Futures and Options

Module 1 Session 5 ,6 , 7 & 8

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Revision of last Session

Cash / Spot Market

- In a Cash/Spot market, the three components of any transaction

✓ **Price Negotiation ;**

✓ **Clearing** (*Determination of funds & asset obligations of each party*), &

✓ **Settlement** (*Exchange of respective obligations*)

takes place with ***'no' or ' with minimal delay'*** (on the spot).

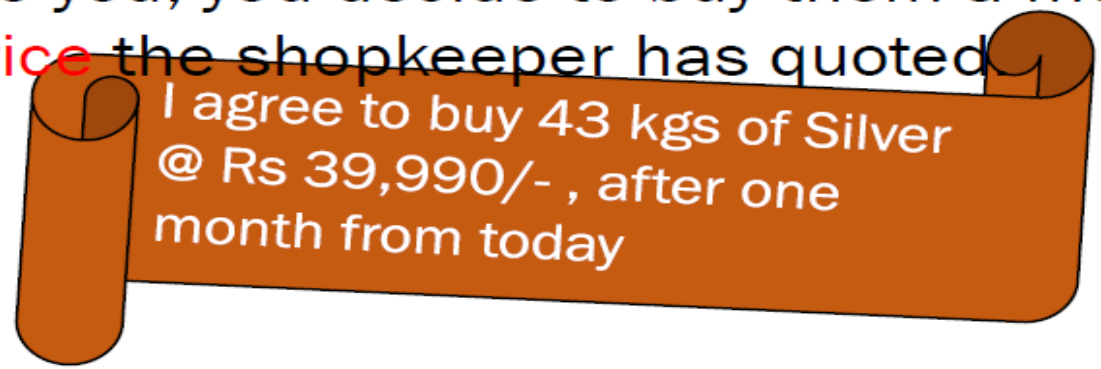
Suppose you wish to buy 43 kgs of Silver today.

- ✓ Enquire about the price of the Silver.
- ✓ The shopkeeper informs that it will cost Rs.37,870/- per kg of Silver.
- ✓ If the price is agreeable to you, you decide to buy.
- ✓ Shopkeeper then calculates how much you have to pay and set aside 43 kgs of Silver.
- ✓ You pay Rs. 16,28,410/- and take delivery of 43 kgs of Silver.

Forward Market

Now, suppose you wish to buy 43 kgs of Silver ***after one month****, instead of today.*

- ✓ You tell the store owner that you want to buy 43 kgs of Silver, ***a month later***.
- ✓ The shopkeeper informs that it will cost Rs. 39,990/- per kg of Silver.
- ✓ If the price is agreeable to you, you decide to buy them a month later at the one-month price the shopkeeper has quoted.



- ✓ *One month later*, you pay Rs 17,19,570/- and take delivery of 43 kgs of Silver.

This is a Forward contract entered into in the Forward market.

Forward Contract

Today — After One Month

Buyer buys a Forward Contract (or goes long) on Silver.
No amount changes hands today.

Buyer pays Rs.17,91,570/- to the Seller, who delivers to the buyer of the Forward contract, 43 kgs of Silver.

- If the price is less than Rs. 39,990/-per kg, the buyer will make a loss.
- If the price is more than Rs. 39,990/-per kg, the buyer will make a profit.

If the Spot price moves up.....

- Pay-off (or Profit) from a Long position in the Forward Contract is: **Spot Price on Maturity (S_T) *less* Forward Price (F_o)**

Pay-offs = profit/loss from the forward contract, as it costs nothing to enter into a Forward contract.

If the Spot price moves down.....

- Pay-off from a Short position in the Forward Contract is:
  Forward Price (F_o) *less* Spot Price on Maturity (S_T)

Futures Exchanges in India

Futures Exchange	Futures Contracts
National Stock Exchange (NSE) www.nseindia.com	• Equity Index Futures • Single Stock Futures • Currency Futures • Interest Rate Futures • VIX Futures
Bombay Stock Exchange (BSE) www.bseindia.com	• Equity Index Futures • Single Stock Futures • Currency Futures • Interest Rate Futures
Metropolitan Stock Exchange of India (MSEI) www.msei.in	• Equity Index Futures • Single Stock Futures • Currency Futures • Interest Rate Futures

Futures Exchanges in India

Futures Exchange	Futures Contracts
National Commodity & Derivatives Exchanges Ltd (NCDEX) www.ncdex.com	• Cereals& Pulses: Bajara, Barley, Chana, Mazie, Wheat • Oil & Oil seeds: Castor seeds, Cotton seeds, Soyabean, Refined Soy Oil, Mustard Seed, Crude Palm Oil. • Fibres: Kapas, Cotton • Soft: Sugar, Gur • Guar Complex: Guar Seed & Gum • Spices: Pepper, Turmeric, Jeera, Chilli, Coriander • Plantation Products: Rubber • Others: Potato • Metals: Steel, Copper • Precious Metals: Gold, Silver • Energy: Crude Oil, Brent Crude Oil.

