5 CHAPTER

Net Present Value and Other Investment Criteria

A company's shareholders prefer to be rich rather than poor. Therefore, they want the firm to invest in every project that is worth more than it costs. The difference between a project's value and its cost is its *net present value (NPV)*. Companies can best help their shareholders by investing in all projects with a positive NPV and rejecting those with a negative NPV.

We start this chapter with a review of the net present value rule. We then turn to some other measures that companies may look at when making investment decisions. The first two of these measures, the project's payback period and its book rate of return, are little better than rules of thumb, easy to calculate and easy to communicate. Although there is a place for rules of thumb in this world, an engineer needs something more accurate when designing a 100-story building, and a financial manager needs more than a rule of thumb when making a substantial capital investment decision.

Instead of calculating a project's NPV, companies often compare the expected rate of return from investing in the project with the return that shareholders could earn on equivalent-risk investments in the financial markets. The company accepts those projects that provide a higher return than shareholders could earn for themselves. If used correctly, this rate of return rule should always identify projects that increase firm value. However, we shall see that the rule sets several traps for the unwary.

We conclude the chapter by showing how to cope with situations when the firm has only limited capital. This raises two problems. One is computational. In simple cases, we just choose those projects that give the highest NPV per dollar invested, but more elaborate techniques are sometimes needed to sort through the possible alternatives. The other problem is to decide whether capital rationing really exists and whether it invalidates the net present value rule. Guess what? NPV, properly interpreted, wins out in the end.

5-1 A Review of the Basics

Vegetron's chief financial officer (CFO) is wondering how to analyze a proposed \$1 million investment in a new venture code-named project X. He asks what you think.

Your response should be as follows: "First, forecast the cash flows generated by project X over its economic life. Second, determine the appropriate opportunity cost of capital (r). This should reflect both the time value of money and the risk involved in project X. Third, use this opportunity cost of capital to discount the project's future cash flows. The sum of the discounted cash flows is called present value (PV). Fourth, calculate *net* present value (NPV) by subtracting the \$1 million investment from PV. If we call the cash flows C_0, C_1, and so on, then

$$\text{NPV} = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots$$

where $C_0 = -\$1$ million. We should invest in project X if its NPV is greater than zero."

However, Vegetron's CFO is unmoved by your sagacity. He asks why NPV is so important.

You: Let us look at what is best for Vegetron stockholders. They want you to make their Vegetron shares as valuable as possible.

Right now Vegetron's total market value (price per share times the number of shares outstanding) is $10 million. That includes $1 million cash, which we can invest in project X. The value of Vegetron's other assets and opportunities must therefore be $9 million. We have to decide whether it is better to keep the $1 million cash and reject project X or to spend the cash and accept the project. Let us call the value of the new project PV. Then the choice is as follows:

	Market Value ($ millions)	
Asset	**Reject Project X**	**Accept Project X**
Cash	1	0
Other assets	9	9
Project X	0	PV
	10	9 + PV

Clearly project X is worthwhile if its present value (PV) is greater than $1 million, that is, if net present value is positive.

CFO: How do I know that the PV of project X will actually show up in Vegetron's market value?

You: Suppose we set up a new, independent firm X, whose only asset is project X. What would be the market value of firm X?

Investors would forecast the dividends that firm X would pay and discount those dividends by the expected rate of return of securities having similar risks. We know that stock prices are equal to the present value of forecasted dividends.

Since project X is the only asset, the dividend payments we would expect firm X to pay are exactly the cash flows we have forecasted for project X. Moreover, the rate that investors would use to discount firm X's dividends is exactly the rate we should use to discount project X's cash flows.

I agree that firm X is hypothetical. But if project X is accepted, investors holding Vegetron stock will really hold a portfolio of project X and the firm's other assets. We know the other assets are worth $9 million considered as a separate venture. Since asset values add up, we can easily figure out the portfolio value once we calculate the value of project X as a separate venture.

By calculating the present value of project X, we are replicating the process by which the common stock of firm X would be valued in the financial markets.

CFO: The one thing I don't understand is where the discount rate comes from.

You: I agree that the discount rate is difficult to measure precisely. But it is easy to see what we are *trying* to measure. The discount rate is the opportunity cost of investing in the project rather than in the financial markets. In other words, instead of accepting a project, the firm can always return the cash to the shareholders and let them invest it in financial assets.

You can see the trade-off (Figure 5.1). The opportunity cost of taking the project is the return shareholders could have earned had they invested the money on their own. When we discount the project's cash flows by the expected rate of return on financial assets, we are measuring how much investors would be prepared to pay for your project.

CFO: But which financial assets? The fact that investors expect only 12% on IBM stock does not mean that we should purchase Fly-by-Night Electronics if it offers 13%.

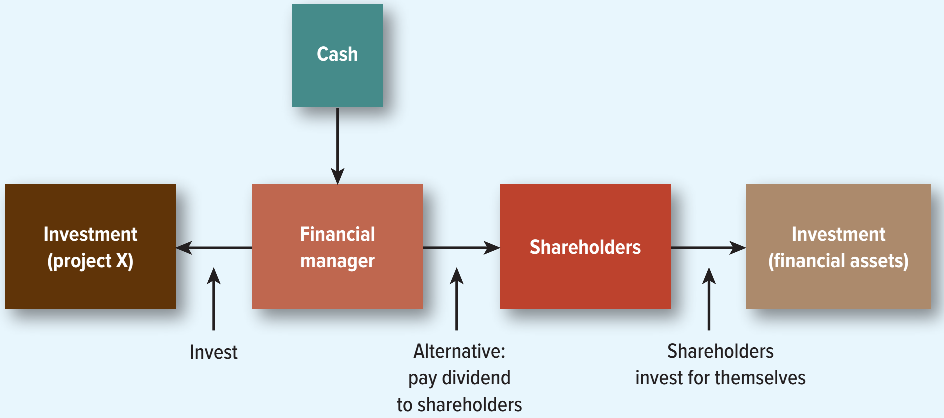

FIGURE 5.1 The firm can either keep and reinvest cash or return it to investors. (Arrows represent possible cash flows or transfers.) If cash is reinvested, the opportunity cost is the expected rate of return that shareholders could have obtained by investing in financial assets.

You: The opportunity-cost concept makes sense only if assets of equivalent risk are compared. In general, you should identify financial assets that have the same risk as your project, estimate the expected rate of return on these assets, and use this rate as the opportunity cost.

Net Present Value's Competitors

When you advised the CFO to calculate the project's NPV, you were in good company. These days, 75% of firms always, or almost always, calculate net present value when deciding on investment projects. However, as you can see from Figure 5.2, NPV is not the only investment criterion that companies use, and firms often look at more than one measure of a project's attractiveness.

About three-quarters of firms calculate the project's internal rate of return (or IRR); that is roughly the same proportion as use NPV. The IRR rule is a close relative of NPV and, when used properly, it will give the same answer. You therefore need to understand the IRR rule and how to take care when using it.

A large part of this chapter is concerned with explaining the IRR rule, but first we look at two other measures of a project's attractiveness—the project's payback and its book rate of return. As we will explain, both measures have obvious defects. Few companies rely on them to make their investment decisions, but they do use them as supplementary measures that may help to distinguish the marginal project from the no-brainer.

Later in the chapter we also come across one further investment measure, the profitability index. Figure 5.2 shows that it is not often used, but you will find that there are circumstances in which this measure has some special advantages.

Three Points to Remember about NPV

As we look at these alternative criteria, it is worth keeping in mind the following key features of the net present value rule. First, the NPV rule recognizes that *a dollar today is worth more than a dollar tomorrow* because the dollar today can be invested to start earning interest immediately. Any investment rule that does not recognize the *time value of money* cannot be sensible. Second, net present value depends solely on the *forecasted cash flows* from the project and the *opportunity cost of capital.* Any investment rule that is affected by the manager's tastes, the company's choice of accounting method, the profitability of the company's existing business, or the profitability of other independent projects will lead to inferior decisions. Third,

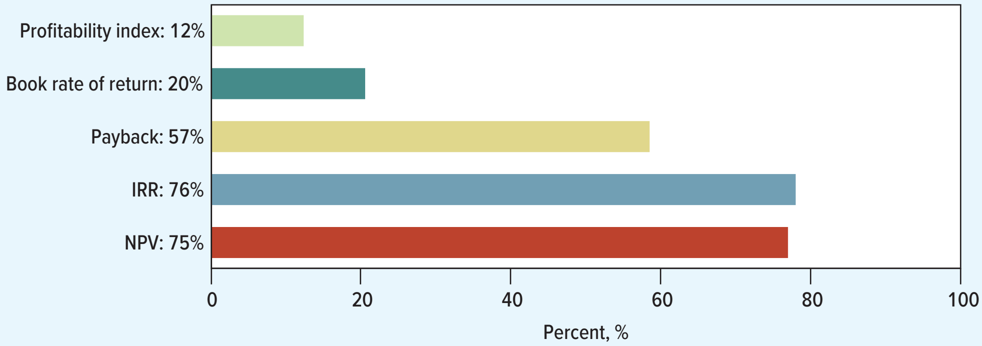

FIGURE 5.2 Survey evidence on the percentage of CFOs who always, or almost always, use a particular technique for evaluating investment projects

Source: J. R. Graham and C. R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," *Journal of Financial Economics* 60 (2001), pp. 187–243.

because present values are all measured in today's dollars, you can add them up. Therefore, if you have two projects A and B, the net present value of the combined investment is

$$\text{NPV}(\text{A} + \text{B}) = \text{NPV}(\text{A}) + \text{NPV}(\text{B})$$

This adding-up property has important implications. Suppose project B has a negative NPV. If you tack it onto project A, the joint project (A + B) must have a lower NPV than A on its own. Therefore, you are unlikely to be misled into accepting a poor project (B) just because it is packaged with a good one (A). As we shall see, the alternative measures do not have this property. If you are not careful, you may be tricked into deciding that a package of a good and a bad project is better than the good project on its own.

5-2 Book Rate of Return and Payback

Book Rate of Return

Net present value depends only on the project's cash flows and the opportunity cost of capital. But when companies report to shareholders, they do not simply show the cash flows. They also report book—that is, accounting—income and book assets.

Financial managers sometimes use these numbers to calculate a book (or accounting) rate of return on a proposed investment. In other words, they look at the prospective book income from the investment as a proportion of the book value of the assets that the firm is proposing to acquire:

$$\text{Book rate of return} = \frac{\text{book income}}{\text{book assets}}$$

They typically compare this figure with the book rate of return that the company is currently earning.

Cash flows and book income are often very different. For example, the accountant labels some cash outflows as *capital investments* and others as *operating expenses.* The operating expenses are, of course, deducted immediately from each year's income. The capital expenditures are put on the firm's balance sheet and then depreciated. The annual depreciation charge is deducted from each year's income. Thus the book rate of return depends on which items the accountant treats as capital investments and how rapidly they are depreciated.[1]

[1]This chapter's mini-case contains simple illustrations of how book rates of return are calculated and of the difference between accounting income and project cash flow. Read the case if you wish to refresh your understanding of these topics. Better still, do the case calculations.

Now the merits of an investment project do not depend on how accountants classify the cash flows[2] and few companies these days make investment decisions just on the basis of the book rate of return. But managers know that the company's shareholders pay considerable attention to book measures of profitability and naturally they think (and worry) about how major projects would affect the company's book return. Those projects that would reduce the company's book return may be scrutinized more carefully by senior management.

You can see the dangers here. The company's book rate of return may not be a good measure of true profitability. It is also an *average* across all of the firm's activities. The average profitability of past investments is not usually the right hurdle for new investments. Think of a firm that has been exceptionally lucky and successful. Say its average book return is 24%, double shareholders' 12% opportunity cost of capital. Should it demand that all *new* investments offer 24% or better? Clearly not: That would mean passing up many positive-NPV opportunities with rates of return between 12 and 24%.

We will come back to the book rate of return in Chapters 12 and 28, when we look more closely at accounting measures of financial performance.

Payback

We suspect that you have often heard conversations that go something like this: "We are spending $6 a week, or around $300 a year, at the laundromat. If we bought a washing machine for $800, it would pay for itself within three years. That's well worth it." You have just encountered the payback rule.

A project's **payback period** is found by counting the number of years it takes before the cumulative cash flow equals the initial investment. For the washing machine the payback period was just under three years. The **payback rule** states that a project should be accepted if its payback period is less than some specified cutoff period. For example, if the cutoff period is four years, the washing machine makes the grade; if the cutoff is two years, it doesn't.

We have no quarrel with those who use payback as a descriptive statistic. It is perfectly fine to say that the washing machine has a three-year payback. But payback should never be a *rule*.

EXAMPLE 5.1 • The Payback Rule

Consider the following three projects:

	Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	Payback Period (years)	NPV at 10%
A	−2,000	500	500	5,000	3	+2,624
B	−2,000	500	1,800	0	2	−58
C	−2,000	1,800	500	0	2	−50

Project A involves an initial investment of $2,000 ($C_0 = -2{,}000$) followed by cash inflows during the next three years. Suppose the opportunity cost of capital is 10%. Then project A has an NPV of +$2,624:

$$\text{NPV(A)} = -2{,}000 + \frac{500}{1.10} + \frac{500}{1.10^2} + \frac{5{,}000}{1.10^3} = +\$2{,}624$$

[2] Of course, the depreciation method used for tax purposes does have cash consequences that should be taken into account in calculating NPV. We cover depreciation and taxes in the next chapter.

Project B also requires an initial investment of $2,000 but produces a cash inflow of $500 in year 1 and $1,800 in year 2. At a 10% opportunity cost of capital project B has an NPV of −$58:

$$\text{NPV(B)} = -2{,}000 + \frac{500}{1.10} + \frac{1{,}800}{1.10^2} = -\$58$$

The third project, C, involves the same initial outlay as the other two projects but its first-period cash flow is larger. It has an NPV of +$50:

$$\text{NPV(C)} = -2{,}000 + \frac{1{,}800}{1.10} + \frac{500}{1.10^2} = +\$50$$

The net present value rule tells us to accept projects A and C but to reject project B.

Now look at how rapidly each project pays back its initial investment. With project A, you take three years to recover the $2,000 investment; with projects B and C, you take only two years. If the firm used the *payback rule* with a cutoff period of two years, it would accept only projects B and C; if it used the payback rule with a cutoff period of three or more years, it would accept all three projects. Therefore, regardless of the choice of cutoff period, the payback rule gives different answers from the net present value rule.

You can see why payback can give misleading answers:

1. *The payback rule ignores all cash flows after the cutoff date.* If the cutoff date is two years, the payback rule rejects project A regardless of the size of the cash inflow in year 3.
2. *The payback rule gives equal weight to all cash flows before the cutoff date.* The payback rule says that projects B and C are equally attractive, but because C's cash inflows occur earlier, C has the higher net present value at any positive discount rate.

To use the payback rule, a firm must decide on an appropriate cutoff date. If it uses the same cutoff regardless of project life, it will tend to accept many poor short-lived projects and reject many good long-lived ones.

We have had little good to say about payback. So why do many companies continue to use it? Senior managers don't truly believe that all cash flows after the payback period are irrelevant. We suggest three explanations. First, payback may be used because it is the simplest way to *communicate* an idea of project profitability. Investment decisions require discussion and negotiation among people from all parts of the firm, and it is important to have a measure that everyone can understand. Second, managers of larger corporations may opt for projects with short paybacks because they believe that quicker profits mean quicker promotion. That takes us back to Chapter 1, where we discussed the need to align the objectives of managers with those of shareholders. Finally, owners of small public firms with limited access to capital may worry about their future ability to raise capital. These worries may lead them to favor rapid payback projects even though a longer-term venture may have a higher NPV.

