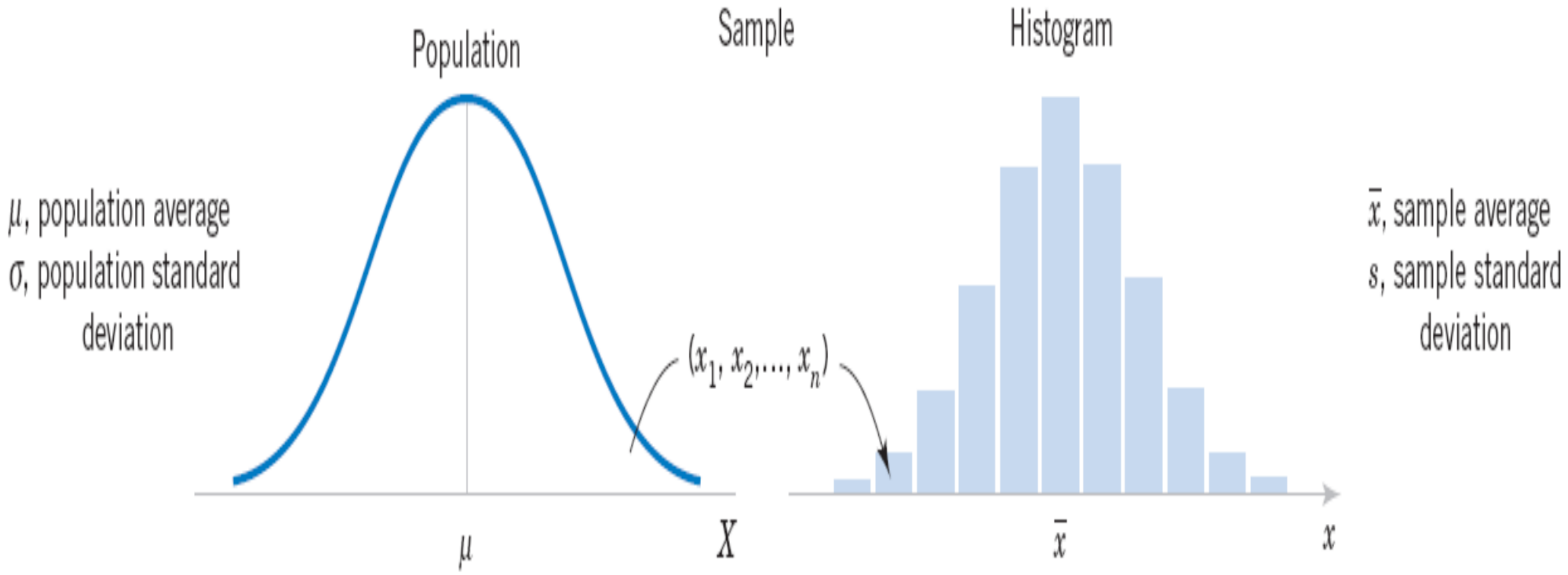
The goal is to turn data into information, and information into insight.

—Carly Fiorina

Statistical Inference

- The field of statistical inference consists of those methods used to make decisions or draw conclusions about a population.
- These methods utilize the information contained in a sample from the population in drawing conclusions.