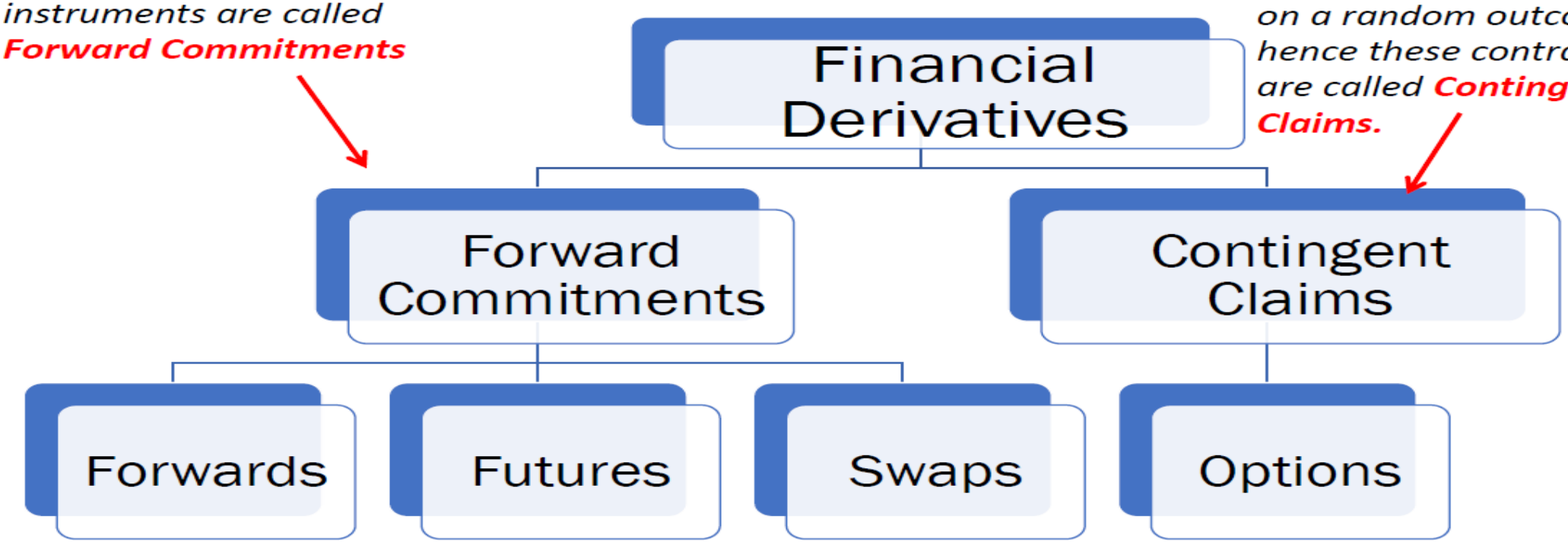
Futures Exchanges in India

Futures Exchange	Futures Contracts
National Multi Commodity Exchange of India Limited (NMCE) www.nmce.com	• Oils & Oil seeds: Castor seed, Copra, Soyabean Oil, Mustard Seed, Guar Seed. • Pulses: Chana • Spices: Pepper, Cardamom • Others: Rubber, Sacking, Coffee Rep Bulk, Isabgul seed
Multi-Commodity Exchange of India Limited (MCX) www.mcxindia.com	• Bullion: Gold, Silver • Base Metals: Aluminum, Copper, Lead, Nickel, Zinc • Energy: Crude Oil, Brent Crude Oil, Natural Gas • Agro-Commodities: Cardamom, Cotton, Crude Palm Oil, Kapas, Mentha Oil

Financial Derivatives

As these contracts force the two parties to transact in the future at the pre-determined price, these instruments are called ***Forward Commitments***

As the choice of buying or selling the underlying vs. doing nothing depends upon on a random outcome, hence these contracts are called ***Contingent Claims.***



Forward Contract

- In a forward contract, the two parties ***irrevocably*** agree on the **price** and **quantity** of goods to buy/ sell today, ***but actual payment and delivery of goods takes place at a future date.***
- No exchange of goods or money takes place at the time of entering into a Forward Contract.
- Buyer of the Contract is said to have a "Long" position while the seller has a "Short" position .
- On the maturity (or expiry) date, the buyer is **obligated** to pay for the goods and take delivery and the seller is **obligated** to deliver goods and receive the payment, *irrespective of the price of the goods on the expiry date.*

Forward Contract

- Each Forward Contract is a Unique Contract, customized to the requirements of the contracting parties.
- Such contract are Over-the-Counter (OTC) products & not traded freely.
- Forward Contracts are mostly settled on maturity with the seller delivering the underlying assets and the buyer paying the specified price.

