Quantitative Analytics tools for financial decisions

Simulation & Bootstrapping

Module 2 Session 5 & 6

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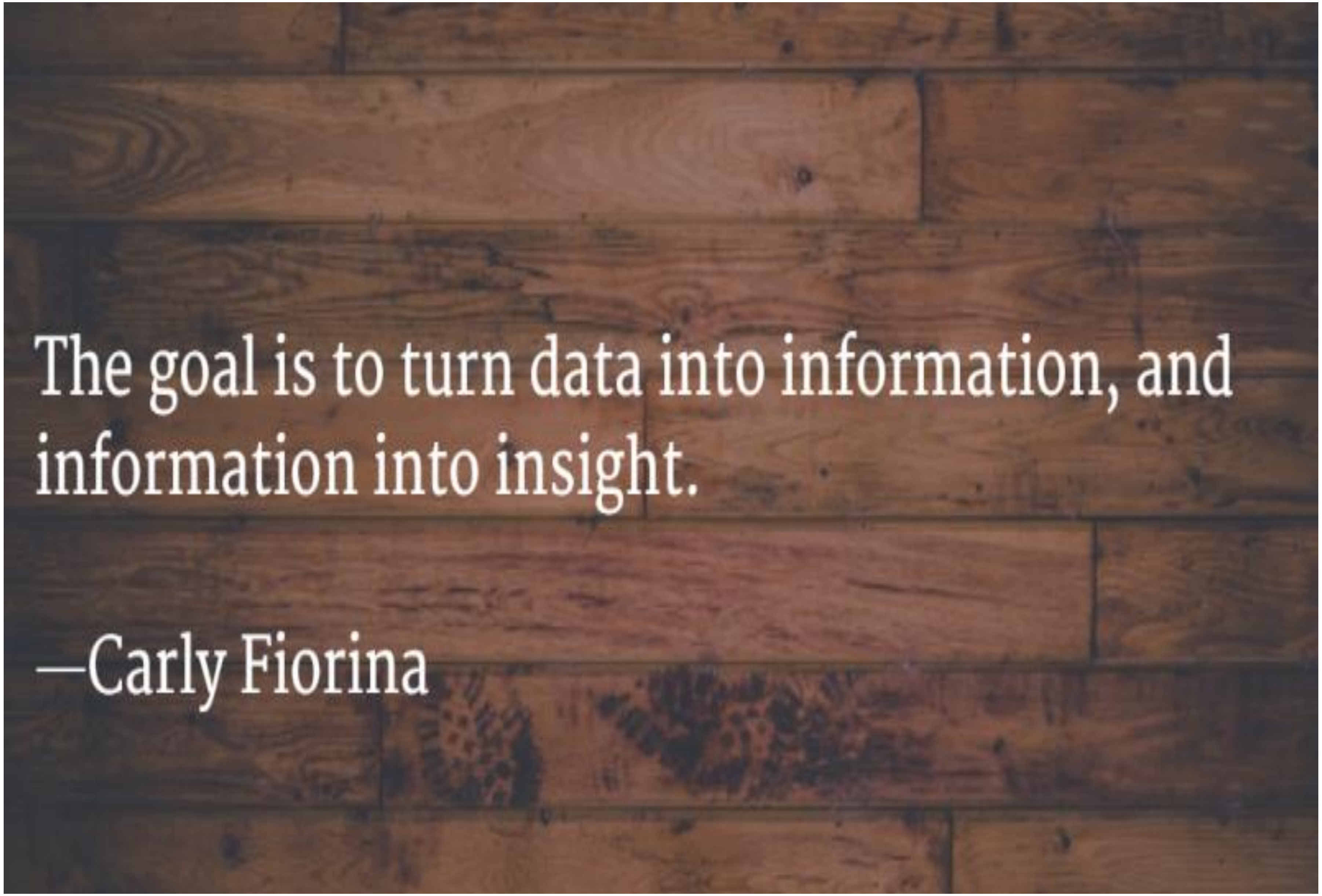

Agenda for Last Session

Regression

Today’s Agenda

Simulation & Bootstrapping

Simple vs. Multiple Regression

- One dependent variable Y predicted from one independent variable X
- One regression coefficient
- **r^2**: proportion of variation in dependent variable Y predictable from X

- One dependent variable Y predicted from a set of independent variables (X1, X2 ….Xk)
- One regression coefficient for each independent variable
- **R^2**: proportion of variation in dependent variable Y predictable by set of independent variables (X’s)

Multiple Correlation Coefficient (R) and Coefficient of Multiple Determination (R^2)

- R = the magnitude of the relationship between the dependent variable and the best linear combination of the predictor variables
- R^2 = the proportion of variation in Y accounted for by the set of independent variables (X's).

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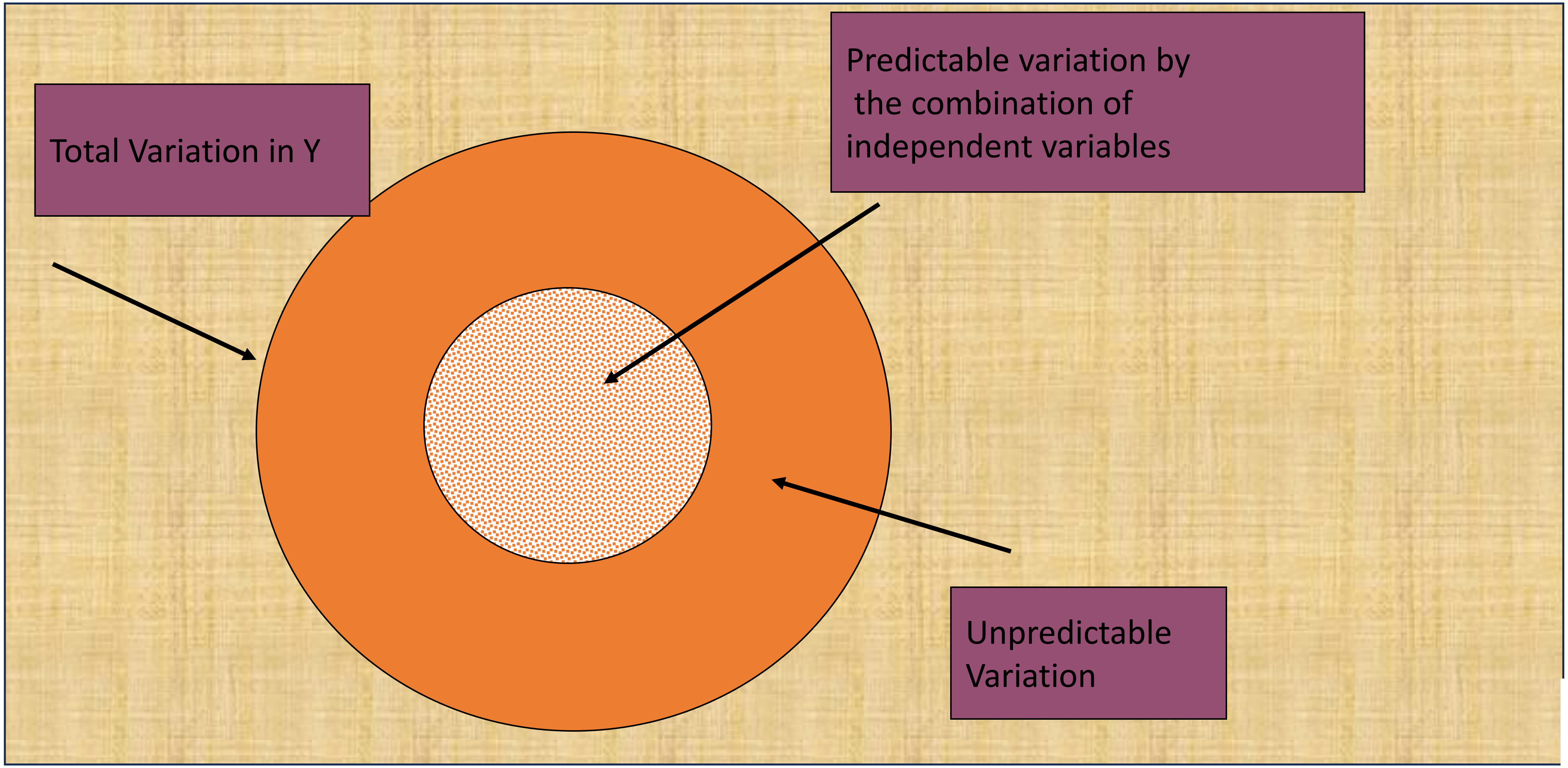
Explaining Variation: How much?



