Probability Distributions

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Applying Probability Distributions

- One study reports that nearly 51% of cell phone owners in India use only cellular phones (no land line). Suppose you randomly select 20 Indians. What is the probability that more than 12 of the sample use only cell phones?
- A study reported that 90% of Indian adults own a cell phone. On the basis of this, if you were to randomly select 25 Indian adults, what is the probability that fewer than 20 own a cell phone?
- Suppose the average person in India receives about 16 texts per day (12 waking hours). This works out to about 4 texts every 3 hours. If this figure is true, what is the probability that the average person in India receives no texts in a 3-hour period? What is the probability that the average person in India receives 10 or more texts in a 3-hour period?

Definitions

Discrete random variable *if the set of all possible values is at most a finite or a countably infinite number of possible values*.

- Examples:
  - Randomly selecting 25 people who consume soft drinks and determining how many people prefer diet soft drinks
  - Determining the number of defective items in a batch of 50 items
  - Counting the number of people who arrive at a store during a 5-minute period
  - Sampling 100 registered voters and determining how many voted for the president in the last election
- Discrete Probability Distributions
  - Binomial Distribution
  - Poisson Distribution
  - Hypergeometric Distribution

Definitions

Continuous random variables *take on values at every point over a given interval*.

- Examples:
  - Sampling the volume of liquid nitrogen in a storage tank
  - Measuring the time between customer arrivals at a retail outlet
  - Measuring the length of newly designed automobiles
  - Measuring the weight of grain in a grain elevator at different points of time
- Continuous Probability Distributions
  - Uniform distribution
  - Normal distribution
  - Exponential distribution
  - *t* distribution
  - Chi-square distribution
  - *F* distribution

Binomial Distribution

- The experiment involves n identical trials.
- Each trial has only two possible outcomes denoted as success or as failure.
- Each trial is independent of the previous trials.
- The terms p and q remain constant throughout the experiment, where the term p is the probability of getting a success on any one trial and the term $q = (1 - p)$ is the probability of getting a failure on any one trial.

Binomial Distribution: Examples

- Data produced by the Ministry of Industries has shown that 34% of all new business establishments fail within the first two years. Suppose 25 business establishments that were newly established two years ago are randomly sampled. What is the probability that exactly 3 have failed?
- Why do clients leave their vendors? According to *Small Business India*, 68% of small-business clients who leave do so because they feel unappreciated, unimportant, and taken for granted. Suppose 20 business clients who have left a small business are randomly interviewed and asked why they left. What is the probability that 18 or more left because they felt unappreciated, unimportant, and taken for granted?
- A study conducted by the Ministry of Labour showed that only 14% of workers feel "very confident" that they will have enough money to live comfortably in retirement. Suppose 16 workers are randomly selected. If the survey is accurate, what is the probability that exactly 6 feel "very confident" they will have enough money to live comfortably in retirement?

Application

An ORG-MARG survey found that 65% of all financial consumers were very satisfied with their primary financial institution. Suppose that 25 financial consumers are sampled. If the Gallup survey result still holds true today, what is the probability that exactly 19 are very satisfied with their primary financial institution?

$$P(x) = {}_{n}C_{x} \,.\, p^{x} \,.\, q^{(n-x)} = [n!/n!(n-x)!] \,.\, p^{x} \,.\, q^{(n-x)}$$

$$P(x = 19) = {}_{25}C_{19} \,.\, 0.65^{19} \,.\, 0.35^{(25-19)} = [25!/25!(25-19)!] \,.\, 0.65^{25} \,.\, 0.35^{6} = 0.0908$$

BINOM.DIST function in Excel

Application

One study by CNNMoney reported that 60% of workers have less than $25,000 in total savings and investments (excluding the value of their home). If this is true and if a random sample of 20 workers is selected, what is the probability that fewer than 10 have less than $25,000 in total savings and investments?

- Cumulative probabilities for x < 10
- P(x=1) + P(x=2) + … P(x=9) = 0.127
- Use BINOM.DIST with “cumulative = TRUE”