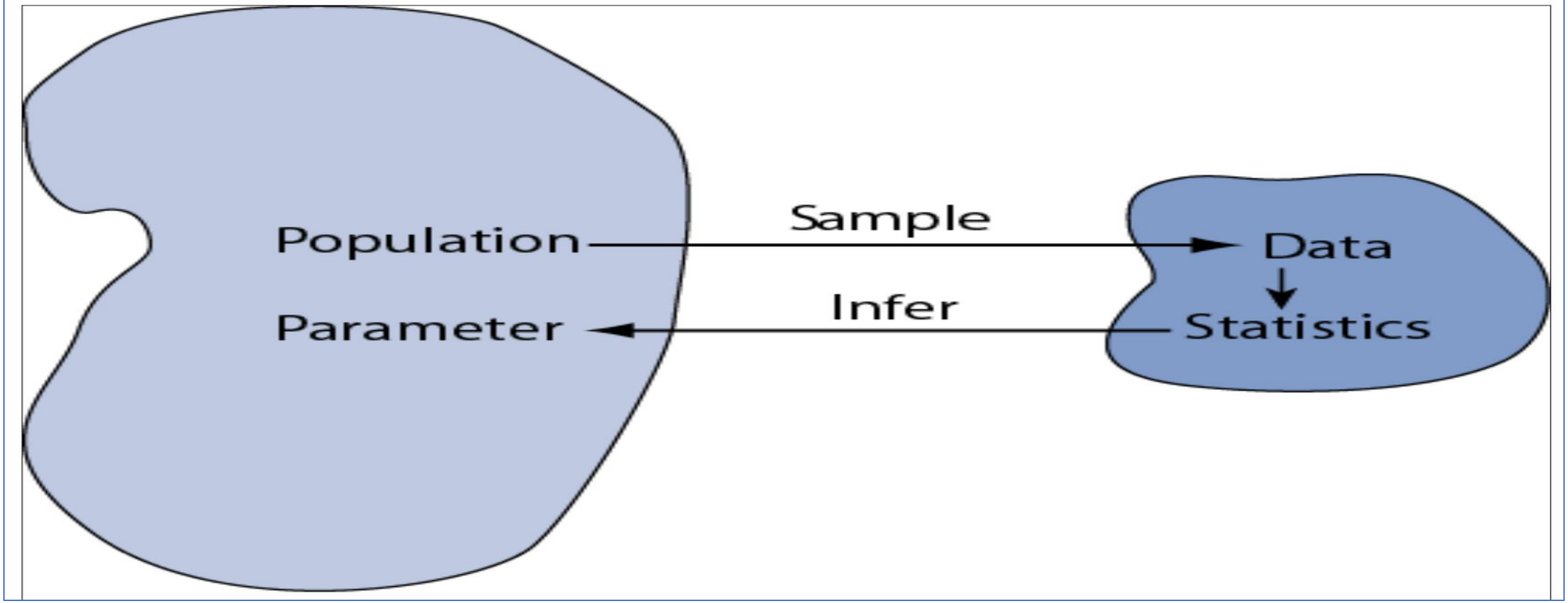
Relationship between a population and sample



Some Important terms

- **Population** ≡ all possible values
- **Sample** ≡ a portion of the population
- **Statistical inference** ≡ generalizing from a sample to a population with calculated degree of certainty
- Two forms of statistical inference
  - **Hypothesis testing**
  - **Estimation**
- **Parameter** ≡ a characteristic of population, e.g., population mean μ
- **Statistic** ≡ calculated from data in the sample, e.g., sample mean ( )

Population and sample



Null and Alternative Hypotheses

Convert the research question to null and alternative hypotheses :

- The **null hypothesis (H_0)** is a claim of "no difference in the population"
- The **alternative hypothesis (H_a)** claims "H_0 is false"
- Collect data and seek evidence against H_0 as a way of bolstering H_a (deduction)

General Procedure of Hypothesis Testing

1. **Parameter of interest:** From the problem context, identify the parameter of interest.
2. **Null hypothesis, H_0:** State the null hypothesis, H_0.
3. **Alternative hypothesis, H_1:** Specify an appropriate alternative hypothesis, H_1.
4. **Test statistic:** State an appropriate test statistic.
5. **Reject H_0 if:** Define the criteria that will lead to rejection of H_0.
6. **Computations:** Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
7. **Conclusions:** Decide whether or not H_0 should be rejected and report that in the problem context. This could involve computing a P-value or comparing the test statistic to a set of critical values.

Sampling Distributions of a Mean

The **sampling distributions of a mean (SDM)** describes the behavior of a sampling mean