Various Significance Tests

Testing R^2

Test R^2 through an F test

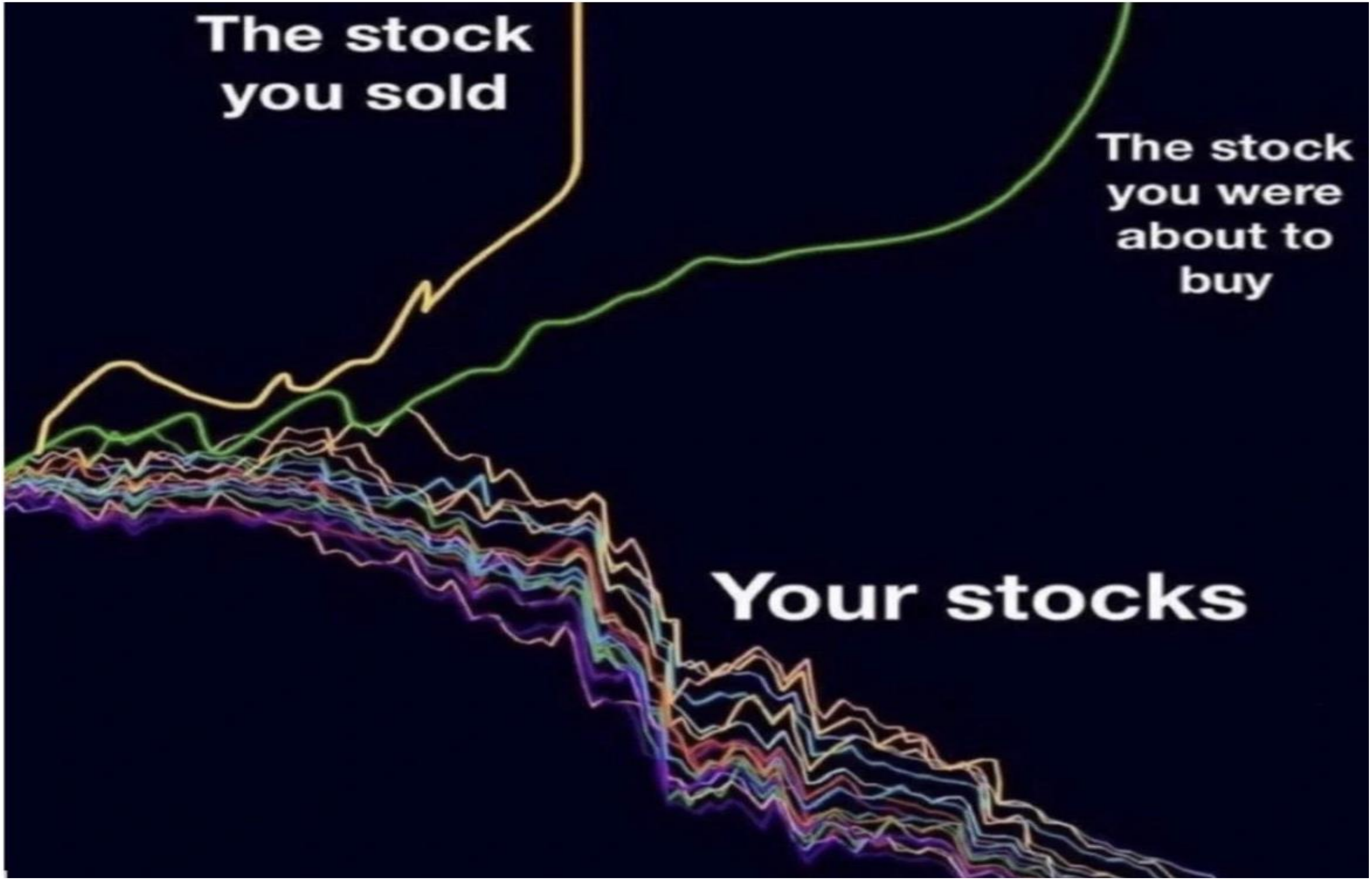
Test of competing models (difference between R^2) through an F test of difference of R^2s

Testing b

Test of each partial regression coefficient (b) by t-tests

Comparison of partial regression coefficients with each other - t-test of difference between **standardized** partial regression coefficients (β)

…We entered into a World of Randomness to see and predict future


The stock
you sold
The stock
you were
about to
buy
Your stocks

Discussion into Randomness takes us to…

STOCHASTIC PROCESS & its calculus…

First...Let’s have a look at the following ...

- What is a VARIABLE?
- What is VARIABLE + PROBABILITY?
- What is VARIABLE + PROBABILITY + TIME DIMENSION?

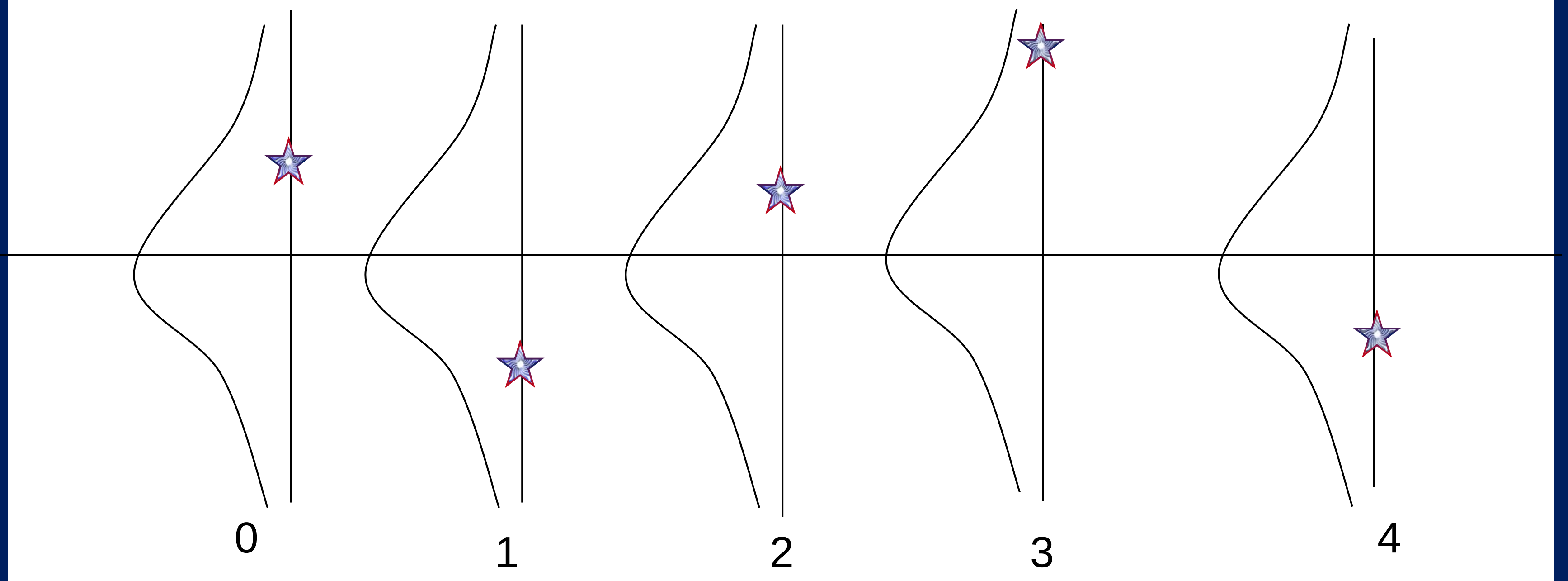
Let's take a diversion before we move ahead…

But, it is important as we are going to learn something important!