Discounted Payback

Occasionally companies discount the cash flows before they compute the payback period. The discounted cash flows for our three projects are as follows:

Discounted Cash Flows ($)						
Project	C_0	C_1	C_2	C_3	Discounted Payback Period (years)	NPV at 10%
A	−2,000	500/1.10 = 455	$500/1.10^2$ = 413	$5{,}000/1.10^3$ = 3,757	3	+2,624
B	−2,000	500/1.10 = 455	$1{,}800/1.10^2$ = 1,488		–	−58
C	−2,000	1,800/1.10 = 1,636	$500/1.10^2$ = 413		2	+50

The *discounted payback measure* asks, How many years does the project have to last in order for it to make sense in terms of net present value? You can see that the value of the cash inflows from project B never exceeds the initial outlay and would always be rejected under the discounted payback rule. Thus a discounted payback rule will never accept a negative-NPV project. On the other hand, it still takes no account of cash flows after the cutoff date, so that good long-term projects such as A continue to risk rejection.

Rather than automatically rejecting any project with a long discounted payback period, many managers simply use the measure as a warning signal. These managers don't unthinkingly reject a project with a long discounted payback period. Instead they check that the proposer is not unduly optimistic about the project's ability to generate cash flows into the distant future. They satisfy themselves that the equipment has a long life and that competitors will not enter the market and eat into the project's cash flows.

5-3 Internal (or Discounted Cash Flow) Rate of Return

Whereas payback and return on book are ad hoc measures, internal rate of return has a much more respectable ancestry and is recommended in many finance texts. If we dwell more on its deficiencies, it is not because they are more numerous but because they are less obvious.

In Chapter 2, we noted that the net present value rule could also be expressed in terms of rate of return, which would lead to the following rule: "Accept investment opportunities offering rates of return in excess of their opportunity costs of capital." That statement, properly interpreted, is absolutely correct. However, interpretation is not always easy for long-lived investment projects.

There is no ambiguity in defining the true rate of return of an investment that generates a single payoff after one period:

$$\text{Rate of return} = \frac{\text{payoff}}{\text{investment}} - 1$$

Alternatively, we could write down the NPV of the investment and find the discount rate that makes NPV = 0.

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{discount rate}} = 0$$

implies

$$\text{Discount rate} = \frac{C_1}{-C_0} - 1$$

Of course C_1 is the payoff and $-C_0$ is the required investment, and so our two equations say exactly the same thing. *The discount rate that makes NPV = 0 is also the rate of return.*

How do we calculate return when the project produces cash flows in several periods? Answer: We use the same definition that we just developed for one-period projects—*the project rate of return is the discount rate that gives a zero NPV.* This discount rate is known as the **discounted cash flow (DCF) rate of return** or **internal rate of return (IRR).** The internal rate of return is used frequently in finance. It can be a handy measure, but, as we shall see, it can also be a misleading measure. You should, therefore, know how to calculate it and how to use it properly.

Calculating the IRR

The internal rate of return is defined as the rate of discount that makes NPV = 0. So to find the IRR for an investment project lasting T years, we must solve for IRR in the following expression:

$$\text{NPV} = C_0 + \frac{C_1}{1 + \text{IRR}} + \frac{C_2}{(1 + \text{IRR})^2} + \cdots + \frac{C_T}{(1 + \text{IRR})^T} = 0$$

Actual calculation of IRR usually involves trial and error. For example, consider a project that produces the following flows:

Cash Flows ($)		
C_0	C_1	C_2
−4,000	+2,000	+4,000

The internal rate of return is IRR in the equation

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1 + \text{IRR}} + \frac{4{,}000}{(1 + \text{IRR})^2} = 0$$

Let us arbitrarily try a zero discount rate. In this case, NPV is not zero but +\$2,000:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.0} + \frac{4{,}000}{(1.0)^2} = +\$2{,}000$$

The NPV is positive; therefore, the IRR must be greater than zero. The next step might be to try a discount rate of 50%. In this case, net present value is −\$889:

$$\text{NPV} = -4{,}000 + \frac{2{,}000}{1.50} + \frac{4{,}000}{(1.50)^2} = -\$889$$

The NPV is negative; therefore, the IRR must be less than 50%. In Figure 5.3, we have plotted the net present values implied by a range of discount rates. From this, we can see that a discount rate of 28.08% gives the desired net present value of zero. Therefore, IRR is 28.08%. (We carry the IRR calculation to two decimal places to avoid confusion from rounding. In practice, no one would worry about the .08%.)[3]

You can always find the IRR by plotting an NPV profile, as in Figure 5.3, but it is quicker and more accurate to let a spreadsheet or specially programmed calculator do the trial and error for you. The Useful Spreadsheet Functions box near the end of the chapter shows how to use the Excel function to calculate an IRR.

Some people confuse the internal rate of return and the opportunity cost of capital because both appear as discount rates in the NPV formula. The internal rate of return is a *profitability measure* that depends solely on the amount and timing of the project cash flows. The

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Calculating the IRR

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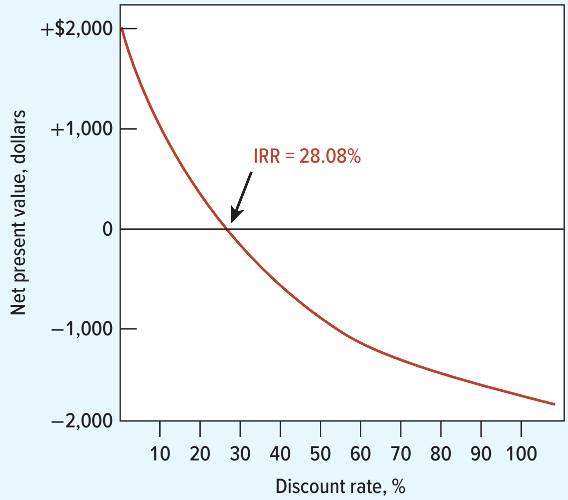
[3]The IRR is a first cousin to the yield to maturity on a bond. Recall from Chapter 3 that the yield to maturity is the discount rate that makes the present value of future interest and principal payments equal to the bond's price. If you buy the bond at that market price and hold it to maturity, the yield to maturity is your IRR on the bond investment.

FIGURE 5.3

This project costs $4,000 and then produces cash inflows of $2,000 in year 1 and $4,000 in year 2. Its internal rate of return (IRR) is 28.08%, the rate of discount at which NPV is zero.



opportunity cost of capital is a *standard of profitability* that we use to calculate how much the project is worth. The opportunity cost of capital is established in the financial markets. It is the expected rate of return offered by other assets with the same risk as the project being evaluated.

The IRR Rule

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Interpreting the IRR
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The internal rate of return rule states that the firm should accept an investment project if the opportunity cost of capital is less than the internal rate of return. You can see the reasoning behind this idea if you look again at Figure 5.3. If the opportunity cost of capital is less than the 28.08% IRR, then the project has a *positive* NPV when discounted at the opportunity cost of capital. If it is equal to the IRR, the project has a *zero* NPV. And if it is greater than the IRR, the project has a *negative* NPV. Therefore, when we compare the opportunity cost of capital with the IRR on our project, we are effectively asking whether our project has a positive NPV. This is true not only for our example. The rule will give the same answer as the net present value rule *whenever the NPV of a project is a steadily declining function of the discount rate.*

The 28.08% internal rate of return on our project tells us how high the opportunity cost of capital must be before the project should be rejected. Although we might not be able to put a precise number on the project's cost of capital, we might nevertheless be confident that it was less than 28.08% and that we can safely go ahead with the project. You can understand, therefore, why a manager may find it helpful to know the project's IRR. Our worries concern those managers who use the internal rate of return as a criterion *in preference to net present value.* Although, properly stated, the two criteria are formally equivalent, the internal rate of return rule contains several pitfalls.

Pitfall 1—Lending or Borrowing?

Not all cash-flow streams have NPVs that decline as the discount rate increases. Consider the following projects A and B:

	Cash Flows ($)			
Project	C_0	C_1	IRR	NPV at 10%
A	−1,000	+1,500	+50%	+364
B	+1,000	−1,500	+50%	−364

Each project has an IRR of 50%. (In other words, –1,000 + 1,500/1.50 = 0 and +1,000 – 1,500/1.50 = 0.)

Does this mean that they are equally attractive? Clearly not, for in the case of A, where we are initially paying out $1,000, we are *lending* money at 50%; in the case of B, where we are initially receiving $1,000, we are *borrowing* money at 50%. When we lend money, we want a *high* rate of return; when we borrow money, we want a *low* rate of return.

If you plot a graph like Figure 5.3 for project B, you will find that NPV increases as the discount rate increases. Obviously the internal rate of return rule, as we stated it above, won't work in this case; we have to look for an IRR *less* than the opportunity cost of capital.

Pitfall 2—Multiple Rates of Return

Helmsley Iron is proposing to develop a new strip mine in Western Australia. The mine involves an initial investment of A$30 billion and is expected to produce a cash inflow of A$10 billion a year for the next nine years. At the end of that time, the company will incur A$65 billion of cleanup costs. Thus, the cash flows from the project are:

Cash Flows (billions of Australian dollars)				
C_0	C_1	. . .	C_9	C_{10}
–30	10		10	–65

Helmsley calculates the project's IRR and its NPV as follows:

IRR (%)	NPV at 10%
+3.50 *and* 19.54	$A2.53 billion

Note that there are *two* discount rates that make NPV = 0. That is, *each* of the following statements holds:

$$\text{NPV} = -30 + \frac{10}{1.035} + \frac{10}{1.035^2} + \cdots + \frac{10}{1.035^9} - \frac{65}{1.035^{10}} = 0$$

$$\text{NPV} = -30 + \frac{10}{1.1954} + \frac{10}{1.1954^2} + \cdots + \frac{10}{1.1954^9} - \frac{65}{1.1954^{10}} = 0$$

In other words, the investment has an IRR of both 3.50% *and* 19.54%. Figure 5.4 shows how this comes about. As the discount rate increases, NPV initially rises and then declines. The reason for this is the double change in the sign of the cash-flow stream. There can be as many internal rates of return for a project as there are changes in the sign of the cash flows.[4]

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Try It! Figure 5.4: Helmsley's multiple IRRs

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Decommissioning and clean-up costs can sometimes lead to huge negative cash flows at the end of a project. The cost of decommissioning oil platforms in the British North Sea has been estimated at $75 billion. It can cost more than $500 million to decommission a nuclear power plant. These are obvious instances where cash flows go from positive to negative, but you can probably think of a number of other cases where the company needs to plan for later expenditures. Ships periodically need to go into dry dock for a refit, hotels may receive a major face-lift, machine parts may need replacement, and so on.

[4]By Descartes's "rule of signs" there can be as many different solutions to a polynomial as there are changes of sign.

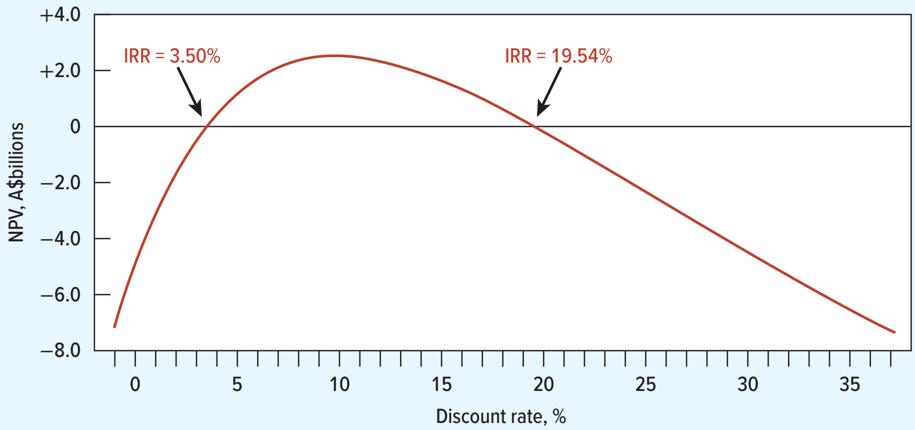

FIGURE 5.4 Helmsley Iron's mine has two internal rates of return. NPV = 0 when the discount rate is +3.50% and when it is +19.54%.

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A project with no IRR
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Whenever the cash-flow stream is expected to change sign more than once, the company typically sees more than one IRR.

As if this is not difficult enough, there are also cases in which *no* internal rate of return exists. For example, project C has a positive net present value at all discount rates:

	Cash Flows ($)				
Project	C_0	C_1	C_2	IRR (%)	NPV at 10%
C	+1,000	−3,000	+2,500	None	+339

A number of adaptations of the IRR rule have been devised for such cases. Not only are they inadequate, but they also are unnecessary, for the simple solution is to use net present value.[5]

Pitfall 3—Mutually Exclusive Projects

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Try It! Calculate the MIRR
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Firms often have to choose between several alternative ways of doing the same job or using the same facility. In other words, they need to choose between **mutually exclusive projects.** Here, too, the IRR rule can be misleading.

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[5]Companies sometimes get around the problem of multiple rates of return by discounting the later cash flows back at the cost of capital until there remains only one change in the sign of the cash flows. A modified internal rate of return (MIRR) can then be calculated on this revised series. In our example, the MIRR is calculated as follows:

1. Calculate the present value in year 5 of all the subsequent cash flows:

$$\text{PV in year 5} = 10/1.1 + 10/1.1^2 + 10/1.1^3 + 10/1.1^4 - 65/1.1^5 = -8.66$$

2. Add to the year 5 cash flow the present value of subsequent cash flows:

$$C_5 + \text{PV(subsequent cash flows)} = 10 - 8.66 = 1.34$$

3. Since there is now only one change in the sign of the cash flows, the revised series has a unique rate of return, which is 13.7%:

$$\text{NPV} = -30 + 10/1.137 + 10/1.137^2 + 10/1.137^3 + 10/1.137^4 + 1.34/1.137^5 = 0$$

Since the MIRR of 13.7% is greater than the cost of capital (and the initial cash flow is negative), the project has a positive NPV when valued at the cost of capital.

Of course, it would be much easier in such cases to abandon the IRR rule and just calculate project NPV.

Consider projects D and E:

	Cash Flows ($)			
Project	C_0	C_1	IRR (%)	NPV at 10%
D	−10,000	+20,000	100	+8,182
E	−20,000	+35,000	75	+11,818

Perhaps project D is a manually controlled machine tool and project E is the same tool with the addition of computer control. Both are good investments, but E has the higher NPV and is, therefore, better. However, the IRR rule seems to indicate that if you have to choose, you should go for D because it has the higher IRR. If you follow the IRR rule, you have the satisfaction of earning a 100% rate of return; if you follow the NPV rule, you are $11,818 richer.

You can salvage the IRR rule in these cases by looking at the internal rate of return on the *incremental* flows. Here is how to do it: First, consider the smaller project (D in our example). It has an IRR of 100%, which is well in excess of the 10% opportunity cost of capital. You know, therefore, that D is acceptable. You now ask yourself whether it is worth making the additional $10,000 investment in E. The incremental flows from undertaking E rather than D are as follows:

	Cash Flows ($)			
Project	C_0	C_1	IRR (%)	NPV at 10%
E − D	−10,000	+15,000	50	+3,636

The IRR on the incremental investment is 50%. While that is not as good as D's IRR, it is well in excess of the 10% opportunity cost of capital. So you should prefer project E to project D.[6]

Unless you look at the incremental expenditure, IRR is unreliable in ranking projects of different scale. It is also unreliable in ranking projects with different patterns of cash flow over time. For example, sometimes it can be worth taking a project that offers a good rate of return for a long period rather than one that offers an even higher rate for just a few years. To illustrate, suppose the firm can take project F or project G but not both:

	Cash Flows ($)										
Project	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	IRR (%)	NPV at 10%
F	−10,000	+6,000	+6,000	+6,000	0	0	0	0	0	36.3	4,921
G	−10,000	+3,000	+3,000	+3,000	+3,000	+3,000	3,000	3,000	3,000	25.0	6,005

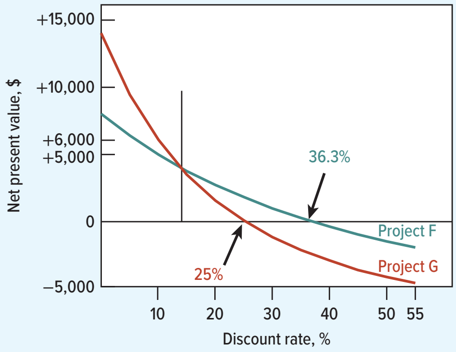
The short-lived project, F, offers the higher IRR, but at a 10% cost of capital, project G has the higher NPV and would therefore make shareholders wealthier.