$$\bar{x} \sim N\left(\mu, SE_{\bar{x}}\right)$$

$$\text{where } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

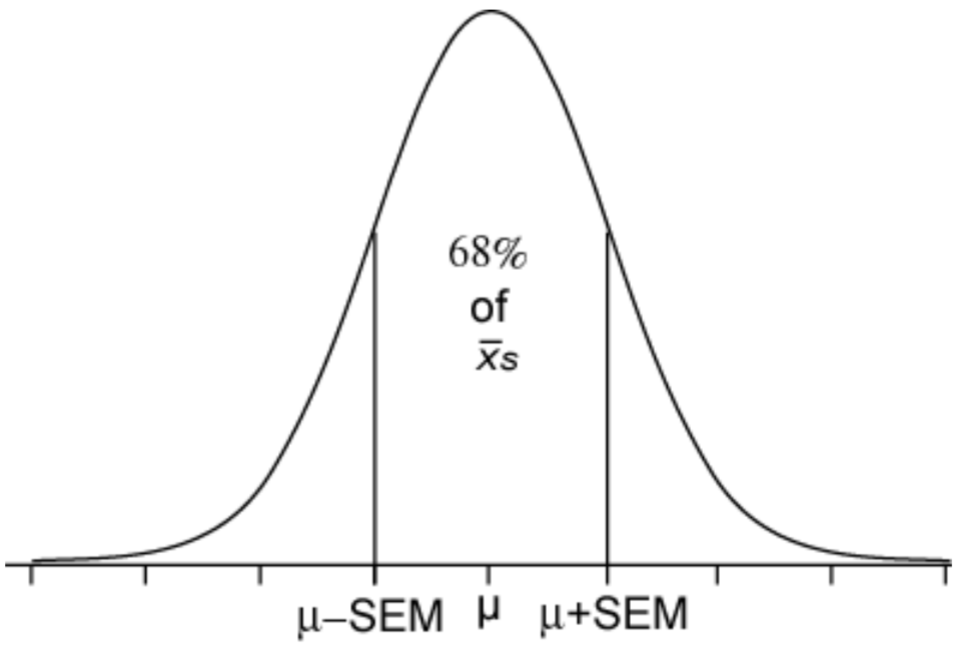


Fig. sdm&se68%.ai

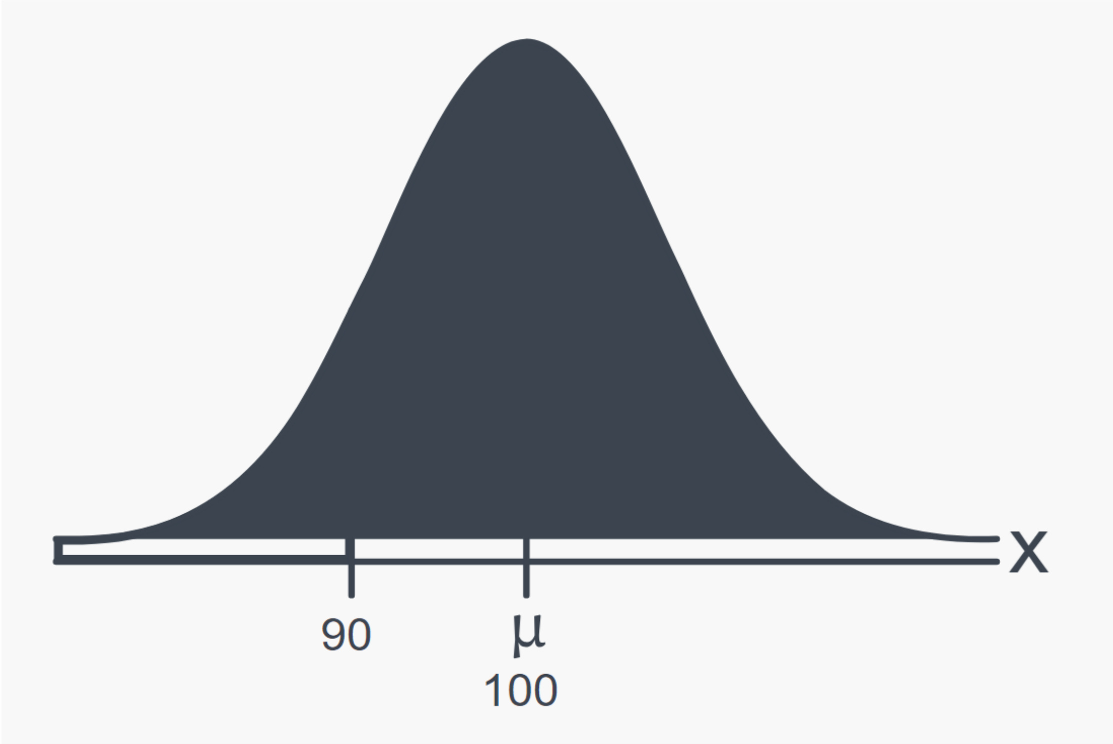
Normal Distribution

- The shape of any normal curve depends on its mean and standard deviation .

- Let X equal the IQ of a randomly selected Indian. Assume mean is 100 and standard deviation is 16. What is the probability that a randomly selected American has an IQ below 90?

If $X \sim N(\mu, \sigma^2)$, then:

$$Z = \frac{X - \mu}{\sigma}$$



Standard Normal Variate

$P\ (X) < 90 \quad , \ z = \frac{100-90}{16}$ =0.69

P(Z<0.69)

0.75490

1-0.75490 = 24%

Hence, there is 24% probability that the IQ is below 90.