Binomial Distribution

- Mean = n . p
- Standard Deviation = $\sigma = \sqrt{n.p.q}$

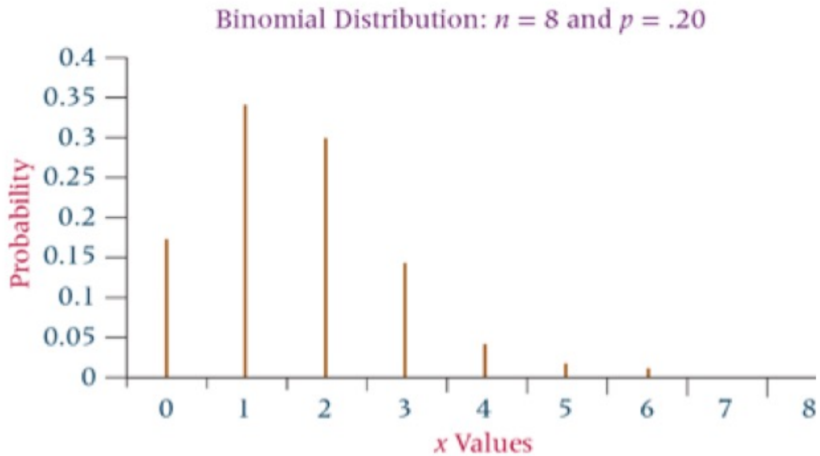
Application

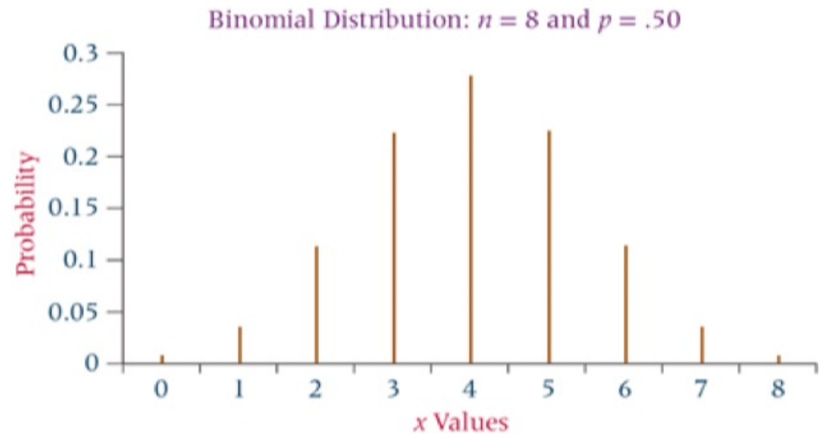
In a study where 14% of workers feel "very confident" that they will have enough money to live comfortably in retirement, if 20 workers are randomly selected, the expected number who feel "very confident" that they will have enough money to live comfortably in retirement is the mean of the binomial distribution $n = 20$ and $p = .14$.

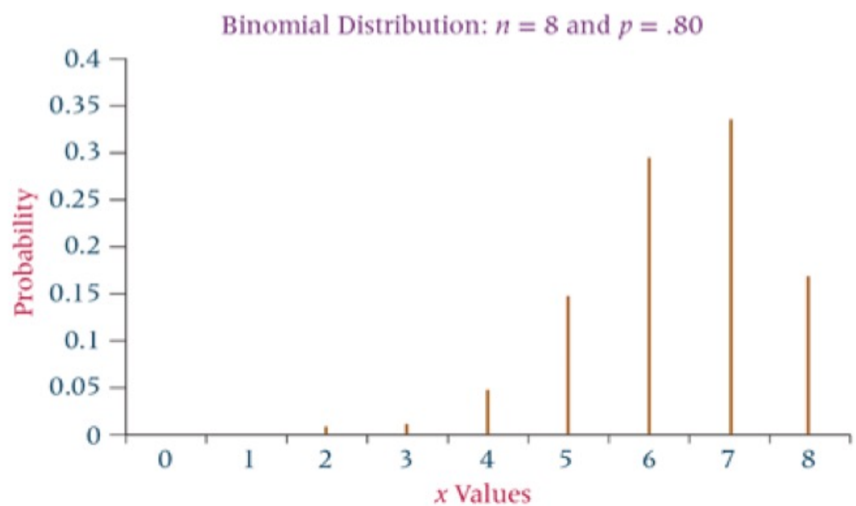
μ = n.p = 20 x 0.14 = 2.8

$$\sigma = \sqrt{20 \times 0.14 \times 0.86} = 1.55$$

Binomial Distribution Graphs







Poisson Distribution

- Characteristics
  - It is a discrete distribution.
  - It describes rare events.
  - Each occurrence is independent of the other occurrences.
  - It describes discrete occurrences over a continuum or interval.
  - The occurrences in each interval can range from zero to infinity.
  - The expected number of occurrences must hold constant throughout the experiment.
- Examples:
  1. Number of telephone calls per minute at a small business
  2. Number of hazardous waste sites per county in the United States
  3. Number of arrivals at a tollbooth per minute between 3 A.M. and 4 A.M. in January on the Kansas Turnpike
  4. Number of sewing flaws per pair of jeans during production
  5. Number of times a tire blows on a passenger car per week

Poisson Distribution

Probability of occurrences over an interval

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- x = Occurrence number → 0, 1, 2, …
- λ = Long-run average
- e = 2.718282

Application

- Suppose bank customers arrive randomly on weekday afternoons at an average of 3.2 customers every 4 minutes. What is the probability of exactly 5 customers arriving in a 4-minute interval on a weekday afternoon?
- $P(x = 5) = [3.2^5 \,.\, 2.718282^{(-3.2)}] / 5! = 0.1140$
- Excel: POISSON.DIST(5, 3.2, cumulative = FALSE)

Application

Suppose that on Saturday mornings, a specialty clothing store averages 2.4 customer arrivals every 10 minutes. What is the probability that, on a given Saturday morning, 2 customers will arrive at the store in a 6-minute interval?

λ = 2.4 customers per 10 minutes

X = 2 customers per 6 minutes

Convert λ and x to equal time intervals

λ = 0.6(2.4) = 1.44 customers per 6 minutes

Use the Poisson Distribution formula

Application

Ship collisions in the Atlantic are rare. Suppose the number of collisions are Poisson distributed, with a mean of 1.2 collisions every four months.

- What is the probability of having no collisions occur over a four-month period?
- What is the probability of having exactly two collisions in a two-month period?
- What is the probability of having one or fewer collisions in a six-month period? If this outcome occurred, what might you conclude about ship channel conditions during this period? What might you conclude about lambda?