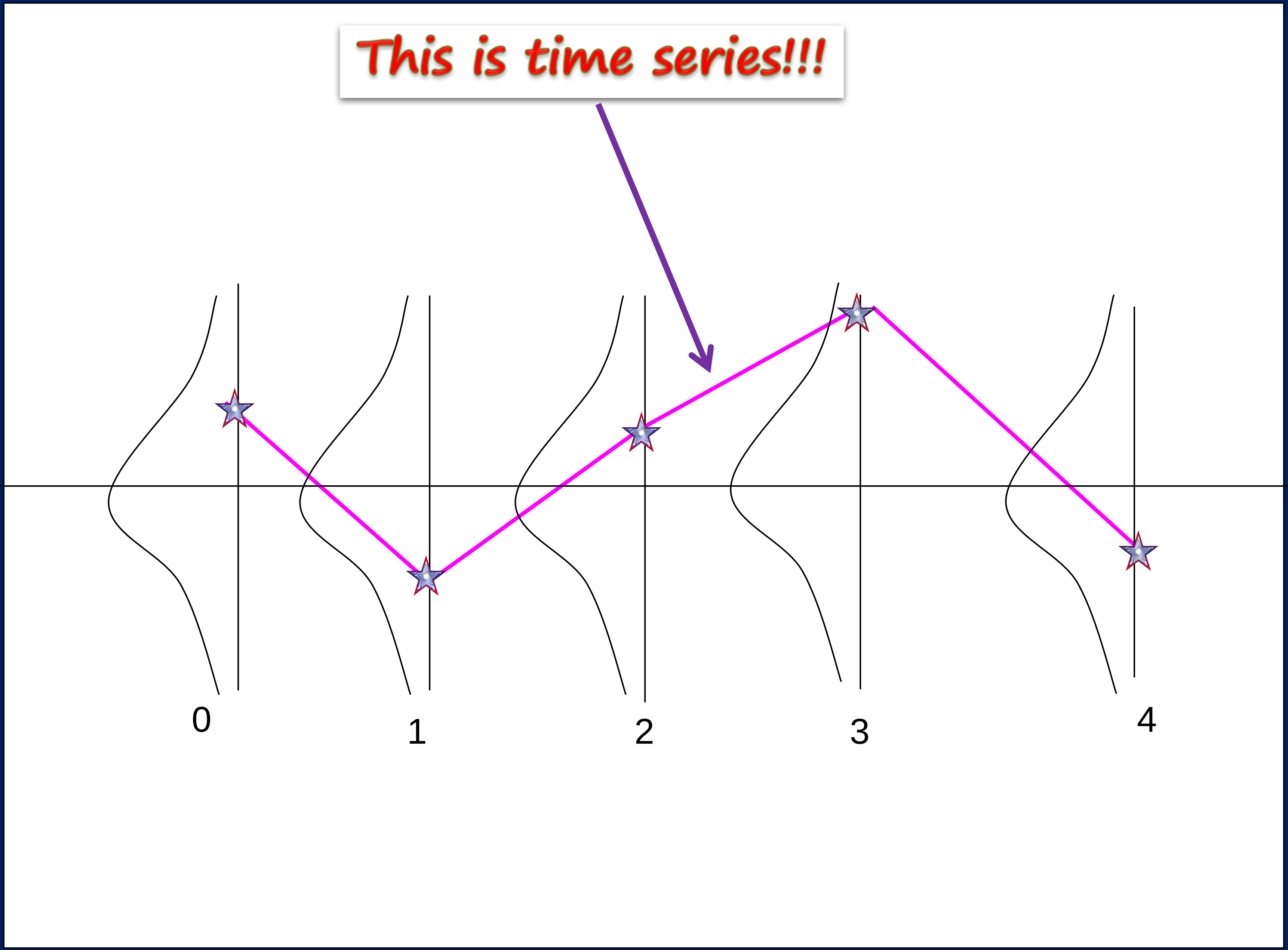
What is TIME SERIES?

Proper understanding about time series is important as most of the data we use in Finance is in the form of a time series. If we do not understand it properly, we cannot analyze a time series data with proper mind set.

How many possible values can happen on day zero?

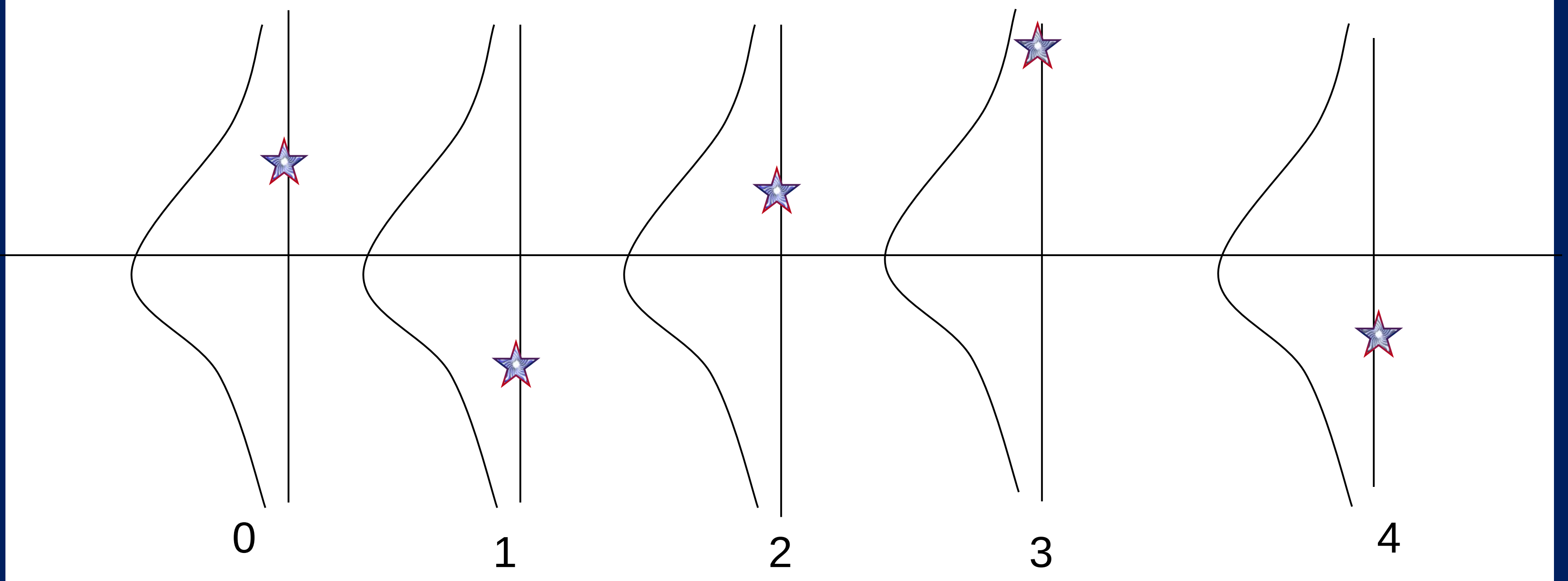


But, we realized only ONE!!! Similarly, for other days.


This is time series!!!
0
1
2
3
4

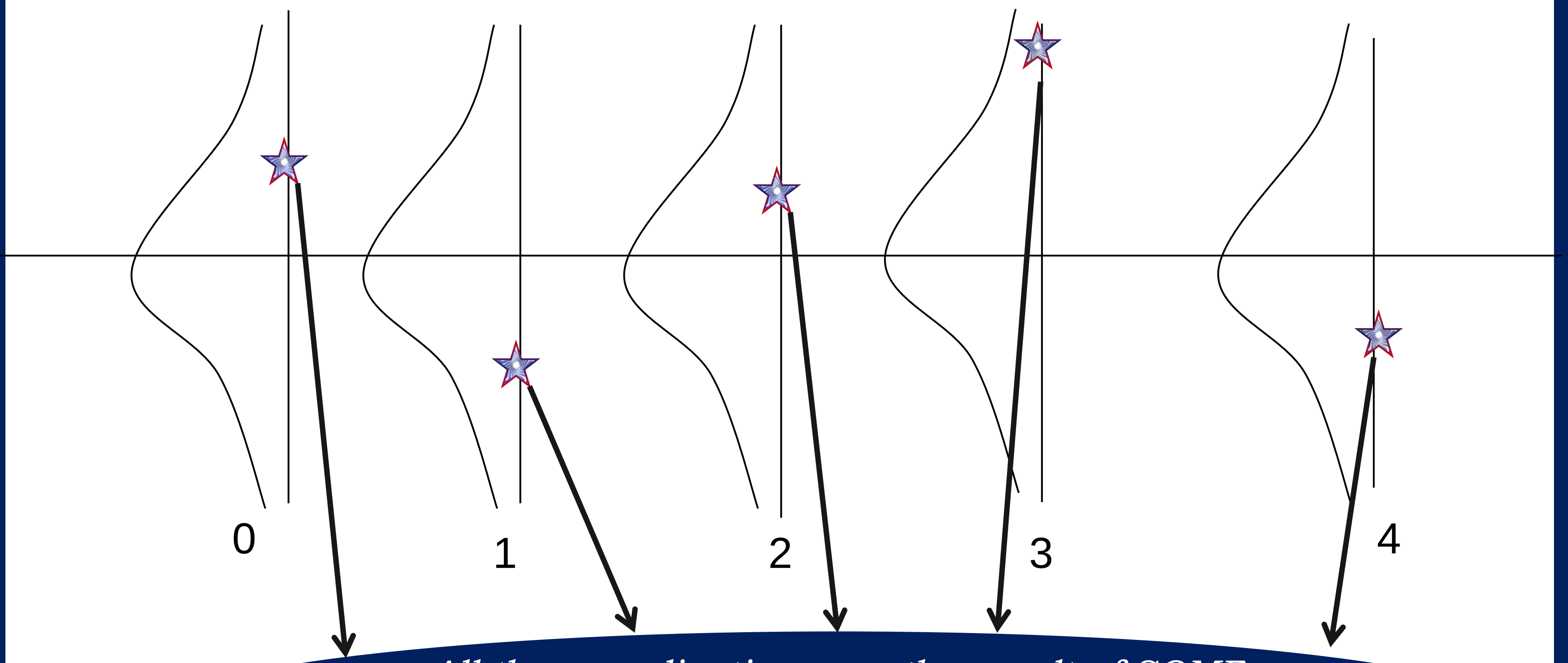

This is time series!!!
0
1
2
3
4

Important thing to note is … that …



Time Series deals with ONE realization out of infinite possibilities.