Hypothesis Testing

- We like to think of statistical hypothesis testing as the data analysis stage of a comparative experiment, in which the Market Expert is interested, for example, in comparing the mean of a population to a specified value (e.g. mean pull strength).

- Suppose we are interested in knowing that price of a stock is more than 1500.

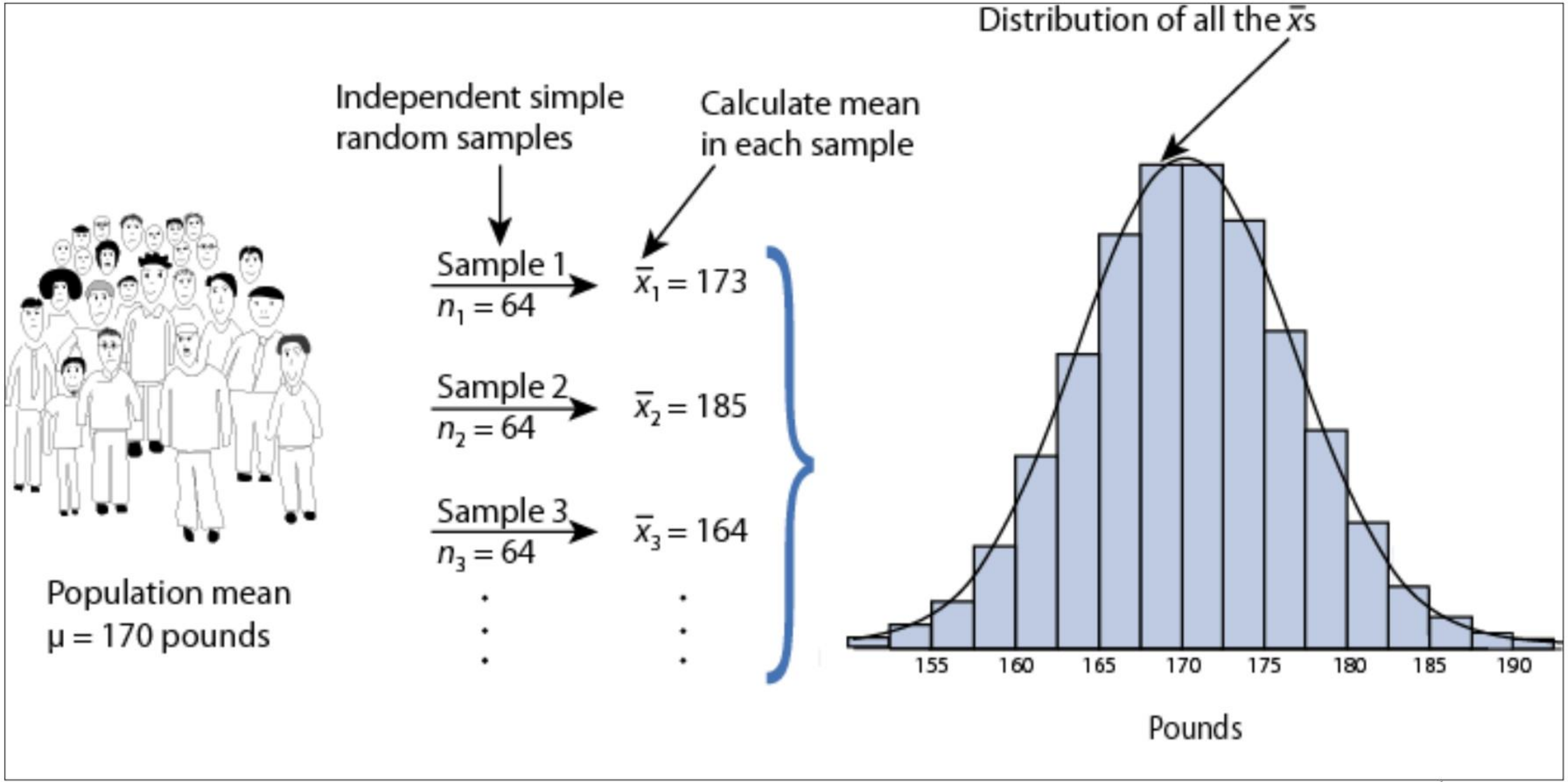
Hypothesis Testing

Two-sided Alternative Hypothesis

$$H_0: \mu = 1500$$

$$H_0: \mu \neq 1500$$

Reasoning Behind μz_{stat}



Sampling distribution of xbar under H_0: μ = 170 for n = 64 ⇒ $\bar{x} \sim N(170,5)$