Figure 5.5 shows how the choice between these two projects depends on the discount rate. Notice that, if investors require a relatively low rate of return (less than 13.9%), they will pay a higher price for project G with its longer life. The short-lived project F is superior only if investors demand a very high rate of return (greater than 13.9%) and therefore place a low

[6] When you examine incremental cash flows, you may find that you have jumped out of the frying pan into the fire. The series of incremental cash flows may involve several changes in sign. In this case there are likely to be multiple IRRs and you will be forced to use the NPV rule after all.

FIGURE 5.5

The IRR of project F exceeds that of project G, but the NPV of project F is higher only if the discount rate is greater than 13.9%.



value on the more distant cash flows. This is not something you could discover by comparing the project IRRs.[7]

The simplest way to choose between projects F and G is to compare their net present values. But if your heart is set on the IRR rule, you can use it as long as you look at the return on the incremental cash flows. The procedure is exactly the same as we showed earlier. First you check that project F has a satisfactory IRR. Then you look at the return on the *incremental* cash flows from G.

The IRR on the incremental cash flows from G is 13.9%. Since this is greater than the opportunity cost of capital, you should undertake G rather than F:[8]

	Cash Flows ($)										
Project	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	IRR (%)	NPV at 10%
G − F	0	−3,000	−3,000	−3,000	+3,000	+3,000	+3,000	+3,000	+3,000	+13.9	+1,084

Pitfall 4—What Happens When There Is More Than One Opportunity Cost of Capital

We have simplified our discussion of capital budgeting by assuming that the opportunity cost of capital is the same for all the cash flows, C_1, C_2, C_3, and so on. Remember our most general formula for calculating net present value:

$$\text{NPV} = C_0 + \frac{C_1}{1 + r_1} + \frac{C_2}{(1 + r_2)^2} + \frac{C_3}{(1 + r_3)^3} + \cdots$$

In other words, we discount C_1 at the opportunity cost of capital for one year, C_2 at the opportunity cost of capital for two years, and so on. The IRR rule tells us to accept a project if the IRR is greater than the opportunity cost of capital. But what do we do when we have several

[7] It is often suggested that the choice between the net present value rule and the internal rate of return rule should depend on the probable reinvestment rate. This is wrong. The prospective return on another independent investment should not be allowed to influence the investment decision.

[8] Because F and G had the same 10% cost of capital, we can choose between the two projects by asking whether the IRR on the incremental cash flows is greater or less than 10%. But suppose that F and G had different risks and, therefore, different costs of capital. In that case, there would be no simple yardstick for assessing whether the IRR on the incremental cash flows was adequate.

opportunity costs? Do we compare IRR with r_1, r_2, r_3, . . .? Actually we would have to compute a complex weighted average of these rates to obtain a number comparable to IRR.

The differences between short- and long-term discount rates can be important when the term structure of interest rates is not "flat." In 2017, for example, long-term U.S. Treasury bonds yielded almost 2% more than short-term Treasury bills. Suppose a financial manager was evaluating leases for new office space. Assume the lease payments were fixed obligations. The manager would not use the same discount rate for a 1-year lease as for a 15-year lease.

But the extra precision from building the term structure of discount rates into discount rates for risky capital investment projects is rarely worth the trouble. The gains from accurately forecasting project cash flows far outweigh the gains from more precise discounting. Thus, the IRR usually survives, even when the term structure is not flat.

The Verdict on IRR

We have given four examples of things that can go wrong with IRR. We spent much less space on payback or return on book. Does this mean that IRR is worse than the other two measures? Quite the contrary. There is little point in dwelling on the deficiencies of payback or return on book. They are clearly ad hoc measures that often lead to silly conclusions. The IRR rule has a much more respectable ancestry. It is less easy to use than NPV, but, used properly, it gives the same answer.

Nowadays, few large corporations use the payback period or return on book as their primary measure of project attractiveness. Most use discounted cash flow (or DCF), and for many companies, DCF means IRR, not NPV. For "normal" investment projects with an initial cash outflow followed by a series of cash inflows, there is no difficulty in using the internal rate of return to make a simple accept/reject decision. However, we think that the financial managers who like to use IRRs need to worry more about pitfall 3. Financial managers never see all possible projects. Most projects are proposed by operating managers. A company that instructs nonfinancial managers to look first at project IRRs prompts a search for those projects with the highest IRRs rather than the highest NPVs. It also encourages managers to *modify* projects so that their IRRs are higher. Where do you typically find the highest IRRs? In short-lived projects requiring little up-front investment. Such projects may not add much to the value of the firm.

We don't know why so many companies pay such close attention to the internal rate of return, but we suspect that it may reflect the fact that management does not trust the forecasts it receives. Suppose that two plant managers approach you with proposals for two new investments. Both have a positive NPV of $1,400 at the company's 8% cost of capital, but you nevertheless decide to accept project A and reject B. Are you being irrational?

The cash flows for the two projects and their NPVs are set out in the accompanying table. You can see that although both proposals have the same NPV, project A involves an investment of $9,000, while B requires an investment of $9 million. Investing $9,000 to make $1,400 is clearly an attractive proposition, and this shows up in A's IRR of nearly 16%. Investing $9 million to make $1,400 might also be worth doing if you could be *sure* of the plant manager's forecasts, but there is almost no room for error in project B. You could spend time and money checking the cash-flow forecasts, but is it really worth the effort? Most managers

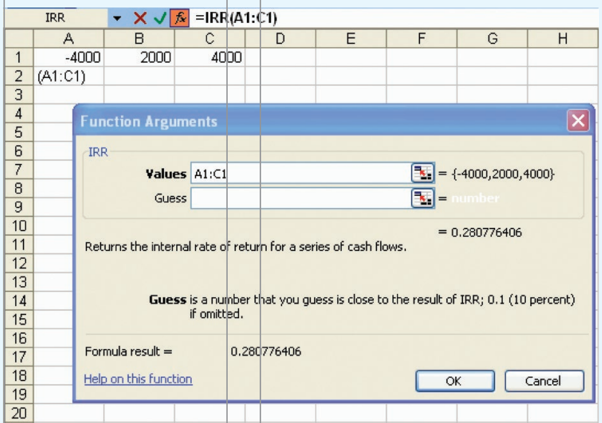
	Cash Flows ($ thousands)					
Project	C_0	C_1	C_2	C_3	NPV at 8%	IRR (%)
A	−9.0	2.9	4.0	5.4	1.4	15.58
B	−9,000	2,560	3,540	4,530	1.4	8.01

Internal Rate of Return

Spreadsheet programs such as Excel provide built-in functions to solve for internal rates of return. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will guide you through the inputs that are required. At the bottom left of the function box there is a Help facility with an example of how the function is used.

Here is a list of useful functions for calculating internal rates of return, together with some points to remember when entering data:

- **IRR:** Internal rate of return on a series of regularly spaced cash flows.
- **XIRR:** The same as IRR, but for irregularly spaced flows.



Source: Microsoft Excel.

Note the following:

- For these functions, you must enter the addresses of the cells that contain the input values.
- The IRR functions calculate (at most) only one IRR even when there are multiple IRRs.

Spreadsheet Questions

The following questions provide an opportunity to practice each of the above functions:

1. (IRR) Check the IRRs for project F in Section 5-3.

2. (IRR) What is the IRR of a project with the following cash flows:

C_0	C_1	C_2	C_3
−$5,000	+$2,200	+$4,650	+$3,330

3. (XIRR) What is the IRR of a project with the following cash flows:

C_0	C_4	C_5	C_6
−$215,000 . . .	+$185,000	+$85,000	+$43,000

(All other cash flows are 0.)

would look at the IRR and decide that if the cost of capital is 8%, a project that offers a return of 8.01% is not worth the worrying time.

Alternatively, management may conclude that project A is a clear winner that is worth undertaking right away, but in the case of project B, it may make sense to wait and see whether the decision looks more clear-cut in a year's time.[9] That is why managers will often postpone the decision on projects such as B by setting a hurdle rate for the IRR that is higher than the cost of capital.

5-4 Choosing Capital Investments When Resources Are Limited

Our entire discussion of methods of capital budgeting has rested on the proposition that the wealth of a firm's shareholders is highest if the firm accepts *every* project that has a positive net present value. Suppose, however, that there are limitations on the investment program that

[9]In Chapter 22, we discuss when it may pay a company to delay undertaking a positive-NPV project. We will see that when projects are "deep-in-the-money" (project A), it generally pays to invest right away and capture the cash flows. However, in the case of projects that are "close-to-the-money" (project B), it makes more sense to wait and see.

prevent the company from undertaking all such projects. Economists call this *capital rationing*. When capital is rationed, we need a method of selecting the package of projects that is within the company's resources yet gives the highest possible net present value.

An Easy Problem in Capital Rationing

Let us start with a simple example. The opportunity cost of capital is 10%, and our company has the following opportunities:

	Cash Flows ($ millions)			
Project	C_0	C_1	C_2	NPV at 10%
A	−10	+30	+5	21
B	−5	+5	+20	16
C	−5	+5	+15	12

All three projects are attractive, but suppose that the firm is limited to spending $10 million. In that case, it can invest *either* in project A *or* in projects B and C, but it cannot invest in all three. Although individually B and C have lower net present values than project A, when taken together they have the higher net present value. Here we cannot choose between projects solely on the basis of net present values. When money is limited, we need to concentrate on getting the biggest bang for our buck. In other words, we must pick the projects that offer the highest net present value per dollar of initial outlay. This ratio is known as the **profitability index:**[10]

$$\text{Profitability index} = \frac{\text{net present value}}{\text{investment}}$$

For our three projects the profitability index is calculated as follows:[11]

Project	Investment ($ millions)	NPV ($ millions)	Profitability Index
A	10	21	2.1
B	5	16	3.2
C	5	12	2.4

Project B has the highest profitability index and C has the next highest. Therefore, if our budget limit is $10 million, we should accept these two projects.[12]

Unfortunately, there are some limitations to this simple ranking method. One of the most serious is that it breaks down whenever more than one resource is rationed.[13] For

[10]If a project requires outlays in two or more periods, the denominator should be the present value of the outlays. A few companies do not discount the benefits or costs before calculating the profitability index. The less said about these companies the better.

[11]Sometimes the profitability index is defined as the ratio of the present value to initial outlay—that is, as PV/investment. This measure is also known as the *benefit–cost ratio*. To calculate the benefit–cost ratio, simply add 1.0 to each profitability index. Project rankings are unchanged.

[12]If a project has a positive profitability index, it must also have a positive NPV. Therefore, firms sometimes use the profitability index to select projects when capital is *not* limited. However, like the IRR, the profitability index can be misleading when used to choose between mutually exclusive projects. For example, suppose you were forced to choose between (1) investing $100 in a project whose payoffs have a present value of $200 or (2) investing $1 million in a project whose payoffs have a present value of $1.5 million. The first investment has the higher profitability index; the second makes you richer.

[13]It may also break down if it causes some money to be left over. It might be better to spend all the available cash even if this involves accepting a project with a slightly lower profitability index.

example, suppose that the firm can raise only $10 million for investment in *each* of years 0 and 1 and that the menu of possible projects is expanded to include an investment next year in project D:

	Cash Flows ($ millions)				
Project	C_0	C_1	C_2	**NPV at 10%**	**Profitability Index**
A	−10	+30	+ 5	21	2.1
B	− 5	+ 5	+20	16	3.2
C	− 5	+ 5	+15	12	2.4
D	0	−40	+60	13	0.4

One strategy is to accept projects B and C; however, if we do this, we cannot also accept D, which costs more than our budget limit for period 1. An alternative is to accept project A in period 0. Although this has a lower net present value than the combination of B and C, it provides a $30 million positive cash flow in period 1. When this is added to the $10 million budget, we can also afford to undertake D next year. A and D have *lower* profitability indexes than B and C, but they have a *higher* total net present value.

The reason that ranking on the profitability index fails in this example is that resources are constrained in each of two periods. In fact, this ranking method is inadequate whenever there is *any* other constraint on the choice of projects. This means that it cannot cope with cases in which two projects are mutually exclusive or in which one project is dependent on another.

For example, suppose that you have a long menu of possible projects starting this year and next. There is a limit on how much you can invest in each year. Perhaps also you can't undertake both project alpha and beta (they both require the same piece of land), and you can't invest in project gamma unless you also invest in delta (gamma is simply an add-on to delta). You need to find the package of projects that satisfies all these constraints and gives the highest NPV.

One way to tackle such a problem is to work through all possible combinations of projects. For each combination you first check whether the projects satisfy the constraints and then calculate the net present value. But it is smarter to recognize that linear programming (LP) techniques are specially designed to search through such possible combinations.

Uses of Capital Rationing Models

BEYOND THE PAGE

Capital rationing models

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Linear programming models seem tailor-made for solving capital budgeting problems when resources are limited. Why then are they not universally accepted either in theory or in practice? One reason is that these models can turn out to be very complex. Second, as with any sophisticated long-range planning tool, there is the general problem of getting good data. It is just not worth applying costly, sophisticated methods to poor data. Furthermore, these models are based on the assumption that all future investment opportunities are known. In reality, the discovery of investment ideas is an unfolding process.

Our misgivings center in part on the basic assumption that capital is limited. This may often be the case in countries such as China and India with financial markets that are not as fully developed as those in the United States, Europe, and Japan, but, when we come to discuss company financing, we shall see that large corporations in the latter economies do not face capital rationing and can raise large sums of money on fair terms. Why then do many company presidents in these countries tell their subordinates that capital is limited? If they are right, the financial markets are seriously imperfect. What then are they doing maximizing

NPV?[14] We might be tempted to suppose that if capital is not rationed, they do not *need* to use linear programming and, if it is rationed, then surely they *ought* not to use it. But that would be too quick a judgment. Let us look at this problem more deliberately.

Soft Rationing Many firms' capital constraints are "soft." They reflect no imperfections in financial markets. Instead they are provisional limits adopted by management as an aid to financial control.

Some ambitious divisional managers habitually overstate their investment opportunities. Rather than trying to distinguish which projects really are worthwhile, headquarters may find it simpler to impose an upper limit on divisional expenditures and thereby force the divisions to set their own priorities. In such instances, budget limits are a rough but effective way of dealing with biased cash-flow forecasts. In other cases, management may believe that very rapid corporate growth could impose intolerable strains on management and the organization. Since it is difficult to quantify such constraints explicitly, the budget limit may be used as a proxy.

Because such budget limits have nothing to do with any inefficiency in the financial market, there is no contradiction in using an LP model in the division to maximize net present value subject to the budget constraint. On the other hand, there is not much point in elaborate selection procedures if the cash-flow forecasts of the division are seriously biased.

Even if capital is not rationed, other resources may be. The availability of management time, skilled labor, or even other capital equipment often constitutes an important constraint on a company's growth. In such cases also there is no contradiction in using an LP model to select the package of projects that maximizes NPV.

Hard Rationing Soft rationing should never cost the firm anything. If capital constraints become tight enough to hurt—in the sense that projects with significant positive NPVs are passed up—then the firm raises more money and loosens the constraint. But what if it *can't* raise more money—what if it faces *hard* rationing?