And, this one realization is the result of some underlying stochastic process!



All these realizations are the result of SOME underlying stochastic process whose behaviour is modeled through Normal Probability Distribution.

Now, we define Time Series…

- Realization of a particular stochastic process is called a **TIME SERIES.**

- Every ***realized*** observation of a stochastic process is understood as a **TIME SERIES.**

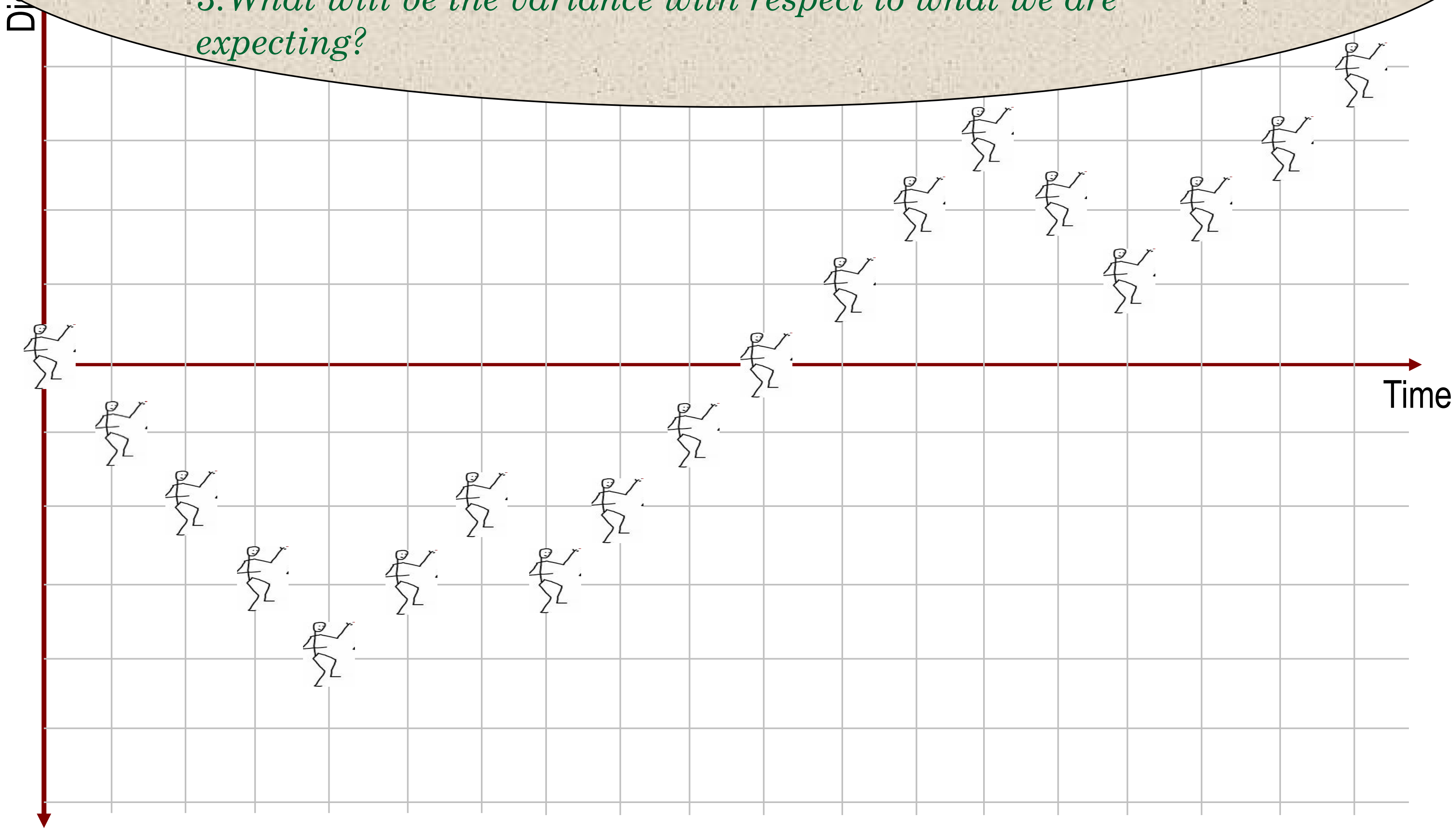
Now, let’s move to the building block of a stochastic process in finance.

RANDOM WALK…

Loc

We have the following interest in the WALK of this person.

1. *What will be the probability that this person will be at a particular position after n steps?*
2. *Where we are expecting him to be FINALLY?*
3. *What will be the variance with respect to what we are expecting?*



Look how a person is walking randomly!!!

- Let's assume that his initial position is S(0) = 0 and he may move up with a distance of +1 and down by -1 with probability ½ of each.
- That is, let X_t be a random variable with the following probability distribution –

$$P(X_t = 1) = P(X_t = -1) = ½$$

- If it is so, then his N^{th} position in this random walk can be described as a partial sum where N>0 as thus:

$$S(N) = S(0)+X_1+X_2+\ldots+X_N$$

$$S(N) - S(0) = X_1+X_2+\ldots+X_N$$

????

What is Random Walk?

Look at the following model:

$$Y_t = Y_{t-1} + \varepsilon_t$$

where:

Y_t *is the value in time period t,*

Y_{t-1} *is the value in time period t-1*

ε_t *is the value of the error term in time period t.*

What do say about it?

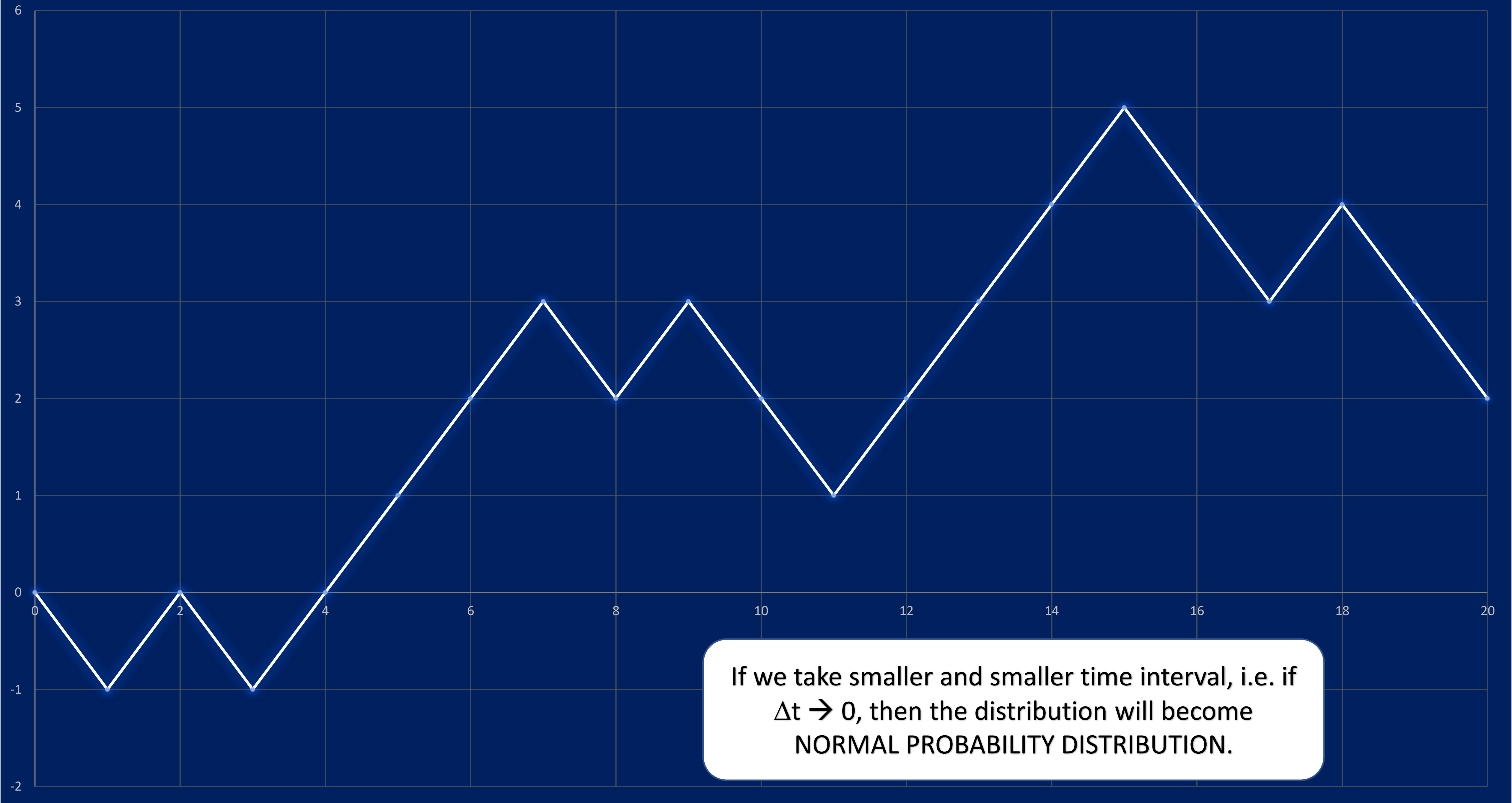
$$Y_t - Y_{t-1} = \varepsilon_t \rightarrow \Delta Y_t = \varepsilon_t$$