Test of a Hypothesis

- A procedure leading to a decision about a particular hypothesis

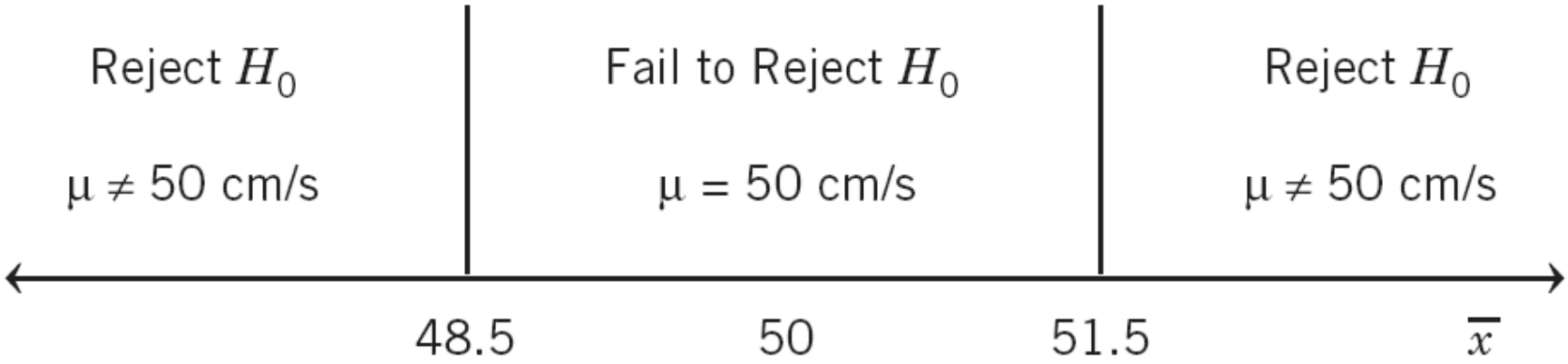
- Hypothesis-testing procedures rely on using the information in a **random sample from the population of interest**.

- If this information is *consistent* with the hypothesis, then we will conclude that the hypothesis is **true**; if this information is *inconsistent* with the hypothesis, we will conclude that the hypothesis is **false**.

Testing Statistical Hypotheses

$$H_0: \mu = 50 \text{ cm/s}$$

$$H_1: \mu \neq 50 \text{ cm/s}$$



Testing Statistical Hypotheses

Rejecting the null hypothesis H_0 when it is true is defined as a **type I error.**

Failing to reject the null hypothesis when it is false is defined as a **type II error.**

Testing Statistical Hypotheses

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Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	No error	Type II error
Reject H_0	Type I error	No error

$$\alpha = P(\text{type I error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ is true})$$

Sometimes the type I error probability is called the **significance level**, or the **α-error**, or the **size** of the test