Hard rationing implies market imperfections, but that does not necessarily mean we have to throw away net present value as a criterion for capital budgeting. It depends on the nature of the imperfection.

Arizona Aquaculture Inc. (AAI) borrows as much as the banks will lend it, yet it still has good investment opportunities. This is not hard rationing so long as AAI can issue stock. But perhaps it can't. Perhaps the founder and majority shareholder vetoes the idea from fear of losing control of the firm. Perhaps a stock issue would bring costly red tape or legal complications.[15]

This does not invalidate the NPV rule. AAI's *shareholders* can borrow or lend, sell their shares, or buy more. They have free access to security markets. The type of portfolio they hold is independent of AAI's financing or investment decisions. The only way AAI can help its shareholders is to make them richer. Thus, AAI should invest its available cash in the package of projects having the largest aggregate net present value.

A barrier between the firm and financial markets does not undermine net present value so long as the barrier is the *only* market imperfection. The important thing is that the firm's *shareholders* have free access to well-functioning financial markets.

The net present value rule *is* undermined when imperfections restrict shareholders' portfolio choice. Suppose that Nevada Aquaculture Inc. (NAI) is solely owned by its founder,

[14]Don't forget that in the Appendix to Chapter 1 we had to assume perfect financial markets to derive the NPV rule.

[15]A majority owner who is "locked in" and has much personal wealth tied up in AAI may be effectively cut off from financial markets. The NPV rule may not make sense to such an owner, though it will to the other shareholders.

Alexander Turbot. Mr. Turbot has no cash or credit remaining, but he is convinced that expansion of his operation is a high-NPV investment. He has tried to sell stock but has found that prospective investors, skeptical of prospects for fish farming in the desert, offer him much less than he thinks his firm is worth. For Mr. Turbot, financial markets hardly exist. It makes little sense for him to discount prospective cash flows at a market opportunity cost of capital.

SUMMARY

If you are going to persuade your company to use the net present value rule, you must be prepared to explain why other rules may *not* lead to correct decisions. That is why we have examined three alternative investment criteria in this chapter.

Some firms look at the book rate of return on the project. In this case, the company decides which cash payments are capital expenditures and picks the appropriate rate to depreciate these expenditures. It then calculates the ratio of book income to the book value of the investment. Few companies nowadays base their investment decision simply on the book rate of return, but shareholders pay attention to book measures of firm profitability, and some managers therefore look with a jaundiced eye on projects that would damage the company's book rate of return.

A few companies use the payback rule to make investment decisions. In other words, they accept only those projects that recover their initial investment within some specified period. Payback is an ad hoc rule. It ignores the timing of cash flows within the payback period, and it ignores subsequent cash flows entirely. It therefore takes no account of the opportunity cost of capital.

The internal rate of return (IRR) is defined as the rate of discount at which a project would have zero NPV. It is a handy measure and widely used in finance; you should therefore know how to calculate it. The IRR rule states that companies should accept any investment offering an IRR in excess of the opportunity cost of capital. Like net present value, the IRR rule is a technique based on discounted cash flows. It will therefore give the correct answer if properly used. The problem is that it is easily misapplied. There are four things to look out for:

1. *Lending or borrowing?* If a project offers positive cash flows followed by negative flows, NPV can *rise* as the discount rate is increased. You should accept such projects if their IRR is *less* than the opportunity cost of capital.
2. *Multiple rates of return.* If there is more than one change in the sign of the cash flows, the project may have several IRRs or no IRR at all.
3. *Mutually exclusive projects.* The IRR rule may give the wrong ranking of mutually exclusive projects that differ in economic life or in the scale of required investment. If you insist on using IRR to rank mutually exclusive projects, you must examine the IRR on each incremental investment.
4. *The cost of capital for near-term cash flows may be different from the cost for distant cash flows.* The IRR rule requires you to compare the project's IRR with the opportunity cost of capital. But sometimes there is more than one opportunity cost of capital. For example, if the term structure of interest rates is steeply upward-sloping, the financial manager may decide to use a lower discount rate for near than for distant cash flows. In these cases there is no simple yardstick for evaluating the IRR of a project.

In developing the NPV rule, we assumed that the company can maximize shareholder wealth by accepting every project that is worth more than it costs. But, if capital is strictly limited, then it may not be possible to take every project with a positive NPV. If capital is rationed in only one period, then the firm should follow a simple rule: Calculate each project's profitability index, which is the project's net present value per dollar of investment. Then pick the projects with the highest profitability indexes until you run out of capital. Unfortunately, this procedure fails when

capital is rationed in more than one period or when there are other constraints on project choice. The only general solution is linear programming.

Hard capital rationing always reflects a market imperfection—a barrier between the firm and financial markets. If that barrier also implies that the firm's shareholders lack free access to a well-functioning financial market, the very foundations of net present value crumble. Fortunately, hard rationing is rare for corporations in the United States. Many firms do use soft capital rationing, however. That is, they set up self-imposed limits as a means of financial planning and control.

FURTHER READING

For a survey of capital budgeting procedures, see:

J. Graham and C. Harvey, "How Do CFOs Make Capital Budgeting and Capital Structure Decisions?" *Journal of Applied Corporate Finance* 15 (Spring 2002), pp. 8–23.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect*. Answers to questions with an "*" are found in the Appendix.

1. **Payback***
   a. What is the payback period on each of the following projects?

Cash Flows ($)					
Project	C_0	C_1	C_2	C_3	C_4
A	−5,000	+1,000	+1,000	+3,000	0
B	−1,000	0	+1,000	+2,000	+3,000
C	−5,000	+1,000	+1,000	+3,000	+5,000

   b. Given that you wish to use the payback rule with a cutoff period of two years, which projects would you accept?
   c. If you use a cutoff period of three years, which projects would you accept?
   d. If the opportunity cost of capital is 10%, which projects have positive NPVs?
   e. "If a firm uses a single cutoff period for all projects, it is likely to accept too many short-lived projects." True or false?
   f. If the firm uses the discounted-payback rule, will it accept any negative-NPV projects? Will it turn down any positive-NPV projects?

2. **Payback** Consider the following projects:

Cash Flows ($)						
Project	C_0	C_1	C_2	C_3	C_4	C_5
A	−1,000	+1,000	0	0	0	0
B	−2,000	+1,000	+1,000	+4,000	+1,000	+1,000
C	−3,000	+1,000	+1,000	0	+1,000	+1,000

   a. If the opportunity cost of capital is 10%, which projects have a positive NPV?
   b. Calculate the payback period for each project.

c. Which project(s) would a firm using the payback rule accept if the cutoff period is three years?

d. Calculate the discounted payback period for each project.

e. Which project(s) would a firm using the discounted payback rule accept if the cutoff period is three years?

3. **Payback and IRR rules** Respond to the following comments:

a. "I like the IRR rule. I can use it to rank projects without having to specify a discount rate."

b. "I like the payback rule. As long as the minimum payback period is short, the rule makes sure that the company takes no borderline projects. That reduces risk."

4. **IRR** Write down the equation defining a project's internal rate of return (IRR). In practice how is IRR calculated?

5. **IRR***

a. Calculate the net present value of the following project for discount rates of 0, 50, and 100%:

Cash Flows ($)		
C_0	C_1	C_2
−6,750	+4,500	+18,000

b. What is the IRR of the project?

6. **IRR** Calculate the IRR (or IRRs) for the following project:

C_0	C_1	C_0	C_0
−3,000	+3,500	+4,000	−4,000

For what range of discount rates does the project have a positive NPV?

7. **IRR rule** You have the chance to participate in a project that produces the following cash flows:

Cash Flows ($)		
C_0	C_1	C_2
+5,000	+4,000	−11,000

The internal rate of return is 13%. If the opportunity cost of capital is 10%, would you accept the offer?

8. **IRR rule*** Consider a project with the following cash flows:

Cash Flows ($)		
C_0	C_1	C_2
−100	+200	−75

a. How many internal rates of return does this project have?

b. Which of the following numbers is the project IRR: (i) −50%; (ii) −12%; (iii) +5%; (iv) +50%?

c. The opportunity cost of capital is 20%. Is this an attractive project? Briefly explain.

9. **IRR rule*** Consider projects Alpha and Beta:

Cash Flows ($)				
Project	C_0	C_1	C_2	IRR (%)
Alpha	−400,000	+241,000	+293,000	21
Beta	−200,000	+131,000	+172,000	31

The opportunity cost of capital is 8%. Suppose you can undertake Alpha or Beta, but not both. Use the IRR rule to make the choice. (*Hint:* What's the incremental investment in Alpha?)

10. **IRR rule** Consider the following two mutually exclusive projects:

Cash flows ($)				
Project	C_0	C_1	C_2	C_3
A	−100	+60	+60	0
B	−100	0	0	+140

a. Calculate the NPV of each project for discount rates of 0%, 10%, and 20%. Plot these on a graph with NPV on the vertical axis and discount rate on the horizontal axis.

b. What is the approximate IRR for each project?

c. In what circumstances should the company accept project A?

d. Calculate the NPV of the incremental investment (B – A) for discount rates of 0%, 10%, and 20%. Plot these on your graph. Show that the circumstances in which you would accept A are also those in which the IRR on the incremental investment is less than the opportunity cost of capital.

11. **IRR rule** Mr. Cyrus Clops, the president of Giant Enterprises, has to make a choice between two possible investments:

Cash Flows ($ thousands)				
Project	C_0	C_1	C_2	IRR (%)
A	−400	+250	+300	23
B	−200	+140	+179	36

The opportunity cost of capital is 9%. Mr. Clops is tempted to take B, which has the higher IRR.

a. Explain to Mr. Clops why this is not the correct procedure.

b. Show him how to adapt the IRR rule to choose the best project.

c. Show him that this project also has the higher NPV.

12. **IRR rule** The Titanic Shipbuilding Company has a noncancelable contract to build a small cargo vessel. Construction involves a cash outlay of $250,000 at the end of each of the next two years. At the end of the third year the company will receive payment of $650,000. The company can speed up construction by working an extra shift. In this case there will be a cash outlay of $550,000 at the end of the first year followed by a cash payment of $650,000 at the end of the second year. Use the IRR rule to show the (approximate) range of opportunity costs of capital at which the company should work the extra shift.

13. IRR rule Plot the NPVs for the following projects for discount rates from 0% to 30%:

Project	C_0	C_1	C_2
A	– 100	20	100
B	−1,000	2260	−1,270
C	100	−50	−80
D	−1,080	2510	−1,500

a. Which one of these projects has no IRR?

b. One of the projects has two IRRs. Which is this project and what are the IRRs?

c. What are the IRRs of the other two projects?

d. Suppose projects A and C are mutually exclusive. If the cost of capital is 6%, which one would you accept?

e. If the cost of capital is very high, would you accept project C? Why or why not?

14. Investment criteria Consider the following two projects:

Cash flows	Project A	Project B
C_0	−$200	−$200
C_1	80	100
C_2	80	100
C_3	80	100
C_4	80	

a. If the opportunity cost of capital is 11%, which of these two projects would you accept (A, B, or both)?

b. Suppose that you can choose only one of these two projects. Which would you choose? The discount rate is still 11%.

c. Which one would you choose if the cost of capital is 16%?

d. What is the payback period of each project?

e. Is the project with the shortest payback period also the one with the highest NPV?

f. What are the internal rates of return on the two projects?

g. Does the IRR rule in this case give the same answer as NPV?

h. If the opportunity cost of capital is 11%, what is the profitability index for each project? Is the project with the highest profitability index also the one with the highest NPV? Which measure should you use to choose between the projects?

15. Profitability index Look again at projects D and E in Section 5-3. Assume that the projects are mutually exclusive and that the opportunity cost of capital is 10%.

a. Calculate the profitability index for each project.

b. Show how the profitability-index rule can be used to select the superior project.

16. Capital rationing* Suppose you have the following investment opportunities, but only $90,000 available for investment. Which projects should you take?

Project	NPV ($)	Investment ($)
1	5,000	10,000
2	5,000	5,000
3	10,000	90,000
4	15,000	60,000
5	15,000	75,000
6	3,000	15,000

17. **Capital rationing** Borgia Pharmaceuticals has $1 million allocated for capital expenditures. Which of the following projects should the company accept to stay within the $1 million budget? How much does the budget limit cost the company in terms of its market value? The opportunity cost of capital for each project is 11%.

Project	Investment ($ thousand)	NPV ($ thousand)	IRR (%)
1	300	66	17.2
2	200	–4	10.7
3	250	43	16.6
4	100	14	12.1
5	100	7	11.8
6	350	63	18.0
7	400	48	13.0

CHALLENGE PROBLEMS

18. **NPV and IRR rules** Some people believe firmly, even passionately, that ranking projects on IRR is OK if each project's cash flows can be reinvested at the project's IRR. They also say that the NPV rule "assumes that cash flows are reinvested at the opportunity cost of capital." Think carefully about these statements. Are they true? Are they helpful?

19. **Modified IRR** Look again at the project cash flows in Problem 6. Calculate the modified IRR as defined in footnote 5 in Section 5-3. Assume the cost of capital is 12%.

    Now try the following variation on the MIRR concept. Figure out the fraction x such that x times C_1 and C_2 has the same present value as (minus) C_3.

$$xC_1 + \frac{xC_2}{1.12} = -\frac{C_3}{1.12^2}$$

    Define the modified project IRR as the solution of

$$C_0 + \frac{(1-x)C_1}{1+\text{IRR}} + \frac{(1-x)C_2}{(1+\text{IRR})^2} = 0$$

    Now you have two MIRRs. Which is more meaningful? If you can't decide, what do you conclude about the usefulness of MIRRs?

20. **Capital rationing** Consider the following capital rationing problem:

    Set up this problem as a linear program and solve it.

You can allow partial investments, that is, $0 \leq x \leq 1$. Calculate and interpret the shadow prices[16] on the capital constraints.

Project	C_0	C_1	C_2	NPV
W	−10,000	−10,000	0	+6,700
X	0	−20,000	+5,000	+9,000
Y	−10,000	+ 5,000	+5,000	+0
Z	−15,000	+ 5,000	+4,000	−1,500
Financing available	20,000	20,000	20,000	

MINI-CASE

Vegetron's CFO Calls Again

(The first episode of this story was presented in Section 5-1.)

Later that afternoon, Vegetron's CFO bursts into your office in a state of anxious confusion. The problem, he explains, is a last-minute proposal for a change in the design of the fermentation tanks that Vegetron will build to extract hydrated zirconium from a stockpile of powdered ore. The CFO has brought a printout (Table 5.1) of the forecasted revenues, costs, income, and book rates of return for the standard, low-temperature design. Vegetron's engineers have just proposed an alternative high-temperature design that will extract most of the hydrated zirconium over a shorter period, five instead of seven years. The forecasts for the high-temperature method are given in Table 5.2.[17]

CFO: Why do these engineers always have a bright idea at the last minute? But you've got to admit the high-temperature process looks good. We'll get a faster payback, and the rate of return beats Vegetron's 9% cost of capital in every year except the first. Let's see, income is \$30,000 per year. Average investment is half the \$400,000 capital outlay, or \$200,000, so the average rate of return is 30,000/200,000, or 15%—a lot better than the 9% hurdle rate. The average rate of return for the low-temperature process is not that good, only 28,000/200,000, or 14%. Of course, we might get a higher rate of return for the low-temperature proposal if we depreciated the investment faster—do you think we should try that?

You: Let's not fixate on book accounting numbers. Book income is not the same as cash flow to Vegetron or its investors. Book rates of return don't measure the true rate of return.

CFO: But people use accounting numbers all the time. We have to publish them in our annual report to investors.

You: Accounting numbers have many valid uses, but they're not a sound basis for capital investment decisions. Accounting changes can have big effects on book income or rate of return, even when cash flows are unchanged.