Random Walk…

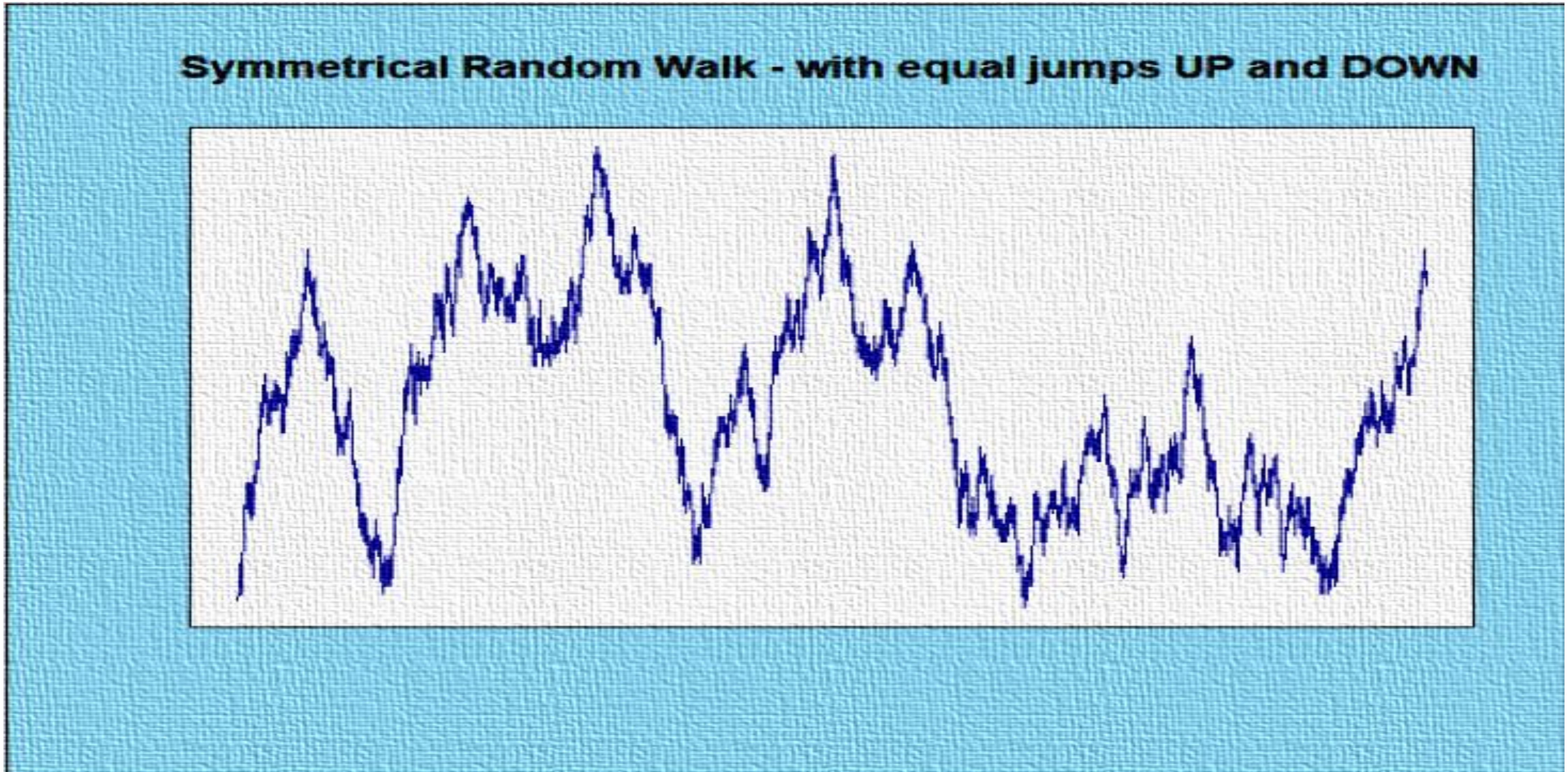
- A series is said to follow a random walk if the first differences are random.

- In a random walk model, *please note that* it is the differences—the changes from one period to the next— which are random.

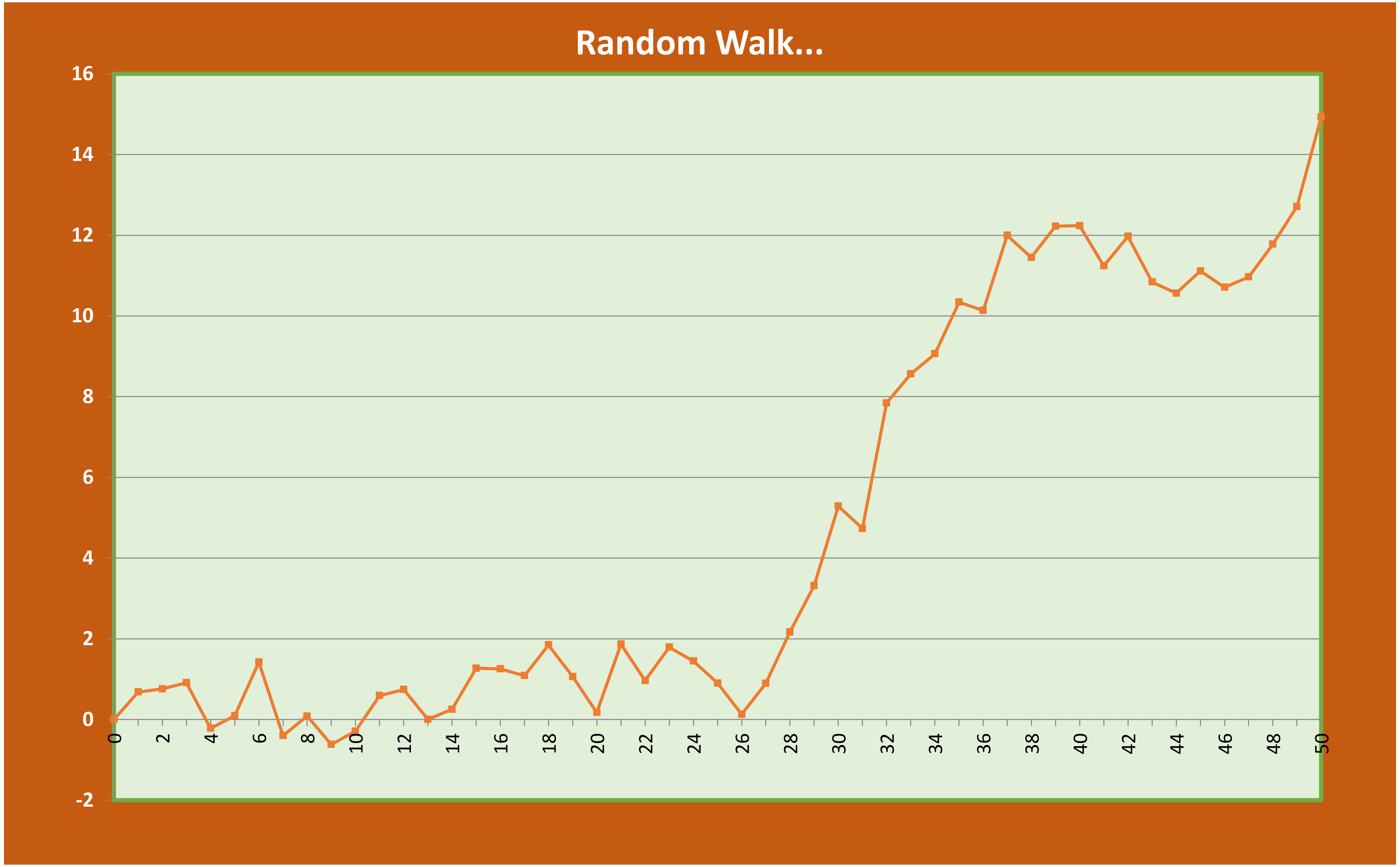
RANDOM WALK WITH JUMPS +1 AND -1 WITH EQUAL PROBABILITY



Try to understand and feel the behaviour of a random walk…



Random Walk



Random Walk...
16
14
12
10
8
6
4
2
0
-2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
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32
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36
38
40
42
44
46
48
50