Testing Statistical Hypotheses

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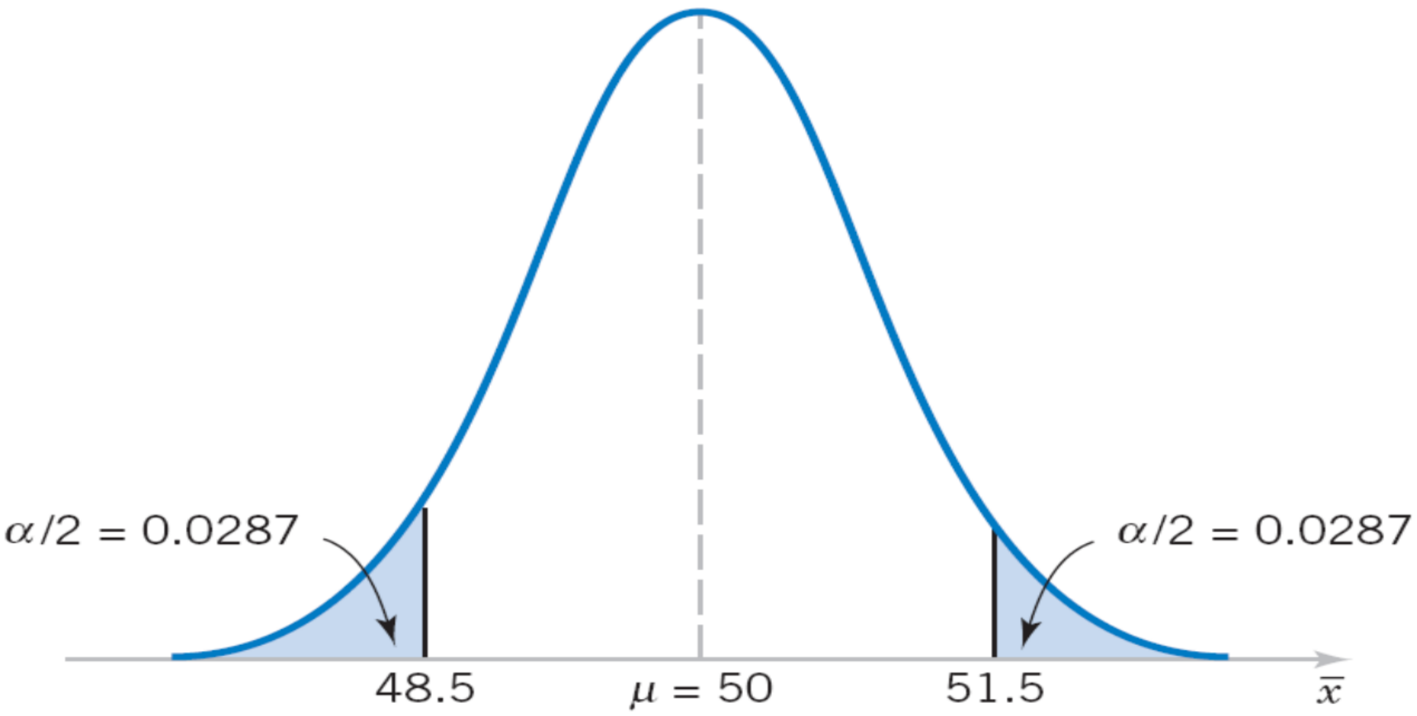


Testing Statistical Hypotheses

The **power** of a statistical test is the probability of rejecting the null hypothesis H_0 when the alternative hypothesis is true.

- The power is computed as 1 - β, and power can be interpreted as *the probability of correctly rejecting a false null hypothesis*. We often compare statistical tests by comparing their **power** properties.

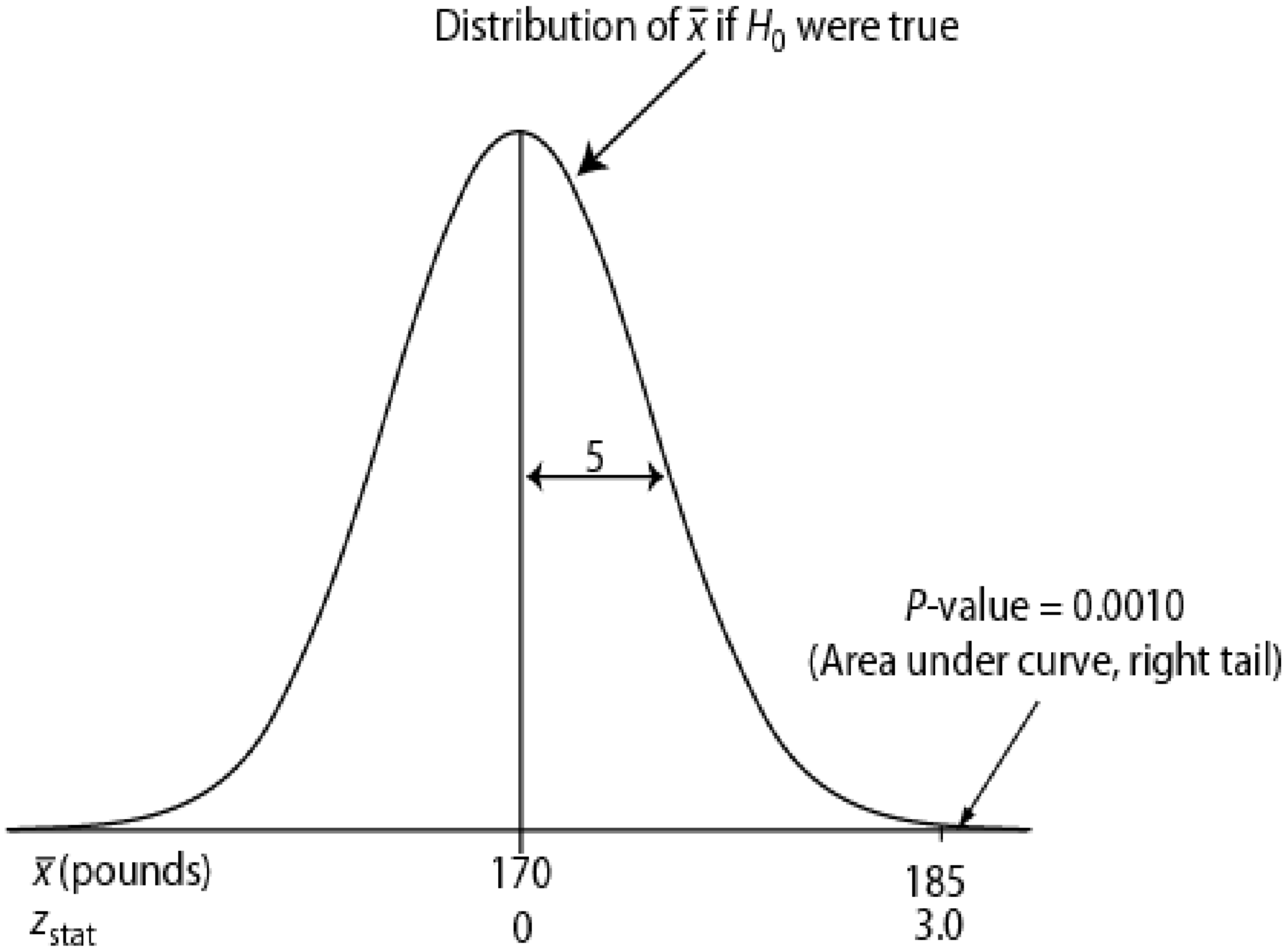
- For example, consider the propellant burning rate problem when we are testing H_0 : μ = 50 centimeters per second against H_1 : μ not equal 50 centimeters per second . Suppose that the true value of the mean is μ = 52. When $n = 10$, we found that β = 0.2643, so the power of this test is 1 - β = 1 - 0.2643 = 0.7357 when μ = 52.

