| B | B1 | B2 | M1 | M2 | 5 | 7 | Scheduled |
| C | C1 | C2 | M1 | M2 | 6 | 9 | |
| D | D1 | D2 | M1 | M2 | 7 | 1 | Scheduled |
| E | E1 | E2 | M1 | M2 | 4 | 6 | Scheduled |

| | | | | |
|---|---|---|---|---|
| E | B | C | A | D |
|---|---|---|---|---|

Sequence E-B-C-A-D

The Gantt chart for the sequence **E-B-C-A-D** is given below. The value of make-span is 31 days.

The best (**global optimal or minimum**) value of make-span for this problem is therefore, 31 days.

| Gantt Chart for Sequence E-B-C-A-D | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Time (Days) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| M1 | E1 | E1 | E1 | E1 | B1 | B1 | B1 | B1 | B1 | C1 | C1 | C1 | C1 | C1 | C1 | A1 | A1 | A1 | A1 | A1 | A1 | A1 | A1 | D1 | D1 | D1 | D1 | D1 | D1 | | | | | | | |
| M2 | | | | | E2 | E2 | E2 | E2 | E2 | E2 | B2 | B2 | B2 | B2 | B2 | B2 | B2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | C2 | A2 | A2 | A2 | | D2 | | | | | |

Case problem:

Consider the following jobs and their processing times at two machines. Machines are connected in series. Each job has to be processed as per the sequence of operations of individual jobs. Identify the best sequence in which the jobs have to be processed so that all the jobs can be finished in minimum time.

| Jobs | Hours to complete Job | |
|------|-----------------------|-------|
| | MC/ 1 | M/C 2 |
| A | 13 | 9 |
| B | 11 | 5 |
| C | 20 | 13 |
| D | 17 | 14 |
| E | 21 | 15 |

Find out the optimal schedule for the given problem and construct Gantt Chart to know the make span.

3. Three Machines Problem

Consider a problem with five jobs (A, B, C, D, and E); and three machines as M₁, M₂, & M₃.

All five jobs consist of three operations each. The first operation of each job is processed on machine M₁; the second operation is processed on machine M₂, and the third operation is processed on machine M₃.

Data for a 5-Job 2-Machine Flow Shop Problem

| Job | Operation # 1 | Operation # 2 | Operation # 3 | Machine for Operation # 1 | Machine for Operation # 2 | Machine for Operation # 3 | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | Time for Operation # 3 (Days) |
|-----|---------------|---------------|---------------|---------------------------|---------------------------|---------------------------|-------------------------------|-------------------------------|-------------------------------|
| A | A1 | A2 | A3 | M1 | M2 | M3 | 8 | 5 | 4 |
| B | B1 | B2 | B3 | M1 | M2 | M3 | 10 | 6 | 9 |
| C | C1 | C2 | C3 | M1 | M2 | M3 | 6 | 2 | 8 |
| D | D1 | D2 | D3 | M1 | M2 | M3 | 7 | 3 | 6 |
| E | E1 | E2 | E3 | M1 | M2 | M3 | 11 | 4 | 5 |

If any of the following condition holds true

$$\text{Min}\{T_{1j}\} \geq \text{Max}\{T_{ij}\}$$

$$\text{and/or } \text{Min}\{T_{3j}\} \geq \text{Max}\{T_{ij}\}$$

Then we can convert 3-Machine problem to a 2-Machine and can apply Johnson's method to get an **optimal sequence**. This will give a minimum production time i.e. make span.