Here's an example. Suppose the accountant depreciates the capital investment for the low-temperature process over six years rather than seven. Then income for years 1 to 6 goes down because depreciation is higher. Income for year 7 goes up because the depreciation for that

[16] A shadow price is the marginal change in the objective for a marginal change in the constraint.

[17] For simplicity we have ignored taxes. There will be plenty about taxes in Chapter 6.

	Year						
	1	2	3	4	5	6	7
1. Revenue	140	140	140	140	140	140	140
2. Operating costs	55	55	55	55	55	55	55
3. Depreciation[a]	57	57	57	57	57	57	57
4. Net income	28	28	28	28	28	28	28
5. Start-of-year book value[b]	400	343	286	229	171	114	57
6. Book rate of return (4 ÷ 5)	7%	8.2%	9.8%	12.2%	16.4%	24.6%	49.1%

TABLE 5.1 Income statement and book rates of return for low-temperature extraction of hydrated zirconium ($ thousands)

[a] Rounded. Straight-line depreciation over seven years is 400/7 = 57.14, or $57,140 per year.
[b] Capital investment is $400,000 in year 0.

	Year				
	1	2	3	4	5
1. Revenue	180	180	180	180	180
2. Operating costs	70	70	70	70	70
3. Depreciation[a]	80	80	80	80	80
4. Net income	30	30	30	30	30
5. Start-of-year book value[b]	400	320	240	160	80
6. Book rate of return (4 ÷ 5)	7.5%	9.4%	12.5%	18.75%	37.5%

TABLE 5.2 Income statement and book rates of return for high-temperature extraction of hydrated zirconium ($ thousands)

[a] Straight-line depreciation over five years is 400/5 = 80, or $80,000 per year.
[b] Capital investment is $400,000 in year 0.

year becomes zero. But there is no effect on year-to-year cash flows because depreciation is not a cash outlay. It is simply the accountant's device for spreading out the "recovery" of the up-front capital outlay over the life of the project.

CFO: So how do we get cash flows?

You: In these cases, it's easy. Depreciation is the only noncash entry in your spreadsheets (Tables 5.1 and 5.2), so we can just leave it out of the calculation. Cash flow equals revenue minus operating costs. For the high-temperature process, annual cash flow is:

Cash flow = revenue − operating cost = 180 − 70 = 110, or $110,000

CFO: In effect you're adding back depreciation because depreciation is a noncash accounting expense.

You: Right. You could also do it that way:

Cash flow = net income + depreciation = 30 + 80 = 110, or $110,000

CFO: Of course. I remember all this now, but book returns seem important when someone shoves them in front of your nose.

You: It's not clear which project is better. The high-temperature process appears to be less efficient. It has higher operating costs and generates less total revenue over the life of the project, but of course, it generates more cash flow in years 1 to 5.

CFO: Maybe the processes are equally good from a financial point of view. If so, we'll stick with the low-temperature process rather than switching at the last minute.

You: We'll have to lay out the cash flows and calculate NPV for each process.

CFO: OK, do that. I'll be back in a half hour—and I also want to see each project's true, DCF rate of return.

QUESTIONS

1. Are the book rates of return reported in Tables 5.1 and 5.2 useful inputs for the capital investment decision?
2. Calculate NPV and IRR for each process. What is your recommendation? Be ready to explain to the CFO.

CHAPTER 6

Making Investment Decisions with the Net Present Value Rule

In 2017, Intel announced plans to invest over $7 billion to produce 7-nanometer chips in its Chandler, Arizona, facility. How does a company such as Intel decide to go ahead with such a massive investment? We know the answer in principle. The company needs to forecast the project's cash flows and discount them at the opportunity cost of capital to arrive at the project's NPV. A project with a positive NPV increases shareholder value.

But those cash flow forecasts do not arrive on a silver platter. For example, Intel's managers would have needed answers to several basic questions. How soon can the new plant be brought into operation? How many semiconductor chips are likely to be sold each year and at what price? How much does the firm need to invest in the new facilities, and what is the likely production cost? How long will the chips stay in production, and what happens to the plant and equipment at the end of that time?

These predictions need to be pulled together to produce a single set of cash-flow forecasts. That requires careful tracking of taxes; changes in working capital; inflation; and the end-of-project salvage values of plant, property, and equipment. The financial manager must also ferret out hidden cash flows and take care to reject accounting entries that look like cash flows but truly are not.

Our first task in this chapter is to look at how to develop a set of project cash flows. We set out several rules of good financial practice. Later in the chapter, we work through a realistic and comprehensive example of a capital investment analysis.

This is the first chapter in which we grapple with the complexities of taxes. Therefore, we have added a section that includes an overview of corporate income taxes and the dramatic recent changes in the U.S. tax code.

We conclude the chapter by looking at how the financial manager should apply the present value rule when choosing between investment in plant and equipment with different economic lives. For example, suppose you must decide between machine Y with a 5-year useful life and a similar machine Z with a 10-year life. The present value of Y's lifetime investment and operating costs is naturally less than Z's because Z will last twice as long. Does that necessarily make Y the better choice? Of course not. You will find that when you are faced with this type of problem, the trick is to transform the present value of the cash flow into an *equivalent annual* flow—that is, the total cash per year from buying and operating the asset.

6-1 Applying the Net Present Value Rule

Many projects require a heavy initial outlay on new production facilities. But often the largest investments involve the acquisition of intangible assets. For example, U.S. banks invest huge sums annually in new information technology (IT) projects. Much of this expenditure goes to intangibles such as system design, programming, testing, and training. Think also of the huge expenditure by pharmaceutical companies on research and development (R&D). Merck, one of the largest pharmaceutical companies, spends more than $7 billion a year on R&D. The R&D cost of bringing *one* new prescription drug to market has been estimated at more than $2 billion.

Expenditures on intangible assets such as IT and R&D are investments just like expenditures on new plant and equipment. In each case, the company is spending money today in the expectation that it will generate a stream of future profits. Ideally, firms should apply the same criteria to all capital investments, regardless of whether they involve a tangible or intangible asset.

We have seen that an investment in any asset creates wealth if the discounted value of the future cash flows exceeds the up-front cost. Up to this point, however, we have glossed over the problem of *what* to discount. When you are faced with this problem, you should stick to five general rules:

1. Discount cash flows, not profits.
2. Discount *incremental* cash flows.
3. Treat inflation consistently.
4. Separate investment and financing decisions.
5. Forecast and deduct taxes.

We discuss each of these rules in turn.

Rule 1: Discount Cash Flows, Not Profits

The first and most important point: Net present value depends on the expected future cash flow. Cash flow is simply the difference between cash received and cash paid out. Many people nevertheless confuse cash flow with accounting income. Accounting income is intended to show how well the company is performing. Therefore, accountants *start* with "dollars in" and "dollars out," but to obtain accounting income, they adjust these inputs in two principal ways.

Capital Expenses When calculating expenditures, the accountant deducts *current* expenses but does not deduct *capital* expenses. There is a good reason for this. If the firm lays out a large amount of money on a big capital project, you do not conclude that the firm is performing poorly, even though a lot of cash is going out the door. Therefore, instead of deducting capital expenditure as it occurs, the accountant depreciates the outlay over several years.

That makes sense when judging firm performance, but it will get you into trouble when working out net present value. For example, suppose that you are analyzing an investment proposal. It costs $2,000 and is expected to provide a cash flow of $1,500 in the first year and $500 in the second. If the accountant depreciates the capital expenditure straight line over the two years, accounting income is $500 in year 1 and −$500 in year 2:

	Year 1	Year 2
Cash inflow	+$1,500	+$ 500
Less depreciation	− 1,000	− 1,000
Accounting income	+$ 500	−$ 500

Suppose you were given this forecast income and naïvely discounted it at 10%. NPV would appear positive:

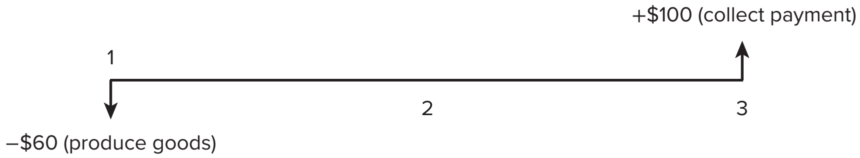
$$\text{Apparent NPV} = \frac{\$500}{1.10} + \frac{-\$500}{1.10^2} = \$41.32$$

This has to be nonsense. The project is obviously a loser. You are laying out $2,000 today and simply getting it back later. At any positive discount rate the project has a negative NPV.

The message is clear: When calculating NPV, state capital expenditures when they occur, not later when they show up as depreciation. To go from accounting income to cash flow, you need to add back depreciation (which is not a cash outflow) and subtract capital expenditure (which is a cash outflow).

Working Capital When measuring income, accountants try to show profit as it is *earned,* rather than when the company and its customers get around to paying their bills.

For example, consider a company that spends \$60 to produce goods in period 1. It sells these goods in period 2 for \$100, but its customers do not pay their bills until period 3. The following diagram shows the firm's cash flows. In period 1 there is a cash *outflow* of \$60. Then, when customers pay their bills in period 3, there is an *inflow* of \$100.



It would be misleading to say that the firm was running at a loss in period 1 (when cash flow was negative) or that it was extremely profitable in period 3 (when cash flow was positive). Therefore, the accountant looks at when the sale was made (period 2 in our example) and gathers together all the revenues and expenses associated with that sale. In the case of our company, the accountant would show for period 2.

Revenue	\$100
Less cost of goods sold	− 60
Income	\$ 40

Of course, the accountant cannot ignore the actual timing of the cash expenditures and payments. So the \$60 cash outlay in the first period will be treated not as an expense but as an *investment* in inventories. Subsequently, in period 2, when the goods are taken out of inventory and sold, the accountant shows a \$60 *reduction* in inventories.

The accountant also does not ignore the fact that the firm has to wait to collect on its bills. When the sale is made in period 2, the accountant will record accounts receivable of \$100 to show that the company's customers owe \$100 in unpaid bills. Later, when the customers pay those bills in period 3, accounts receivable are reduced by that \$100.

To go from the figure for income to the actual cash flows, you need to add back these changes in inventories and receivables:

	Period		
	1	2	3
Accounting income	0	+40	0
− Investment in inventories	−60	+60	0
− Investment in receivables	0	−100	+100
= Cash flow	−60	0	+100

Net working capital (often referred to simply as *working capital*) is the difference between a company's short-term assets and liabilities. Accounts receivable and inventories of raw materials, work in progress, and finished goods are the principal short-term assets. The principal short-term liabilities are accounts payable (bills that *you* have not paid) and taxes that have been incurred but not yet paid.[1]

Most projects entail an investment in working capital. Each period's change in working capital should be recognized in your cash-flow forecasts.[2] By the same token, when the project comes to an end, you can usually recover some of the investment. This results in a cash inflow. (In our simple example the company made an investment in working capital of $60 in period 1 and $40 in period 2. It made a *disinvestment* of $100 in period 3, when the customers paid their bills.)

Working capital is a common source of confusion in capital investment calculations. Here are the most common mistakes:

1. *Forgetting about working capital entirely.* We hope that you do not fall into that trap.
2. *Forgetting that working capital may change during the life of the project.* Imagine that you sell $100,000 of goods a year and customers pay on average six months late. You therefore have $50,000 of unpaid bills. Now you increase prices by 10%, so revenues increase to $110,000. If customers continue to pay six months late, unpaid bills increase to $55,000, and so you need to make an *additional* investment in working capital of $5,000.
3. *Forgetting that working capital is recovered at the end of the project.* When the project comes to an end, inventories are run down, any unpaid bills are (you hope) paid off, and you recover your investment in working capital. This generates a cash *inflow.*

Rule 2: Discount Incremental Cash Flows

The value of a project depends on *all* the additional cash flows that follow from project acceptance. Here are some things to watch for when you are deciding which cash flows to include.

Include All Incidental Effects It is important to consider a project's effects on the remainder of the firm's business. For example, suppose Sony proposes to launch PlayStation X, a new version of its videogame console. Demand for the new product will almost certainly cut into sales of Sony's existing consoles. This incidental effect needs to be factored into the incremental cash flows. Of course, Sony may reason that it needs to go ahead with the new product because its existing product line is likely to come under increasing threat from competitors. So, even if it decides not to produce the new PlayStation, there is no guarantee that sales of the existing consoles will continue at their present level. Sooner or later, they will decline.

Sometimes a new project will *help* the firm's existing business. Suppose that you are the financial manager of an airline that is considering opening a new short-haul route from Harrisburg, Pennsylvania, to Chicago's O'Hare Airport. When considered in isolation, the new route may have a negative NPV. But once you allow for the additional business that the new route brings to your other traffic out of O'Hare, it may be a very worthwhile investment.

Do Not Confuse Average with Incremental Payoffs Most managers naturally hesitate to throw good money after bad. For example, they are reluctant to invest more money in a losing division. But occasionally you will encounter turnaround opportunities in which the *incremental* NPV from investing in a loser is strongly positive.

Conversely, it does not always make sense to throw good money after good. A division with an outstanding past profitability record may have run out of good opportunities. You would not pay a large sum for a 20-year-old horse, sentiment aside, regardless of how many races that horse had won or how many champions it had sired.

[1] If you delay paying *your* bills, your investment in net working capital is reduced. When you finally pay up, it is increased.

[2] Holdings of cash and marketable securities are also short-term assets and debt due within a year is a short-term liability. These are *not* relevant to your capital budgeting calculations.

Here is another example illustrating the difference between average and incremental returns: Suppose that a railroad bridge is in urgent need of repair. With the bridge the railroad can continue to operate; without the bridge it can't. In this case, the payoff from the repair work consists of all the benefits of operating the railroad. The incremental NPV of such an investment may be enormous. Of course, these benefits should be net of all other costs and all subsequent repairs; otherwise, the company may be misled into rebuilding an unprofitable railroad piece by piece.

Forecast Product Sales but also Recognize After-Sales Cash Flows Financial managers should forecast all incremental cash flows generated by an investment. Sometimes these incremental cash flows last for decades. When GE commits to the design and production of a new jet engine, the cash inflows come first from the sale of engines and then from service and spare parts. A jet engine will be in use for 30 years. Over that period revenues from service and spare parts will be roughly seven times the engine's purchase price.

Many other manufacturing companies depend on the revenues that come *after* their products are sold. For example, the consulting firm Accenture estimates that services and parts typically account for about 25% of revenues and 50% of profits for auto companies.[3]

Include Opportunity Costs The cost of a resource may be relevant to the investment decision even when no cash changes hands. For example, suppose a new manufacturing operation uses land that could otherwise be sold for $100,000. This resource is not free: It has an opportunity cost, which is the cash it could generate for the company if the project were rejected and the resource were sold or put to some other productive use.

This example prompts us to warn you against judging projects on the basis of "before versus after." The proper comparison is "with or without." A manager comparing before versus after might not assign any value to the land because the firm owns it both before and after:

Before	Take Project	After	Cash Flow, Before versus After
Firm owns land	→	Firm still owns land	0

The proper comparison, with or without, is as follows:

With	Take Project	After	Cash Flow, with Project
Firm owns land	→	Firm still owns land	0

Without	Do Not Take Project	After	Cash Flow, without Project
	→	Firm sells land for $100,000	$100,000

Comparing the two possible "afters," we see that the firm gives up $100,000 by undertaking the project. This reasoning still holds if the land will not be sold but is worth $100,000 to the firm in some other use.

Sometimes opportunity costs may be very difficult to estimate; however, where the resource can be freely traded, its opportunity cost is simply equal to the market price. Consider a widely used aircraft such as the Boeing 737. Secondhand 737s are regularly traded, and their prices

[3]Accenture, "Refocusing on the After-Sales Market," 2010.

are quoted on the web. So, if an airline needs to know the opportunity cost of continuing to use one of its 737s, it just needs to look up the market price of a similar plane. The opportunity cost of using the plane is equal to the cost of buying an equivalent aircraft to replace it.