P-value

What is the probability of the observed test statistic … ***when H_0 is true***?

This corresponds to the AUC in the tail of the Standard Normal distribution beyond the z_{stat}

P-value



Distribution of $\bar{x}$ if H_0 were true
5
P-value = 0.0010
(Area under curve, right tail)
$\bar{x}$ (pounds)
z_{stat}
170
0
185
3.0

P-Value

- Thus, smaller and smaller *P*-values provide stronger and stronger evidence against H_0

- Small *P*-value $\Rightarrow$ strong evidence

α-Level

- Let $\alpha \equiv$ probability of erroneously rejecting H_0
- Set α threshold (e.g., let α = .10, .05, *or whatever*)
- Reject H_0 when $P \leq \alpha$
- Retain H_0 when $P > \alpha$
- Example: Set α = .10. Find P = 0.27 $\Rightarrow$ retain H_0
- Example: Set α = .01. Find P = .001 $\Rightarrow$ reject H_0

One-Sample z Test

A. Hypothesis statements
   H_0: $\mu = \mu_0$ vs.
   H_a: $\mu \neq \mu_0$ (two-sided) or
   H_a: $\mu < \mu_0$ (left-sided) or
   H_a: $\mu > \mu_0$ (right-sided)

B. Test statistic

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} \text{ where } SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

C. P-value: convert z_{stat} to P value

D. Significance statement (usually not necessary)

Conditions for z test

- σ known (not from data)

- Population approximately Normal or large sample (central limit theorem)

Statistical Inference

- Let X represent Trader Intelligence scores for stock price forecasting.
- Typically, $X \sim N(100, 15)$
- Take SRS of $n = 9$ from trader population
- Data $\Rightarrow$ {116, 128, 125, 119, 89, 99, 105, 116, 118}
- Calculate: x-bar = 112.8
- Does sample mean provide strong evidence that population mean $\mu > 100$?