| Job | Time for Dummy Operation # 1 (Days) | Time for Dummy Operation # 2 (Days) | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | Time for Operation # 3 (Days) |
|-----|-------------------------------------|-------------------------------------|-------------------------------|-------------------------------|-------------------------------|
| A | 13 | 9 | 8 | 5 | 4 |
| B | 16 | 15 | 10 | 6 | 9 |
| C | 8 | 10 | 6 | 2 | 8 |
| D | 10 | 9 | 7 | 3 | 6 |
| E | 15 | 9 | 11 | 4 | 5 |

| Job | Time for Dummy Operation # 1 (Days) | Time for Dummy Operation # 2 (Days) | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | Time for Operation # 3 (Days) |
|-----|--|--|-------------------------------------|-------------------------------------|-------------------------------------|
| A | 13 | 9 | 8 | 5 | 4 |
| B | 16 | 15 | 10 | 6 | 9 |
| C | 8 | 10 | 6 | 2 | 8 |
| D | 10 | 9 | 7 | 3 | 6 |
| E | 15 | 9 | 11 | 4 | 5 |

Applying Johnson's rule we can get an optimal sequence

| | | | | |
|----------|----------|----------|----------|----------|
| C | B | E | D | A |
|----------|----------|----------|----------|----------|

| | | | | |
|----------|----------|----------|----------|----------|
| C | B | D | E | A |
|----------|----------|----------|----------|----------|

| | | | | |
|---|---|---|---|---|
| C | B | E | A | D |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | A | E | D |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | A | D | E |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | D | A | E |
|---|---|---|---|---|

So, we get total six optimal sequence for the 3-Machine problem

| | | | | |
|---|---|---|---|---|
| C | B | E | D | A |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | D | E | A |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | E | A | D |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | A | E | D |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | A | D | E |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| C | B | D | A | E |
|---|---|---|---|---|

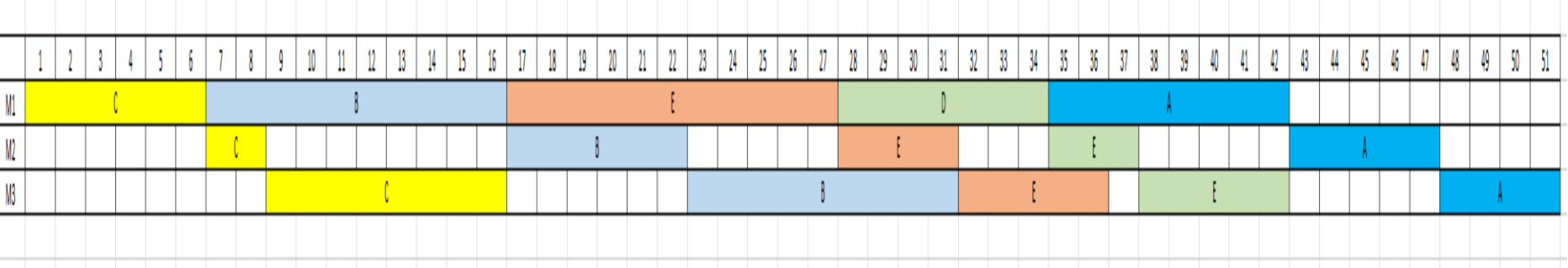
All these six sequence will give an optimized production schedule for these five jobs on three different machines.

Let's take first sequence to construct production schedule. Similarly, it can done for other sequence.

| | | | | |
|----------|----------|----------|----------|----------|
| C | B | E | D | A |
|----------|----------|----------|----------|----------|

| Job | Time for Dummy Operation # 1 (Days) | Time for Dummy Operation # 2 (Days) | Time for Operation # 1 (Days) | Time for Operation # 2 (Days) | Time for Operation # 3 (Days) |
|-----|-------------------------------------|-------------------------------------|-------------------------------|-------------------------------|-------------------------------|
| A | 13 | 9 | 8 | 5 | 4 |
| B | 16 | 15 | 10 | 6 | 9 |
| C | 8 | 10 | 6 | 2 | 8 |
| D | 10 | 9 | 7 | 3 | 6 |
| E | 15 | 9 | 11 | 4 | 5 |

| | | | | |
|----------|----------|----------|----------|----------|
| C | B | E | D | A |
|----------|----------|----------|----------|----------|



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EXAMPLES
SCHEDULING

ON

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