Forget Sunk Costs Sunk costs are like spilled milk: They are past and irreversible outflows. Because sunk costs are bygones, they cannot be affected by the decision to accept or reject the project, and so they should be ignored.

Take the case of the James Webb Space Telescope. It was originally supposed to launch in 2011 and cost $1.6 billion. But the project became progressively more expensive and further behind schedule. Latest estimates put the cost at $8.8 billion and a launch date of 2019. When Congress debated whether to cancel the program, supporters of the project argued that it would be foolish to abandon a project on which so much had already been spent. Others countered that it would be even more foolish to continue with a project that had proved so costly. Both groups were guilty of the *sunk-cost fallacy;* the money that had already been spent by NASA was irrecoverable and, therefore, irrelevant to the decision to terminate the project.

Beware of Allocated Overhead Costs We have already mentioned that the accountant's objective is not always the same as the investment analyst's. A case in point is the allocation of overhead costs. Overheads include such items as supervisory salaries, rent, heat, and light. These overheads may not be related to any particular project, but they have to be paid for somehow. Therefore, when the accountant assigns costs to the firm's projects, a charge for overhead is usually made. Now our principle of incremental cash flows says that in investment appraisal we should include only the *extra* expenses that would result from the project. A project may generate extra overhead expenses; then again, it may not. We should be cautious about assuming that the accountant's allocation of overheads represents the true extra expenses that would be incurred.

Remember Salvage Value When the project comes to an end, you may be able to sell the plant and equipment or redeploy the assets elsewhere in the business. If the equipment is sold, you must pay tax on the difference between the sale price and the book value of the asset. The salvage value (net of any taxes) represents a positive cash flow to the firm.

Some projects have significant shutdown costs, in which case the final cash flows may be *negative.* For example, the mining company, FCX, has earmarked $451 million to cover the future reclamation and closure costs of its New Mexico mines.

Rule 3: Treat Inflation Consistently

As we pointed out in Chapter 3, interest rates are usually quoted in *nominal* rather than *real* terms. For example, if you buy an 8% Treasury bond, the government promises to pay you $80 interest each year, but it does not promise what that $80 will buy. Investors take inflation into account when they decide what is an acceptable rate of interest.

If the discount rate is stated in nominal terms, then consistency requires that cash flows should also be estimated in nominal terms, taking account of trends in selling price, labor and materials costs, and so on. This calls for more than simply applying a single assumed inflation rate to all components of cash flow. Labor costs per hour of work, for example, normally increase at a faster rate than the consumer price index because of improvements in productivity. Tax savings from depreciation do *not* increase with inflation; they are constant in nominal terms because tax law in most countries allows only the original cost of assets to be depreciated.

Of course, there is nothing wrong with discounting real cash flows at a real discount rate. In fact, this is standard procedure in countries with high and volatile inflation. Here is a simple example showing that real and nominal discounting, properly applied, always give the same present value.

Suppose your firm usually forecasts cash flows in nominal terms and discounts at a 15% nominal rate. In this particular case, however, you are given project cash flows in real terms, that is, current dollars:

Real Cash Flows ($ thousands)			
C_0	$_1$	C_2	C_3
−100	+35	+50	+30

It would be inconsistent to discount these real cash flows at the 15% nominal rate. You have two alternatives: Either restate the cash flows in nominal terms and discount at 15%, or restate the discount rate in real terms and use it to discount the real cash flows.

Assume that inflation is projected at 10% a year. Then the cash flow for year 1, which is $35,000 in current dollars, will be 35,000 × 1.10 = $38,500 in year-1 dollars. Similarly, the cash flow for year 2 will be 50,000 × $(1.10)^2$ = $60,500 in year-2 dollars, and so on. If we discount these nominal cash flows at the 15% nominal discount rate, we have

$$\text{NPV} = -100 + \frac{38.5}{1.15} + \frac{60.5}{(1.15)^2} + \frac{39.9}{(1.15)^3} = 5.5, \text{ or } \$5{,}500$$

Instead of converting the cash-flow forecasts into nominal terms, we could convert the discount rate into real terms by using the following relationship:

$$\text{Real discount rate} = \frac{1 + \text{nominal discount rate}}{1 + \text{inflation rate}} - 1$$

In our example, this gives

$$\text{Real discount rate} = \frac{1.15}{1.10} - 1 = .045, \text{ or } 4.5\%$$

If we now discount the real cash flows by the real discount rate, we have an NPV of $5,500, just as before:

$$\text{NPV} = -100 + \frac{35}{1.045} + \frac{50}{(1.045)^2} + \frac{30}{(1.045)^3} = 5.5, \text{ or } \$5{,}500$$

The message of all this is quite simple. Discount nominal cash flows at a nominal discount rate. Discount real cash flows at a real rate. *Never* mix real cash flows with nominal discount rates or nominal flows with real rates.

Rule 4: Separate Investment and Financing Decisions

Suppose you finance a project partly with debt. How should you treat the proceeds from the debt issue and the interest and principal payments on the debt? Answer: You should *neither* subtract the debt proceeds from the required investment *nor* recognize the interest and principal payments on the debt as cash outflows. Regardless of the actual financing, you should view the project as if it were all-equity-financed, treating all cash outflows required for the project as coming from stockholders and all cash inflows as going to them.

This procedure focuses exclusively on the *project* cash flows, not the cash flows associated with alternative financing schemes. It, therefore, allows you to separate the analysis of the investment decision from that of the financing decision. We explain how to recognize the effect of financing choices on project values in Chapter 19.

Rule 5: Remember to Deduct Taxes

Taxes are an expense just like wages and raw materials. Therefore, cash flows should be estimated on an after-tax basis. Subtract cash outflows for taxes from pretax cash flows and discount the net amount.

Some firms do not deduct tax payments. They try to offset this mistake by discounting the pretax cash flows at a rate that is higher than the cost of capital. Unfortunately, there is no reliable formula for making such adjustments to the discount rate.

Be careful to subtract cash taxes. Cash taxes paid are usually different from the taxes reported on the income statement provided to shareholders. For example, the shareholder accounts typically assume straight-line depreciation instead of the accelerated depreciation allowed by the U.S. tax code. We will highlight the differences between straight-line and accelerated depreciation later in this chapter.

The next section takes a broader look at corporate income taxes and the recent changes in the U.S. tax code.

6-2 Corporate Income Taxes

Look at Table 6.1, which shows corporate income tax rates in 11 countries. These are the tax rates imposed by the national governments, but corporations may also need to pay tax to a regional government. For example, in Canada, the provincial governments levy an additional tax of between 11% and 16%. In the United States, states and some municipalities also impose an extra layer of corporate tax that averages around 4%. To complicate matters further, in many countries, the first tranche of income may be taxed at a lower rate, or special arrangements may apply to some types of business.

Tax rates change over time, sometimes dramatically. For example, the U.K. has cut its corporate tax rate from 30% in 1998 to 19% today. The U.S. reduced its rate from 35% to 21% starting in 2018. This rate reduction was one of several important changes in U.S. corporate income taxes. We summarize the changes now.

Country	Corporate Tax Rate (%)
Australia	30
Brazil	34
Canada	15
China	25
France	33
Germany	16
India	30
Ireland	13
Japan	34
United Kingdom	19
United States	21

TABLE 6.1 National corporate tax rates

Source: PWC, Worldwide Tax Summaries: Corporate Taxes, 2017/2018, www.pwc.com/taxsummaries.

U.S. Corporate Income Tax Reform

The U.S. Tax Cuts and Jobs Act was passed in December 2017 and implemented immediately in 2018. Suddenly, the corporate tax rate dropped from 35% to 21%. But there were several other important changes.[4]

Depreciation Before 2018, when calculating taxable income, U.S. corporations were allowed to deduct an immediate bonus depreciation of 50% of the asset's cost. The fraction of the investment not covered by this bonus depreciation was then depreciated over the following years using the modified accelerated cost recovery system (MACRS), a form of accelerated depreciation. ("Accelerated" means that depreciation is front-loaded: higher in the early years of an asset's life but lower as the asset ages. Straight-line depreciation is the same in all years.) But the new tax law allows companies to take bonus depreciation sufficient to write off 100% of investment immediately—the ultimate in accelerated depreciation. With 100% bonus depreciation, the firm can treat investments in plant and equipment as immediate expenses.

Bonus depreciation is a temporary provision, however. It is scheduled for phase-out starting in 2023. By 2027, it will be gone. We will have to wait and see what depreciation schedules apply to investments not covered by 100% bonus depreciation. Perhaps it will be that old standby MACRS. We discuss MACRS and other forms of accelerated depreciation in the next section.

Investment in real estate does not qualify for bonus or accelerated depreciation. It is depreciated straight-line over periods of 15 years or more.

Amortization of Research Expenses U.S. companies can now could write off most outlays for R&D as immediate expenses. Starting in 2022, most R&D investments must be amortized (depreciated) over a five-year period. Many observers were puzzled by this change. If investments in plant and equipment now (2018–2022) qualify for immediate expensing, why must investments in R&D, which used to be expensed, be put on the balance sheet and amortized?

Tax Carry-Forwards When a corporation makes a profit, it pays tax. But what happens when it suffers a loss? In 2017 and earlier, U.S. corporations could carry back losses to recover taxes paid on the prior two years' income. Starting in 2018, carry-backs are no longer allowed. But corporations can carry forward losses indefinitely, using the losses to offset up to 80% of future years' income. Suppose, for example, that a manufacturer of gargle blasters loses $100,000 in 2018 but earns $100,000 in 2019 and 2020. It pays no tax in 2018, but carries forward the loss. In 2019, it uses $80,000 of the loss to offset income, paying tax of $4,200 (21% of $20,000). In 2020, it uses the remaining $20,000 carried forward, paying tax of $16,800 (21% of $80,000).

Limits on Interest Deductions U.S. tax law treats interest on debt as a tax-deductible expense. In Chapters 17 and 18, we will show that the resulting *interest tax shields* favor debt over equity financing. But interest deductions are now (2018–2021) limited to 30% of taxable income before depreciation and amortization, though unused deductions can be carried forward and used in later years. From 2022 on, interest deductions are limited to 30% of taxable income *after* depreciation and amortization. (There are exceptions for small businesses, car dealerships, farmers, and some other taxpayers.) In other words, the limit from 2018–2021 is 30% of taxable EBITDA (earnings before interest, taxes, depreciation, and amortization); from 2022, it is 30% of taxable EBIT (earnings before interest and taxes). EBIT is smaller than EBITDA, so the restriction on interest deductions is tighter post-2021.

It appears that most large U.S. corporations will be safely below the 30% limits. But those corporations that do hit the limits may have to rethink their valuation methods and financing strategies. We cover these issues in Chapters 18, 19, and 25.

[4] The Tax Cuts and Jobs Act is much more extensive and complicated than the changes that we outline here. For example, we say nothing about changes in personal income taxes. We are not offering comprehensive tax advice, just noting the most important changes for corporate finance.

Territorial versus Worldwide Taxation Most countries have *territorial* corporate income taxes: They tax income earned in their own countries but not outside their borders. The United States switched over to a territorial system in 2018.

Before the switch, the United States taxed U.S. corporations' *worldwide* income, which had some unfortunate consequences. To see why it mattered, think of a U.S. and a Canadian company, both operating in the United States and in Canada before the U.S. tax reform. Both companies paid U.S. taxes at 35% on their U.S. income and Canadian taxes at 15% on their Canadian income. But the U.S company owed an additional 20% in U.S. taxes when its Canadian income was repatriated. Thus, the U.S. company's total tax rate on its Canadian profits added up to 35%, far in excess of the 15% rate paid by the Canadian company on its Canadian profits.

The U.S. company could defer payment of the 20% additional U.S. tax by refusing to bring its Canadian profits home. That is exactly what U.S. corporations did. As we will see in Chapter 30, Apple, Microsoft, Alphabet, and several large pharmaceutical companies stored up mountains of cash in low-tax foreign jurisdictions. Once the U.S. switched to a territorial tax in 2018, these companies had no incentive to make their cash mountains higher. They were, however, subject to a one-time tax of 15.5% on overseas profits accumulated through the end of 2017. For example, Apple announced that it would pay a tax of $38 billion to repatriate cumulative foreign profits of $252 billion.

U.S. taxation of worldwide income also affected mergers and acquisitions. Suppose the U.S. company in our example bought the Canadian company before 2018, when the United States moved to the territorial system. The Canadian company's home operations would then be owned by the U.S. company and subject to the U.S. worldwide tax. But if the Canadian company bought the U.S. company, the profits from the Canadian operations that the U.S. company used to own would escape the worldwide tax. Only the Canadian tax of 15% is paid. If there were a merger, it was clearly better for the Canadian company to be the buyer.

Thus, worldwide taxation rewarded foreign acquisitions of U.S. companies. Some U.S. companies arranged *inversions,* which were takeovers designed so that the foreign party was treated as the buyer. For example, Pfizer's proposed 2016 merger with the smaller Irish company Allergen was designed to move the combined company's headquarters to Ireland, where the corporate tax rate was only 13%. The deal was abandoned after stubborn resistance by the U.S. Treasury. But if the Pfizer-Allergen deal resurfaced today, there would be no tax motive to move the headquarters to Ireland because the United States no longer taxes Pfizer's foreign profits.

6-3 Example—IM&C's Fertilizer Project

The Three Elements of Project Cash Flows

You can think of an investment project's cash flow as composed of three elements:

$$\begin{aligned}\text{Total cash flow} &= \text{cash flow from capital investment}\\ &\quad + \text{operating cash flow}\\ &\quad + \text{cash flow from changes in working capital}\end{aligned}$$

Capital Investment To get a project off the ground, a company typically makes an up-front investment in plant, equipment, research, start-up costs, and diverse other outlays. This expenditure is a negative cash flow—negative because cash goes out the door.

When the project comes to an end, the company can either sell the plant and equipment or redeploy it elsewhere in its business. This salvage value (net of any taxes if the plant and equipment is sold) is a positive cash flow. However, remember our earlier comment that final cash flows can be negative if there are significant shutdown costs.

Operating Cash Flow Operating cash flow consists of the net increase in sales revenue brought about by the new project less outlays for production, marketing, distribution, and other incremental costs. Incremental taxes are likewise subtracted.

$$\text{Operating cash flow} = \text{revenues} - \text{expenses} - \text{taxes}$$

Many investments do not produce any additional revenues; they are simply designed to reduce the costs of the company's existing operations. Such projects also contribute to the firm's operating cash flow. The after-tax cost saving is a positive addition to the cash flow.

Don't forget that the depreciation charge is not a cash flow. It affects the tax that the company pays, but the company does not send anyone a check for depreciation, and it should not be deducted when calculating operating cash flow.

Investment in Working Capital When a company builds up inventories of raw materials or finished products, this investment in inventories requires cash. Cash is also absorbed when customers are slow to pay their bills; in this case the firm makes an investment in accounts receivable. On the other hand, cash is preserved when the firm can delay paying its bills. Accounts payable are in a way a source of financing.

Investment in working capital, just like investment in plant and equipment, represents a negative cash flow. On the other hand, later in the project's life, as inventories are sold and accounts receivable are collected, working capital is reduced and the firm enjoys a positive cash flow.

Forecasting the Fertilizer Project's Cash Flows

As the newly appointed financial manager of International Mulch and Compost Company (IM&C), you are about to analyze a proposal for marketing guano as a garden fertilizer. (IM&C's planned advertising campaign features a rustic gentleman who steps out of a vegetable patch singing, "All my troubles have guano way.")[5]

Table 6.2 shows the forecasted cash flows from the project. All the entries in the table are nominal. In other words, the forecasts that you have been given take into account the likely effect of inflation on revenues and costs. We assume initially that for tax purposes the company uses straight-line depreciation. In other words, when it calculates each year's taxable income, it deducts one-sixth of the initial investment.