A. Hypotheses:

H_0: μ = 100 versus

H_a: μ > 100 (one-sided)

H_a: μ ≠ 100 (two-sided)

B. Test statistic:

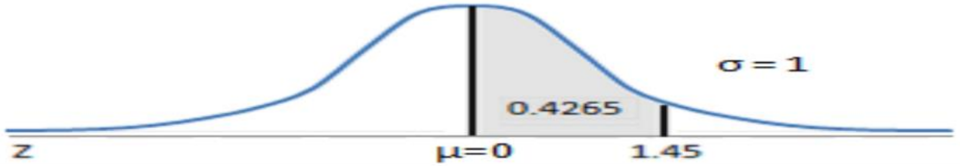
A. Test statistic:

$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{9}} = 5$$

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{112.8 - 100}{5} = 2.56$$

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

A. Test statistic:

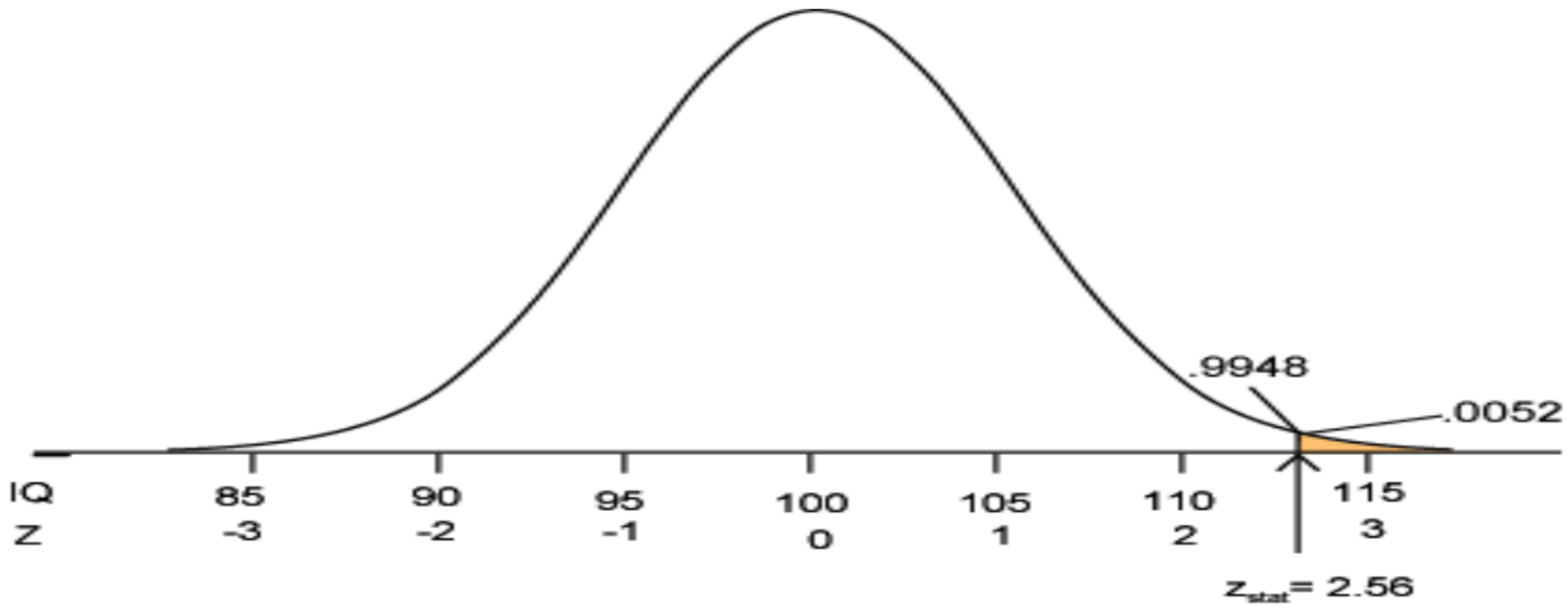
$$SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{9}} = 5$$

$$z_{\text{stat}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{112.8 - 100}{5} = 2.56$$

Calculated Value = 2.56 , tabulated value = 1.96, reject null hypothesis

Statistical Inference

***P*-value:** $P = \Pr(Z \geq 2.56) = 0.0052$



P =.0052 ⇒ it is unlikely the sample came from this null distribution ⇒ strong evidence against H_0

Two-sided P value

H_a: $\mu \neq 100$

Considers random deviations "up" and "down" from $\mu_0 \Rightarrow$ tails above and below $\pm z_{stat}$

Thus, two-sided *P*

$= 2 \times 0.0052$

$= 0.0104$

Question

Suppose you start up a company that has developed a drug that is supposed to increase IQ. You know that the standard deviation of IQ in the general population is 15. You test your drug on 36 patients and obtain a mean IQ of 97.65. Using an alpha value of 0.05, is this IQ significantly different than the population mean of 100?

Question

$$z = \frac{97.65 - 100}{2.5}$$

$$= -0.94$$

Level of Significance =0.05, two tailed , 0.05/2 =0.025, Z value = -1., Since calculated value is less than tab null accepted.