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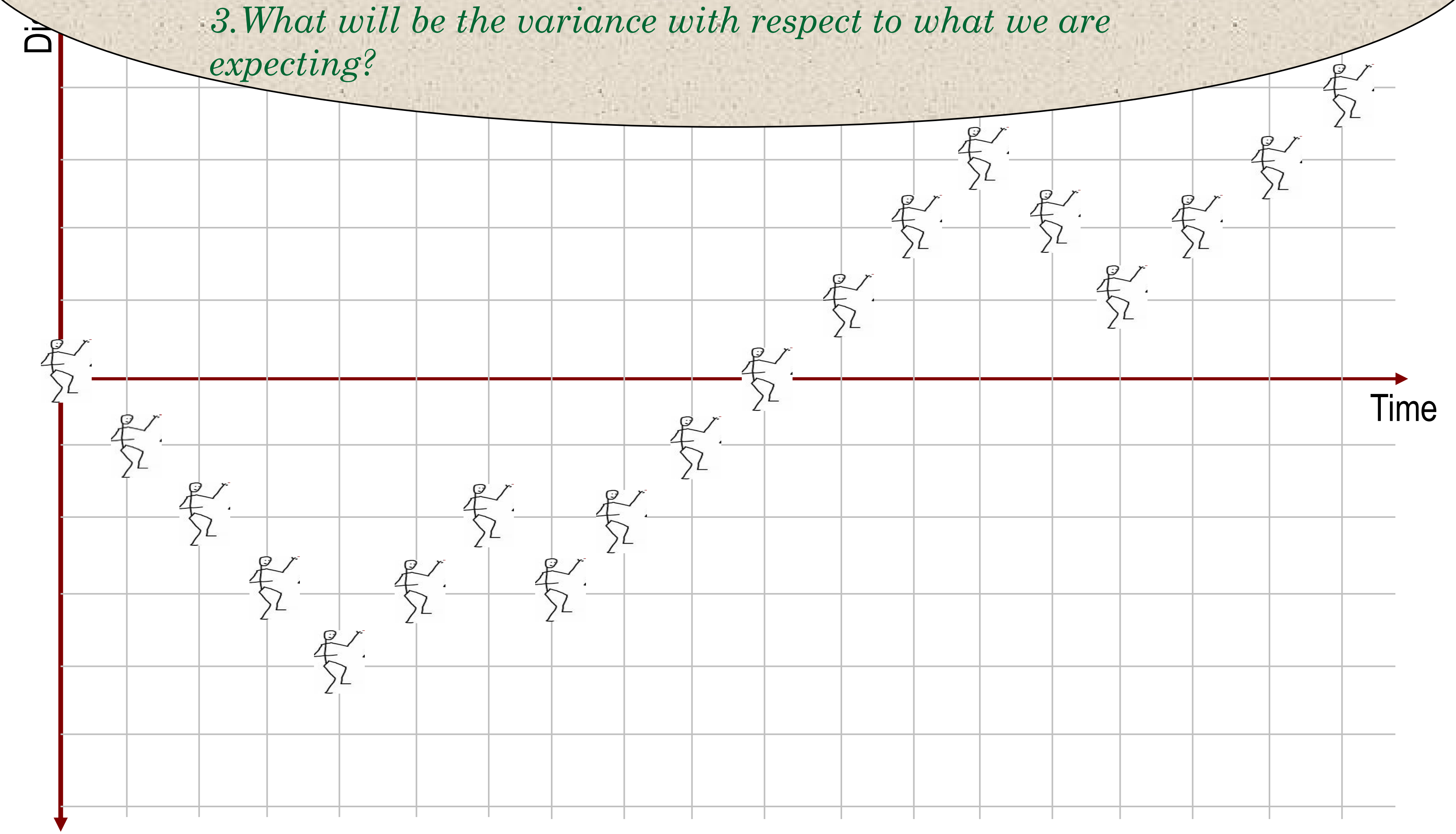

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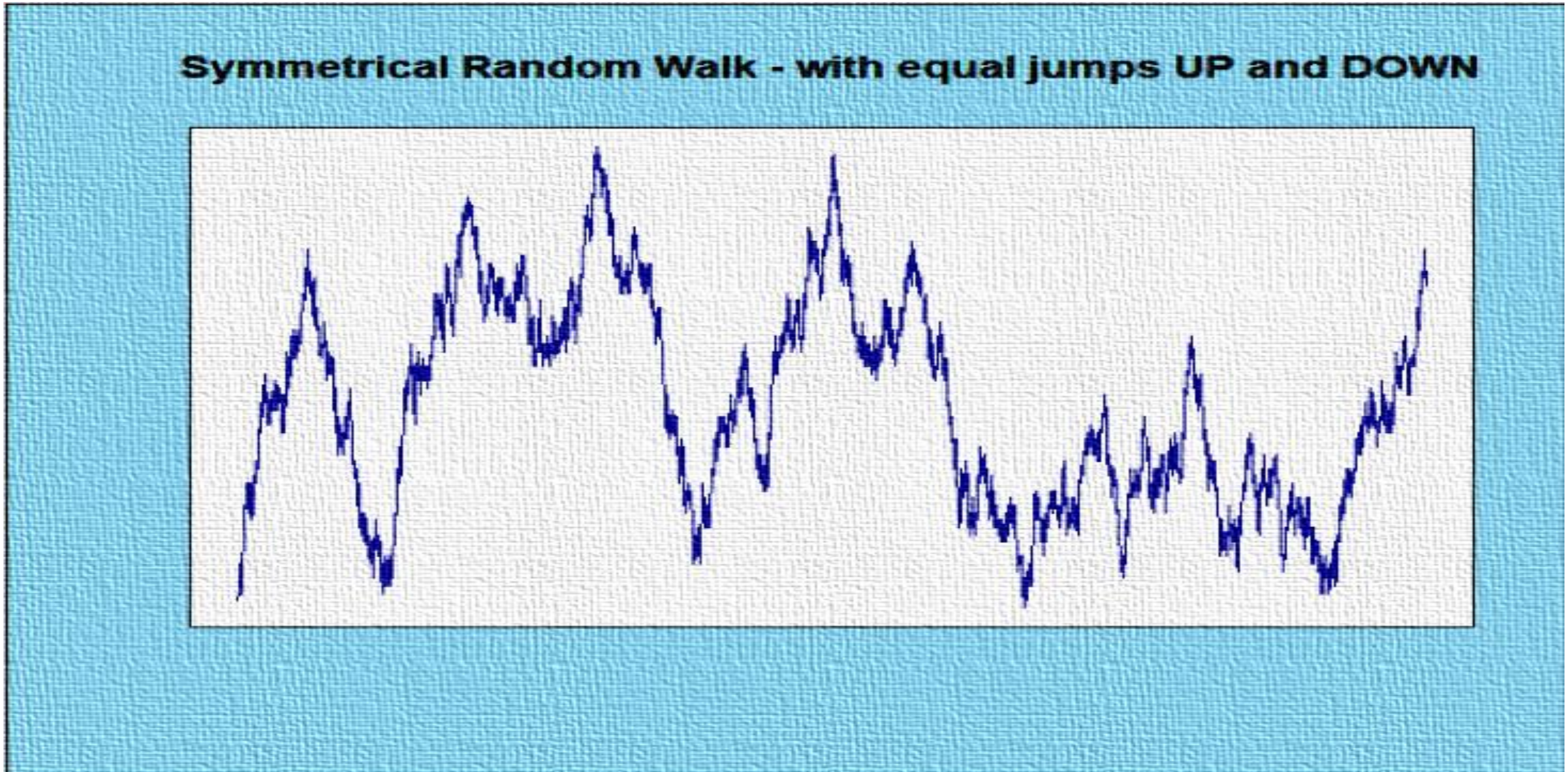
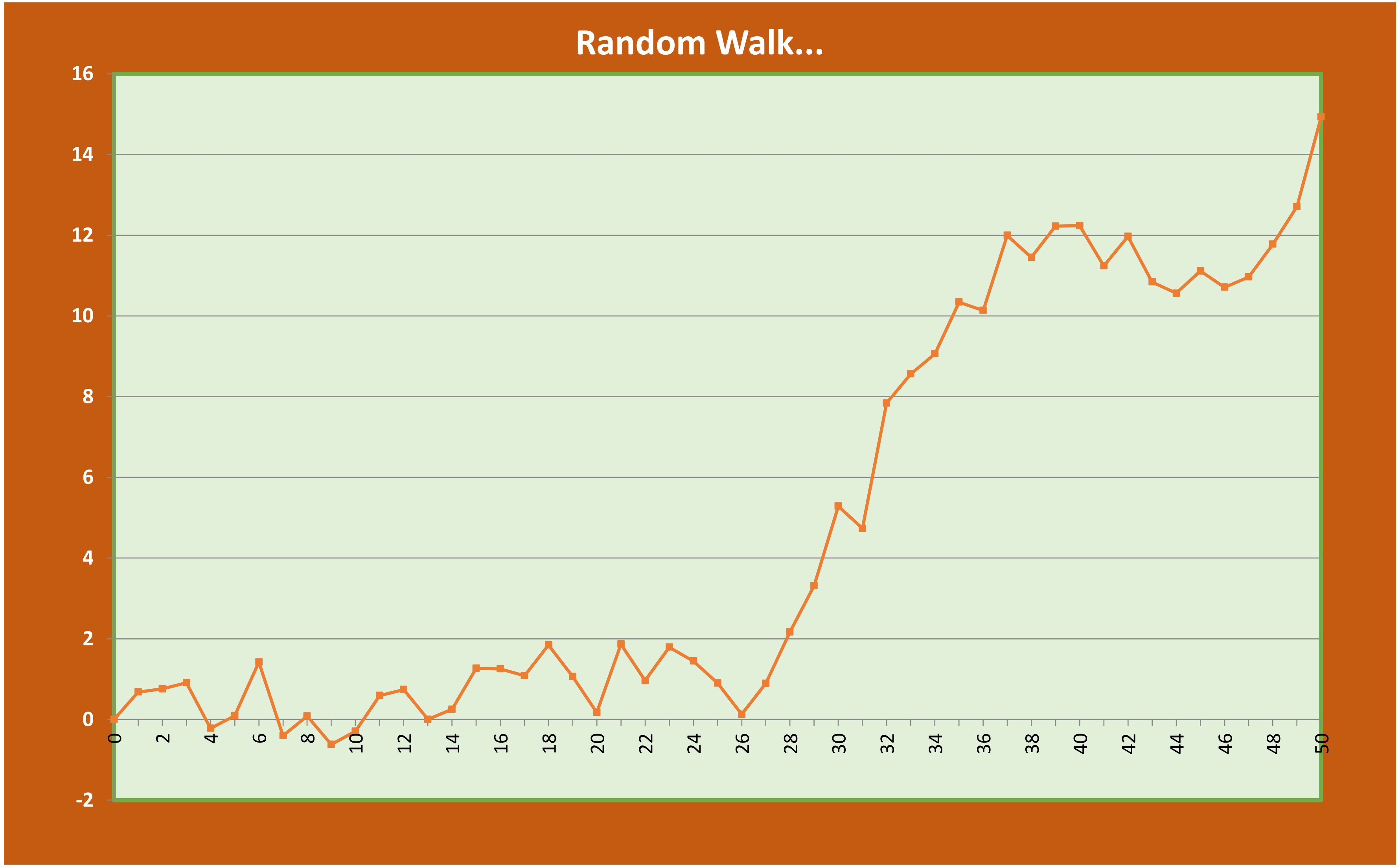