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The calculation in panel B of profit after tax is similar to the calculation in IM&C's financial statements. There is one important difference. When calculating the depreciation figure in the published income statement, IM&C may choose to depreciate the plant and equipment to its likely salvage value. By contrast, IRS rules for calculating the company's tax liability always assume that the plant and equipment has a salvage value of zero.

Capital Investment Rows 1 through 4 of Table 6.2 show the cash flows from the investment in fixed assets. The project requires an investment of $12 million in plant and machinery. IM&C expects to sell the equipment in year 7 for $1.949 million. Any difference between this figure and the book value of the equipment is a taxable gain. By year 7, IM&C has fully depreciated the equipment, so the company will be taxed on a capital gain of $1.949 million. If the tax rate is 21%, the company will pay tax of $.21 \times 1.949 = \$0.409$ million, and the net cash flow from the sale of equipment will be $1.949 - 0.409 = \$1.540$ million. This is shown in rows 2 and 3 of the table.

Operating Cash Flow Panel B of Table 6.2 show the calculation of the operating cash flow from the guano project. Operating cash flow consists of revenues from the sale of guano less

[5]Sorry.

	Year:	0	1	2	3	4	5	6	7
	Panel A Capital Investment								
1	Cash flow from investment in fixed assets	–12,000							
2	Sale of fixed assets								1,949
3	Less tax on sale								409[a]
4	Cash flow from capital investment (1 + 2 – 3)	–12,000							1,540
	Panel B Operating Cash Flow								
5	Revenues		523	12,887	32,610	48,901	35,834	19,717	
6	Cost of goods sold		837	7,729	19,552	29,345	21,492	11,830	
7	Other costs[b]	4,000	2,200	1,210	1,331	1,464	1,611	1,772	
8	Depreciation[c]		2,000	2,000	2,000	2,000	2,000	2,000	
9	Pretax profit (5 – (6 + 7) – 8)	–4,000	–4,514	1,948	9,727	16,092	10,731	4,115	
10	Tax (.21 × 9)	–840[d]	–948	409	2,043	3,379	2,254	864	
11	Profit after tax (9 – 10)	–3,160	–3,566	1,539	7,684	12,713	8,477	3,251	
12	Operating cash flow (11 + 8)	–3,160	–1,566	3,539	9,684	14,713	10,477	5,251	
	Panel C Investment in Working Capital								
13	Working capital		550	1,289	3,261	4,890	3,583	2,002	0
14	Change in working capital		550	739	1,972	1,629	–1,307	–1,581	–2,002
15	Cash flow from investment in working capital (–14)		–550	–739	–1,972	–1,629	1,307	1,581	2,002
	Panel D Project Valuation								
16	Total project cash flow (4 + 12 + 15)	–15,160	–2,116	2,800	7,712	13,084	11,784	6,832	3,542
17	Discount factor at 20%	1.0	0.833	0.694	0.579	0.482	0.402	0.335	0.279
18	Discounted cash flows (16 × 17)	–15,160	–1,763	1,944	4,463	6,310	4,736	2,288	988
19	NPV	+3,806							

TABLE 6.2 Calculating the cash flows and net present value of IM&C's guano project assuming straight-line depreciation ($ thousands)

[a] The asset has been entirely depreciated for tax purposes and the entire sales price is subject to tax.
[b] Start-up costs in years 0 and 1, and general and administrative costs in years 1–6.
[c] Depreciation is calculated straight line on the initial investment of $12 million.
[d] A negative tax payment means a cash inflow, assuming that IM&C can use the tax loss on the guano project to shield income from the rest of its business.

the cash expenses of production and any taxes. Taxes are calculated on profits net of depreciation. Thus, if the tax rate is 21%,

$$\text{Tax} = .21 \times (\text{sales} - \text{cash expenses} - \text{depreciation})$$

We assume in this first-pass table that the company uses straight-line depreciation. This means that, if the depreciable life of the equipment is six years, IM&C can deduct from profits one-sixth of the initial $12 million investment. Thus, row 8 shows that straight-line depreciation in each year is

$$\text{Annual depreciation} = (1/6 \times 12.0) = \$2.0 \text{ million}$$

Pretax profits and taxes are shown in rows 9 and 10. For example, in year 2

$$\text{Pretax profit} = 12.887 - (7.729 + 1.210) - 2.000 = \$1.948 \text{ million}$$

$$\text{Tax} = .21 \times 1.948 = \$0.409 \text{ million}$$

Once we have calculated taxes, it is a simple matter to calculate operating cash flow. Thus,

$$\text{Operating cash flow in year 2} = \text{revenues} - \text{cash expenses} - \text{taxes}$$

$$= 12.887 - (7.729 + 1.210) - 0.409 = \$3.539 \text{ million}^{6}$$

Notice that, when calculating operating cash flow, we ignored the possibility that the project may be partly financed by debt. Following our earlier Rule 4, we did not deduct any debt proceeds from the original investment, and we did not deduct interest payments from the cash inflows. Standard practice forecasts cash flows as if the project is all-equity financed. Any additional value resulting from financing decisions is considered separately.

Investment in Working Capital You can see from Table 6.2 that working capital increases in the early and middle years of the project. Why is this? There are several possible reasons:

1. Sales recorded on the income statement overstate actual cash receipts from guano shipments because sales are increasing and customers are slow to pay their bills. Therefore, accounts receivable increase.
2. It takes several months for processed guano to age properly. Thus, as projected sales increase, larger inventories have to be held in the aging sheds.
3. An offsetting effect occurs if payments for materials and services used in guano production are delayed. In this case accounts payable will increase.

Thus, the additional investment in working capital can be calculated as:

$$\begin{array}{c} \text{Additional} \\ \text{investment in} \\ \text{working capital} \end{array} = \begin{array}{c} \text{increase in} \\ \text{inventory} \end{array} + \begin{array}{c} \text{increase in} \\ \text{accounts} \\ \text{receivable} \end{array} - \begin{array}{c} \text{increase in} \\ \text{accounts} \\ \text{payable} \end{array}$$

There is an alternative to worrying about changes in working capital. You can estimate cash flow directly by counting the dollars coming in from customers and deducting the dollars going out to suppliers. You would also deduct all cash spent on production, including cash spent for goods held in inventory. In other words,

1. If you replace each year's sales with that year's cash payments received from customers, you don't have to worry about accounts receivable.
2. If you replace cost of goods sold with cash payments for labor, materials, and other costs of production, you don't have to keep track of inventory or accounts payable.

However, you would still have to construct a projected income statement to estimate taxes.

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[6] There are several alternative ways to calculate operating cash flow. For example, you can add depreciation back to the after-tax profit:

$$\text{Operating cash flow} = \text{after-tax profit} + \text{depreciation}$$

Thus, in year 2 of the guano project:

$$\text{Operating cash flow} = 1.539 + 2.000 = \$3.539 \text{ million}$$

Another alternative is to calculate after-tax profit assuming *no* depreciation, and then to add back the tax saving provided by the depreciation allowance:

$$\text{Operating cash flow} = (\text{revenues} - \text{expenses}) \times (1 - \text{tax rate}) + (\text{depreciation} \times \text{tax rate})$$

Thus, in year 2 of the guano project:

$$\text{Operating cash flow} = (12.887 - 7.729 - 1.210) \times (1 - .21) + (2.000 \times .21) = \$3.539 \text{ million.}$$

Project Valuation Rows 16 to 19 of Table 6.2 show the calculation of project NPV. Row 16 shows the total cash flow from IM&C's project as the sum of the capital investment, operating cash flow, and investment in working capital. IM&C estimates the opportunity cost of capital for projects of this type as 20%.

Remember that to calculate the present value of a cash flow in year t you can either divide the cash flow by $(1 + r)^t$ or you can multiply by a discount factor that is equal to $1/(1 + r)^t$. Row 17 shows the discount factors for a 20% discount rate, and Row 18 multiplies the discount factor by the cash flow to give each flow's present value. When all the cash flows are discounted and added up, the project is seen to offer a net present value of $3.806 million.

Accelerated Depreciation and First-Year Expensing

Depreciation is a noncash expense; it is important only because it reduces taxable income. It provides an annual *tax shield* equal to the product of depreciation and the marginal tax rate. In the case of IM&C:

$$\text{Annual tax shield} = \text{depreciation} \times \text{tax rate} = 2{,}000 \times .21 = 420.0, \text{ or } \$420{,}000.$$

The present value of these tax shields ($420,000 for six years) is $1,397,000 at a 20% discount rate.

In Table 6.2 we assumed that IM&C was required to use straight-line depreciation, which allowed it to write off a fixed proportion of the initial investment each year. This is the most common method of depreciation, but some countries, including the United States, permit firms to depreciate their investments more rapidly.

There are several different methods of accelerated depreciation. For example, firms may be allowed to use the double-declining-balance method. Suppose that IM&C is permitted to use double-declining-balance depreciation. In this case, it can deduct not one-sixth, but 2 × 1/6 = 1/3 of the remaining book value of the investment in each year.[7] Therefore, in year 1, it deducts depreciation of 12/3 = $4 million, and the written-down value of the equipment falls to 12 – 4 = $8 million. In year 2, IM&C deducts depreciation of 8/3 = $2.7 million, and the written-down value is further reduced to $8 – 2.7 = $5.3 million. In year 5, IM&C observes that depreciation would be higher if it could switch to straight-line depreciation and write off the balance of $2.4 million over the remaining two years of the equipment's life. If this is permitted, IM&C's depreciation allowance each year would be as follows:

	Year					
	1	2	3	4	5	6
Written-down value, start of year ($ millions)	12	8	5.3	3.6	2.4	1.2
Depreciation ($ millions)	12/3 = 4	8/3 = 2.7	5.3/3 = 1.8	3.6/3 = 1.2	2.4/2 = 1.2	1.2
Written-down value, end of year ($ millions)	12 – 4 = 8	8 – 2.7 = 5.3	5.3 – 1.8 = 3.6	3.6 – 1.2 = 2.4	2.4 – 1.2 = 1.2	1.2 – 1.2 = 0

The present value of the tax shields with double-declining-balance depreciation is $1.608 million, $212,000 million higher than if IM&C was restricted to straight-line depreciation.

[7] IM&C's new plant and equipment has a life of six years. Therefore, with double-declining-balance, it can depreciate each year 2 × (1/6) = 1/3 of the asset's written-down value. If, for example, IM&C was allowed only 150% declining balance, it would be able to depreciate each year 1.5 × (1/6) = 1/4 of the written-down value.

From 1986 to the end of 2017, U.S. companies used a slight variation of the double-declining balance method, called the modified accelerated cost recovery system (MACRS).[8] But the 2017 Tax Cuts and Jobs Act offered companies bonus depreciation sufficient to write off 100% of their investment expenditures in the year that they come on line. Table 6.3 recalculates the NPV of the guano project, assuming that the full $12 million investment can be depreciated immediately.

We initially assumed that the guano project could be depreciated straight-line over six years. This resulted in an NPV of $3.806 million. We then calculated that if IM&C could use the double-declining-balance method, NPV would increase by $212,000 to $4.018 million. Finally, Table 6.3 shows that full first-year expensing introduced in the 2017 tax reform would increase NPV further to $4.929 million.

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MACRS depreciation system

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		Period							
		0	1	2	3	4	5	6	7
	Panel A Capital Investment								
1	Investment in fixed assets	–12,000							
2	Sale of fixed assets								1,949
3	Less tax on sale								409
4	Cash flow from capital investment (1 + 2 + 3)	–12,000							1,540
	Panel B Operating Cash Flow								
5	Revenues	0	523	12,887	32,610	48,901	35,834	19,717	
6	Cost of goods sold	0	837	7,729	19,552	29,345	21,492	11,830	
7	Other costs	4,000	2,200	1,210	1,331	1,464	1,611	1,772	
8	Depreciation	12,000	0	0	0	0	0	0	
9	Pretax profit (5 – 6 – 7 – 8)	–16,000	–2,514	3,948	11,727	18,092	12,731	6,115	
10	Tax (.21 × 9)	–3,360	–528	829	2,463	3,799	2,674	1,284	
11	Profit after tax (9 – 10)	–12,640	–1,986	3,119	9,264	14,293	10,057	4,831	
12	Operating cash flow (8 + 11)	–640	–1,986	3,119	9,264	14,293	10,057	4,831	
	Panel C Investment in Working Capital								
13	Working capital		550	1,289	3,261	4,890	3,583	2,002	0
14	Change in working capital		550	739	1,972	1,629	–1,307	–1,581	–2,002
15	Cash flow from investment in working capital (–14)		–550	–739	–1,972	–1,629	1,307	1,581	2,002
	Panel D Project Valuation								
16	Total project cash flow (4 + 12 + 15)	–12,640	–2,536	2,380	7,292	12,664	11,364	6,412	3,542
17	Discount factor	1.000	0.833	0.694	0.579	0.482	0.402	0.335	0.279
18	Discounted cash flows (16 × 17)	–12,640	–2.113	1,653	4,220	6,107	4,567	2,147	988
19	NPV	4,929							

TABLE 6.3 IM&C's guano project. Revised analysis with immediate expensing of investment expenditures.

[8]The only difference between MACRS and our example of double-declining-balance is that MACRS assumes that the investment is made halfway through the year and, therefore, receives only half the allowance in the first year.

Final Comments on Taxes

Two final comments. First, note that all of the guano project's $12 million capital investment is in plant and equipment, which, under current U.S. tax law, can be expensed immediately. But suppose the project also requires an up-front R&D outlay of $500,000. Under the Tax Cuts and Jobs Act, R&D expenditures after 2021 cannot be expensed but must be written off over five years.

Second, all large U.S. corporations keep two separate sets of books, one for stockholders and one for the Internal Revenue Service (IRS). It is common to use straight-line depreciation on the stockholder books and accelerated depreciation on the tax books. The IRS doesn't object to this, and it makes the firm's reported earnings higher than if accelerated depreciation were used everywhere. There are many other differences between tax books and shareholder books.[9]

The financial analyst must be careful to remember which set of books he or she is looking at. In capital budgeting only the tax books are relevant, but to an outside analyst only the shareholder books are available.

Project Analysis

Let us review. Earlier in this section, you embarked on an analysis of IM&C's guano project. You drew up a series of cash-flow forecasts assuming straight-line depreciation. Then you remembered accelerated depreciation and recalculated cash flows and NPV. Finally, you recognized that under the Tax Cuts and Jobs Act, IM&C could write off the capital expenditure in the year that it was incurred.

You were lucky to get away with just three NPV calculations. In real situations, it often takes several tries to purge all inconsistencies and mistakes. Then you may want to analyze some alternatives. For example, should you go for a larger or smaller project? Would it be better to market the fertilizer through wholesalers or directly to the consumer? Should you build 90,000-square-foot aging sheds for the guano in northern South Dakota rather than the planned 100,000-square-foot sheds in southern North Dakota? In each case, your choice should be the one offering the highest NPV. Sometimes the alternatives are not immediately obvious. For example, perhaps the plan calls for two costly, high-speed packing lines. But, if demand for guano is seasonal, it may pay to install just one high-speed line to cope with the base demand and two slower but cheaper lines simply to cope with the summer rush. You won't know the answer until you have compared NPVs.

You will also need to ask some "what if clear" questions. How would NPV be affected if inflation rages out of control? What if technical problems delay start-up? What if gardeners prefer chemical fertilizers to your natural product? Managers employ a variety of techniques to develop a better understanding of how such unpleasant surprises could damage NPV. For example, they might undertake a *sensitivity analysis,* in which they look at how far the project could be knocked off course by bad news about one of the variables. Or they might construct different *scenarios* and estimate the effect of each on NPV. Another technique, known as *break-even analysis,* is to explore how far sales could fall short of forecast before the project goes into the red.