Random Walk



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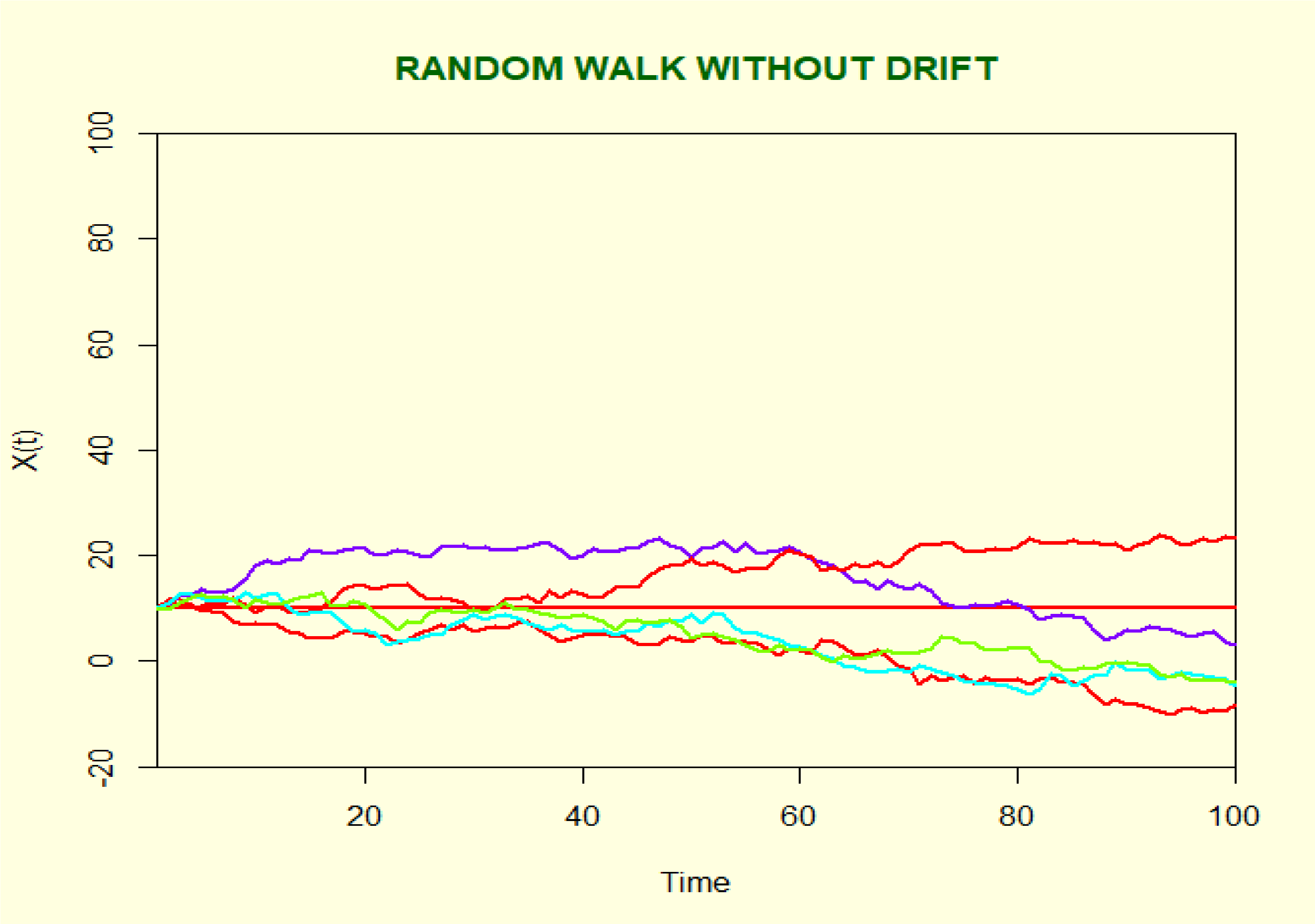
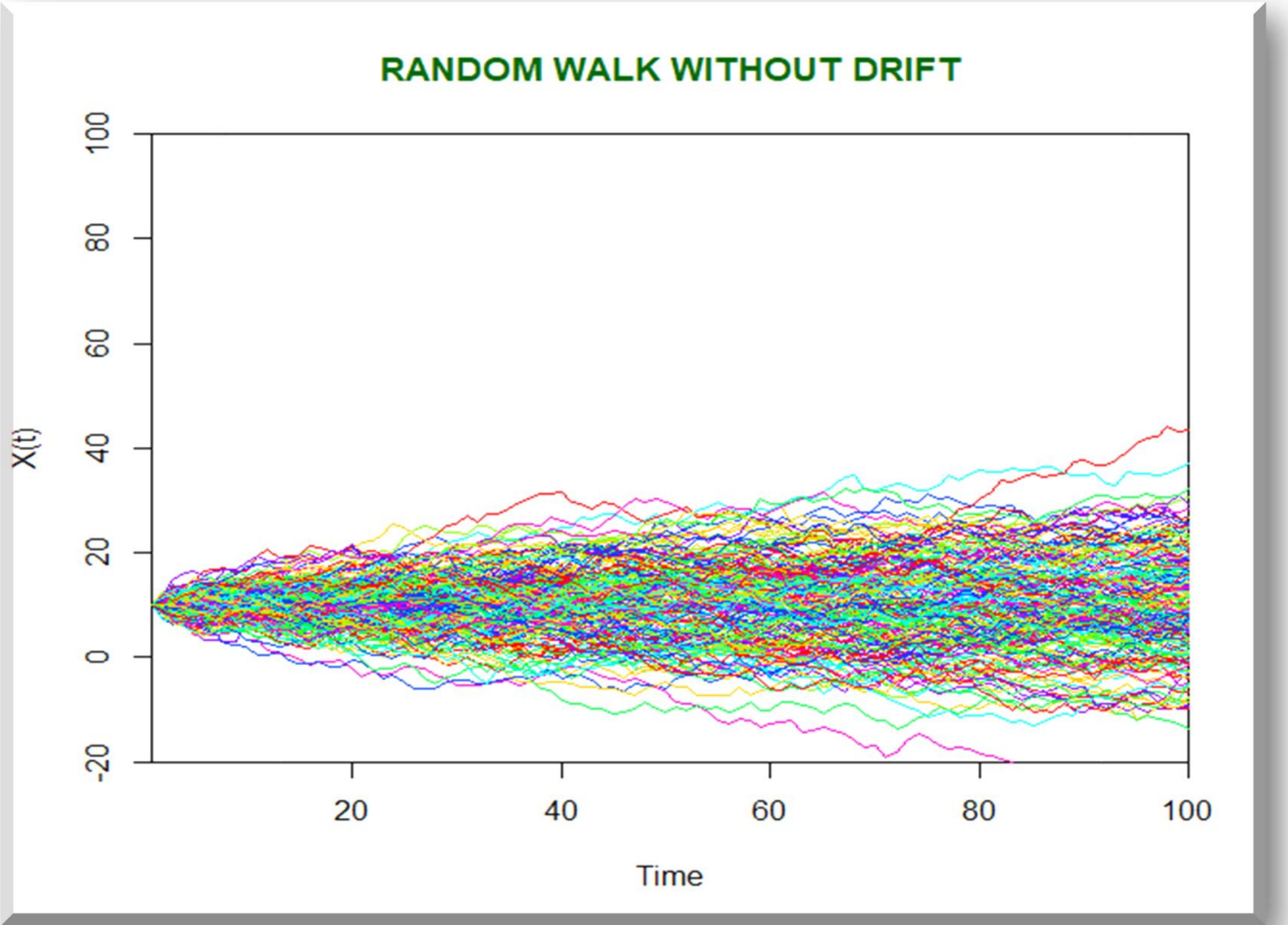
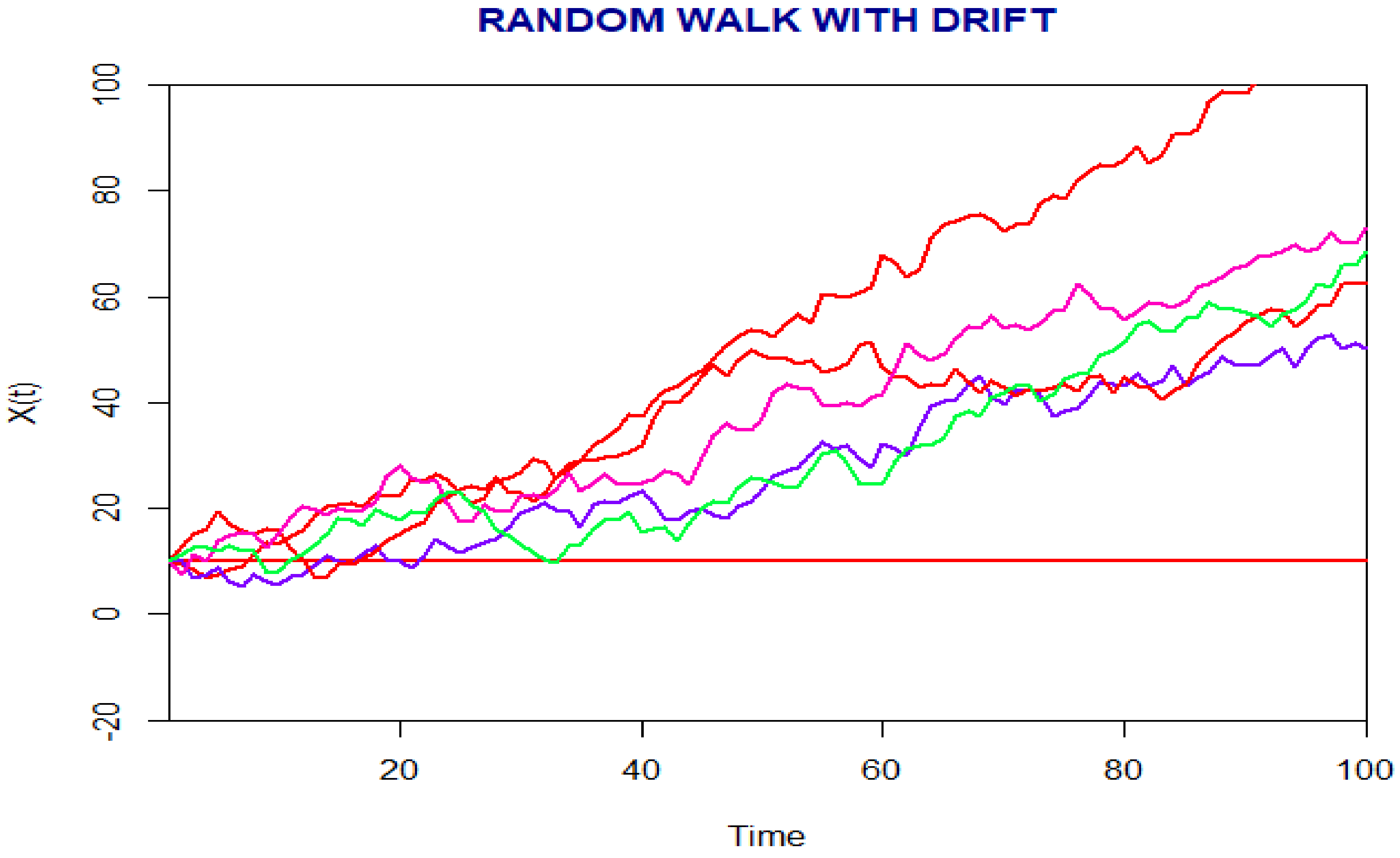
Random Walk Model without drift



Random Walk Model without drift



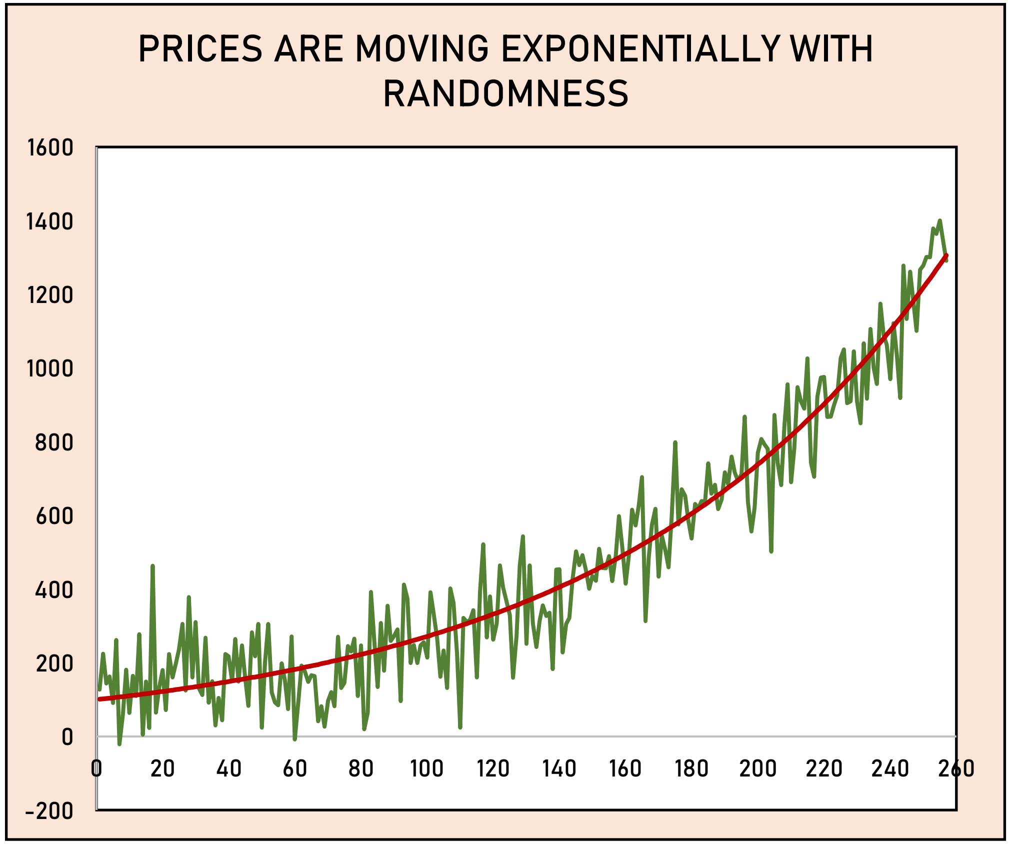
Random Walk Model with drift



What will happen if we stochastic component in the equation?

- $P_t = P_0 e^{rt+\sigma W_t}$ How can we model it?

- Please note the following about the above equation:
    - Prices will still increase in exponential manner but will have fluctuations around the mean value.
    - And, we have to learn how to model it.



Let’s take the equation…

- $P_t = P_0 e^{rt+\sigma dW} \Leftrightarrow \log P_t = \log(P_0) + rt + \sigma W_t$

- Taking derivative of the equation, we get –

$$\frac{dP}{P} = r\, dt + \sigma dW_t$$

$$dP = r\, P\, dt + \sigma\, P\, dW_t$$

What is this equation?

- Also, try to understand what this equation represents

$$\log P_t - \log(P_0) = rt + \sigma W_t$$