In Chapter 10, we practice using each of these "what if clear" techniques. You will find that project analysis is much more than one or two NPV calculations.[10]

[9]This separation of tax accounts from shareholder accounts is not found worldwide. In Japan, for example, taxes reported to shareholders must equal taxes paid to the government; ditto for France and many other European countries.

[10]In the meantime, you might like to get ahead of the game by viewing the spreadsheets for the guano project and seeing how NPV would change with a shortfall in sales or an unexpected rise in costs.

Calculating NPV in Other Countries and Currencies

Our guano project was undertaken in the United States by a U.S. company. But the principles of capital investment are the same worldwide. For example, suppose that you are the financial manager of the German company, K.G.R. Ökologische Naturdüngemittel GmbH (KGR), that is faced with a similar opportunity to make a €10 million investment in Germany. What changes?

1. KGR must also produce a set of cash-flow forecasts, but in this case the project cash flows are stated in euros, the eurozone currency.
2. In developing these forecasts, the company needs to recognize that prices and costs will be influenced by the German inflation rate.
3. Profits from KGR's project are liable to the German rate of corporate tax, which is currently 15.8% plus a large municipal trade tax.
4. KGR must use the German system of depreciation allowances. In common with many other countries, Germany requires firms to use the straight-line system. KGR, therefore, writes off one-sixth of the capital outlay each year.
5. Finally, KGR discounts the project's euro cash flows at the German cost of capital measured in euros.

Now suppose you are the financial manager of a U.S. company considering the same investment in Germany. You would go through exactly the same steps as KGR. You would not have to worry about U.S. taxes on your company's German profits because the United States now has a territorial corporate income tax. You would probably convert the project NPV from euros to U.S. dollars, however, and you might use a different cost of capital. We discuss cross-border capital investment decisions in Chapter 27.

6-4 Using the NPV Rule to Choose among Projects

Almost all real-world investment decisions entail either-or choices. Such choices are said to be *mutually exclusive*. We came across an example of mutually exclusive investments in Chapter 2. There we looked at whether it was better to build an office block for immediate sale or to rent it out and sell it at the end of two years. To decide between these alternatives, we calculated the NPV of each and chose the one with the higher NPV.

That is the correct procedure as long as the choice between the two projects does not affect any future decisions that you might wish to make. But sometimes the choices that you make today *will* have an impact on future opportunities. When that is so, choosing between competing projects is trickier. Here are four important, but often challenging, problems:

- *The investment timing problem.* Should you invest now or wait and think about it again next year? (Here, today's investment is competing with possible future investments.)
- *The choice between long- and short-lived equipment.* Should the company save money today by choosing cheaper machinery that will not last as long? (Here, today's decision would accelerate a later investment in machine replacement.)
- *The replacement problem.* When should existing machinery be replaced? (Using it another year could delay investment in more modern equipment.)
- *The cost of excess capacity.* What is the cost of using equipment that is temporarily not being used? (Increasing use of the equipment may bring forward the date at which additional capacity is required.)

We will look at each of these problems in turn.

Problem 1: The Investment Timing Decision

The fact that a project has a positive NPV does not mean that it is best undertaken now. It might be even more valuable if undertaken in the future. The question of optimal timing is not difficult when the cash flows are certain. You must first examine alternative start dates (t) for the investment and calculate the net *future* value at each of these dates. Then, to find which of the alternatives would add most to the firm's *current* value, you must discount these net future values back to the present:

$$\text{Net present value of investment if undertaken at date } t = \frac{\text{net future value at date } t}{(1+r)^t}$$

For example, suppose you own a large tract of inaccessible timber. To harvest it, you need to invest a substantial amount in access roads and other facilities. The longer you wait, the higher the investment required. On the other hand, lumber prices may rise as you wait, and the trees will keep growing, although at a gradually decreasing rate.

Let us suppose that the net present value of the harvest at different *future* dates is as follows:

	Year of Harvest					
	0	1	2	3	4	5
Net *future* value ($ thousands)	50	64.4	77.5	89.4	100	109.4
Change in value from previous year (%)		+ 28.8	+20.3	+15.4	+11.9	+9.4

As you can see, the longer you defer cutting the timber, the more money you will make. However, your concern is with the date that maximizes the net *present* value of your investment, that is, its contribution to the value of your firm *today*. You therefore need to discount the net future value of the harvest back to the present. Suppose the appropriate discount rate is 10%. Then, if you harvest the timber in year 1, it has a net *present* value of $58,500:

$$\text{NPV if harvested in year 1} = \frac{64.4}{1.10} = 58.5, \text{ or } \$58{,}500$$

The net present value for other harvest dates is as follows:

	Year of Harvest					
	0	1	2	3	4	5
Net present value ($ thousands)	50	58.5	64.0	67.2	68.3	67.9

The optimal point to harvest the timber is year 4 because this is the point that maximizes NPV.

Notice that before year 4, the net future value of the timber increases by more than 10% a year: The gain in value is greater than the cost of the capital tied up in the project. After year 4, the gain in value is still positive but less than the required return. So delaying the harvest further just reduces shareholder wealth.[11]

[11] Our timber-cutting example conveys the right idea about investment timing, but it misses an important practical point: The sooner you cut the first crop of trees, the sooner the second crop can start growing. Thus, the value of the second crop depends on when you cut the first. This more complex and realistic problem can be solved in one of two ways:

1. Find the cutting dates that maximize the present value of a series of harvests, taking into account the different growth rates of young and old trees.
2. Repeat our calculations, counting the future market value of cut-over land as part of the payoff to the first harvest. The value of cut-over land includes the present value of all subsequent harvests.

The second solution is far simpler if you can figure out what cut-over land will be worth.

The investment timing problem is much more complicated when you are unsure about future cash flows. We return to the problem of investment timing under uncertainty in Chapters 10 and 22.

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Investment timing decisions

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Problem 2: The Choice between Long- and Short-Lived Equipment

An advertising agency needs to choose between two digital presses. Let's call them machines A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs $15,000 and will last three years. It costs $5,000 per year to run. Machine B is an "economy" model, costing only $10,000, but it will last only two years and costs $6,000 per year to run.

The only way to choose between these two machines is on the basis of cost. The present value of each machine's cost is as follows:

Costs ($ thousands)					
Year:	0	1	2	3	PV at 6% ($ thousands)
Machine A	15	5	5	5	$28.37
Machine B	10	6	6	—	21.00

Should the agency take machine B, the one with the lower present value of costs? Not necessarily. All we have shown is that machine B offers two years of service for a lower total cost than three years of service from machine A. But is the *annual* cost of using B lower than that of A?

Suppose the financial manager agrees to buy machine A and pay for its operating costs out of her budget. She then charges the annual amount for use of the machine. There will be three equal payments starting in year 1. The financial manager has to make sure that the present value of these payments equals the present value of the costs of each machine.

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Equivalent annual costs

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When the discount rate is 6%, the payment stream with such a present value turns out to be $10,610 a year. In other words, the cost of buying and operating machine A over its three-year life is equivalent to an annual charge of $10,610 a year for three years.

Costs ($ thousands)					
Year:	0	1	2	3	PV at 6% ($ thousands)
Machine A	15	5	5	5	28.37
Equivalent annual cost		10.61	10.61	10.61	28.37

We calculated this *equivalent annual cost* by finding the three-year annuity with the same present value as A's lifetime costs.

$$\begin{aligned}\text{PV of annuity} &= \text{PV of A's costs} = 28.37\\ &= \text{annuity payment} \times \text{3-year annuity factor}\end{aligned}$$

At a 6% cost of capital, the annuity factor is 2.673 for three years, so

$$\text{Annuity payment} = \frac{28.37}{2.673} = 10.61$$

A similar calculation for machine B gives an equivalent annual cost of $11,450:

Costs ($ thousands)				
Year:	0	1	2	PV at 6% ($ thousands)
Machine B	10	6	6	21.00
Equivalent annual cost		11.45	11.45	21.00

Machine A is better because its equivalent annual cost is less ($10,610 versus $11,450 for machine B).

Equivalent Annual Cash Flow, Inflation, and Technological Change When we calculated the equivalent annual costs of machines A and B, we implicitly assumed that inflation is zero. But, in practice, the cost of buying and operating the machines is likely to rise with inflation. If so, the *nominal* costs of operating the machines will rise, while the *real* costs will be constant. Therefore, when you compare the equivalent annual costs of two machines, we strongly recommend doing the calculations in real terms. Do *not* calculate equivalent annual cash flows as level *nominal* annuities. This procedure can give incorrect rankings of true equivalent annual flows at high inflation rates. See Challenge Problem 37 at the end of this chapter for an example.[12]

There will also be circumstances in which even the real cash flows of buying and operating the two machines are not expected to be constant. For example, suppose that thanks to technological improvements, new machines cost 20% less each year in *real* terms to buy and operate. In this case, future owners of brand-new, lower-cost machines will be able to cut their (real) rental cost by 20%, and owners of old machines will be forced to match this reduction. Thus, we now need to ask: If the real level of rents declines by 20% a year, how much will it cost to rent each machine?

If the real rent for year 1 is rent_1, then the real rent for year 2 is $\text{rent}_2 = 0.8 \times \text{rent}_1$. Rent_3 is $0.8 \times \text{rent}_2$, or $0.64 \times \text{rent}_1$. The owner of each machine must set the real rents sufficiently high to recover the present value of the costs. If the real cost of capital is 6%:

$$\text{PV of renting machine A} = \frac{\text{rent}_1}{1.06} + \frac{\text{rent}_2}{1.06^2} + \frac{\text{rent}_3}{1.06^3} = 28.37$$

$$= \frac{\text{rent}_1}{1.06} + \frac{0.8(\text{rent}_1)}{1.06^2} + \frac{0.64(\text{rent}_1)}{1.06^3} = 28.37$$

$\text{rent}_1 = 12.94$, or $12,940

For machine B:

$$\text{PV of renting machine B} = \frac{\text{rent}_1}{1.06} + \frac{0.8(\text{rent}_1)}{1.06^2} = 21.00$$

$\text{rent}_1 = 12.69$, or $12,690

The merits of the two machines are now reversed. Once we recognize that technology is expected to reduce the real costs of new machines, then it pays to buy the shorter-lived machine B rather than become locked into an aging technology with machine A in year 3.

[12]If you actually rent out the machine to the plant manager, or anyone else, be careful to specify that the rental payments be "indexed" to inflation. If inflation runs on at 5% per year and rental payments do not increase proportionally, then the real value of the rental payments must decline and will not cover the full cost of buying and operating the machine.

You can imagine other complications. Perhaps machine C will arrive in year 1 with an even lower equivalent annual cost. You would then need to consider scrapping or selling machine B at year 1 (more on this decision follows). The financial manager could not choose between machines A and B in year 0 without taking a detailed look at what each machine could be replaced with.

Comparing equivalent annual cash flows should never be a mechanical exercise; always think about the assumptions that are implicit in the comparison. Finally, remember why equivalent annual cash flows are necessary in the first place. It is because A and B will be replaced at different future dates. The choice between them therefore affects future investment decisions. If subsequent decisions are not affected by the initial choice (e.g., because neither machine will be replaced), then we do *not need to take future decisions into account.*[13]

Equivalent Annual Cash Flow and Taxes We have not mentioned taxes. But you surely realized that machine A and B's lifetime costs should be calculated after-tax, recognizing that operating costs are tax-deductible and that capital investment generates depreciation tax shields.

Problem 3: When to Replace an Old Machine

Our earlier comparison of machines A and B took the life of each machine as fixed. In practice, the point at which equipment is replaced reflects economics, not physical collapse. *We* must decide when to replace. The machine will rarely decide for us.

Here is a common problem. You are operating an elderly machine that is expected to produce a net cash *inflow* of $4,000 in the coming year and $4,000 next year. After that it will give up the ghost. You can replace it now with a new machine, which costs $15,000 but is much more efficient and will provide a cash inflow of $8,000 a year for three years. You want to know whether you should replace your equipment now or wait a year.

We can calculate the NPV of the new machine and also its *equivalent annual cash flow*—that is, the three-year annuity that has the same net present value:

	Cash Flows ($ thousands)				
	C_0	C_1	C_2	C_3	NPV at 6% ($ thousands)
New machine	−15	+8	+8	+8	6.38
Equivalent annual cash flow		+2.387	+2.387	+2.387	6.38

In other words, the cash flows of the new machine are equivalent to an annuity of $2,387 per year. So we can equally well ask at what point we would want to replace our old machine with a new one producing $2,387 a year. When the question is put this way, the answer is obvious. As long as your old machine can generate a cash flow of $4,000 a year, who wants to put in its place a new one that generates only $2,387 a year?

It is a simple matter to incorporate salvage values into this calculation. Suppose that the current salvage value is $8,000 and next year's value is $7,000. Let us see where you come out next year if you wait and then sell. On one hand, you gain $7,000, but you lose today's salvage value *plus* a year's return on that money. That is $8,000 \times 1.06 = \$8,480$. Your net loss is $8,480 - 7,000 = \$1,480$, which only partly offsets the operating gain. You should not replace yet.

Remember that the logic of such comparisons requires that the new machine be the best of the available alternatives and that it in turn be replaced at the optimal point.

[13]However, if neither machine will be replaced, then we have to consider the extra revenue generated by machine A in its third year, when it will be operating but B will not.

Problem 4: Cost of Excess Capacity

Any firm with a centralized information system (computer servers, storage, software, and telecommunication links) encounters many proposals for using it. Recently installed systems tend to have excess capacity, and since the immediate marginal costs of using them seem to be negligible, management often encourages new uses. Sooner or later, however, the load on a system increases to the point at which management must either terminate the uses it originally encouraged or invest in another system several years earlier than it had planned. Such problems can be avoided if a proper charge is made for the use of spare capacity.

Suppose we have a new investment project that requires heavy use of an existing information system. The effect of adopting the project is to bring the purchase date of a new, more capable system forward from year 4 to year 3. This new system has a life of five years, and at a discount rate of 6%, the present value of the cost of buying and operating it is $500,000.

We begin by converting the $500,000 present value of the cost of the new system to an equivalent annual cost of $118,700 for each of five years.[14] Of course, when the new system in turn wears out, we will replace it with another. So we face the prospect of future information-system expenses of $118,700 a year. If we undertake the new project, the series of expenses begins in year 4; if we do not undertake it, the series begins in year 5. The new project, therefore, results in an *additional* cost of $118,700 in year 4. This has a present value of $118,700/(1.06)^4$, or about $94,000. This cost is properly charged against the new project.

When we recognize it, the NPV of the project may prove to be negative. If so, we still need to check whether it is worthwhile undertaking the project now and abandoning it later, when the excess capacity of the present system disappears.

[14] The present value of $118,700 a year for five years discounted at 6% is $500,000.

SUMMARY

By now present value calculations should be a matter of routine. However, forecasting project cash flows will never be routine. Here is a checklist that will help you to avoid mistakes:

1. Discount cash flows, not profits.
   a. Remember that depreciation is not a cash flow (though it affects tax payments).
   b. Remember to track investment in working capital. As sales increase, the firm will probably make additional investments in working capital, and as the project comes to an end, it will recover those investments.
   c. Beware of allocated overhead charges. These may not reflect the incremental costs of the project.
2. Estimate the project's *incremental* cash flows—that is, the difference between the cash flows with the project and those without the project.
   a. Include all indirect effects of the project, such as its impact on the sales of the firm's other products.
   b. Forget sunk costs.
   c. Include *opportunity costs,* such as the value of land that you would otherwise sell.
3. Treat inflation consistently.
   a. If cash flows are forecasted in nominal terms, use a nominal discount rate.
   b. Discount real cash flows at a real rate.
4. Forecast cash flows as if the project is all-equity-financed. Thus, project cash flows should exclude debt interest or the cost of repaying any loans. This enables you to separate the investment from the financing decision.