Issues with Forward Contracts

- Counterparty Risk: As Forward contracts are bilateral relationship between two parties, unfavourable price movement, may tempt one adversely affected party to declare itself bankrupt and avoid performance. →Counterparty risk.
- Illiquidity: Forward Contracts are Custom-designed and situation specific contracts, involve too much of "flexibility" which makes the contracts non-tradable. as others might not find those specific terms useful to them.
- Often, forward markets turn into small private clubs of dealers, who earn high intermediation fees.
- This makes forward markets an illiquid market.

Futures Contract

- Futures contract is an agreement to buy or sell an asset for a certain price at a certain future date.
- A Futures contract ISa forward contract, except that it is standardized and hence are traded on an Exchange.
- It is characterized by the following:
- Standardization of the Contract

Traded on Exchange means:

- Centralization of Trading
- Enhanced liquidity
- Novation by Clearinghouse
- Mark-to-Market (MTM) margining system

Standardisation of Futures Contract

- Futures Exchange specifies in detail the nature of a Futures contract, subject to regulatory approval.
- Futures contract are specified in terms of:
- Underlying asset
- Contract size
- Delivery arrangements
- Delivery month
- Price quotes
- Price & Position Limits

Underlying Asset

- In case of Commodity Futures, the Exchange stipulates the underlying assets and grade(s) that are acceptable.

✓ Silver(MCX): Grade 999, Fine 999 (as per IS2112:1981)

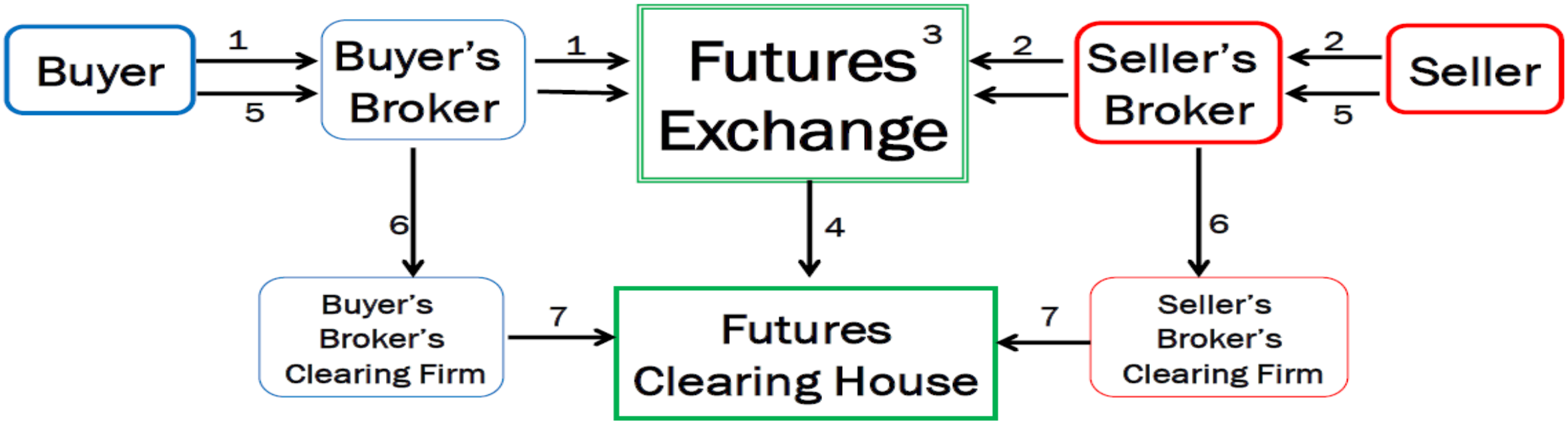
✓ Aluminum (MCX) : Min purity 99.7%

✓ Copper (MCX): Grade 1 electrolytic copper (as per B115 specification)

- For some commodities, a range of grades may be delivered with adjustment in price based on the grade delivered.
  - ❑ Standard Grade: No. 2 Yellow for Corn Futures (CME)
  - ❑ Acceptable Grade: No. 1 Yellow with 1.5 cents per bushel more or No. 3 Yellow with 1.5 cents per bushel less than No. 2 Yellow.
- In case of Financial assets - Futures contracts are well defined & unambiguous.

✓ Enough to say 'Futures on BSE Sensex' or 'Futures on Infosys'

Transaction on a Futures Exchange



1: Buyer places a BUY order with his Broker who in turn places it with the Futures Exchange.
2: Seller places a SELL order with his Broker who in turn places it with the Futures Exchange
3: Futures Exchange matches the trade through a computerized system.
4: Information about the trade is reported to the Clearing House.
5: Buyer and Seller deposit margin with their respective brokers.
6: Buyer's and Seller's Brokers deposit the margins with their respective clearing firms.
7: Clearing firms deposit the margins with the Clearing House.

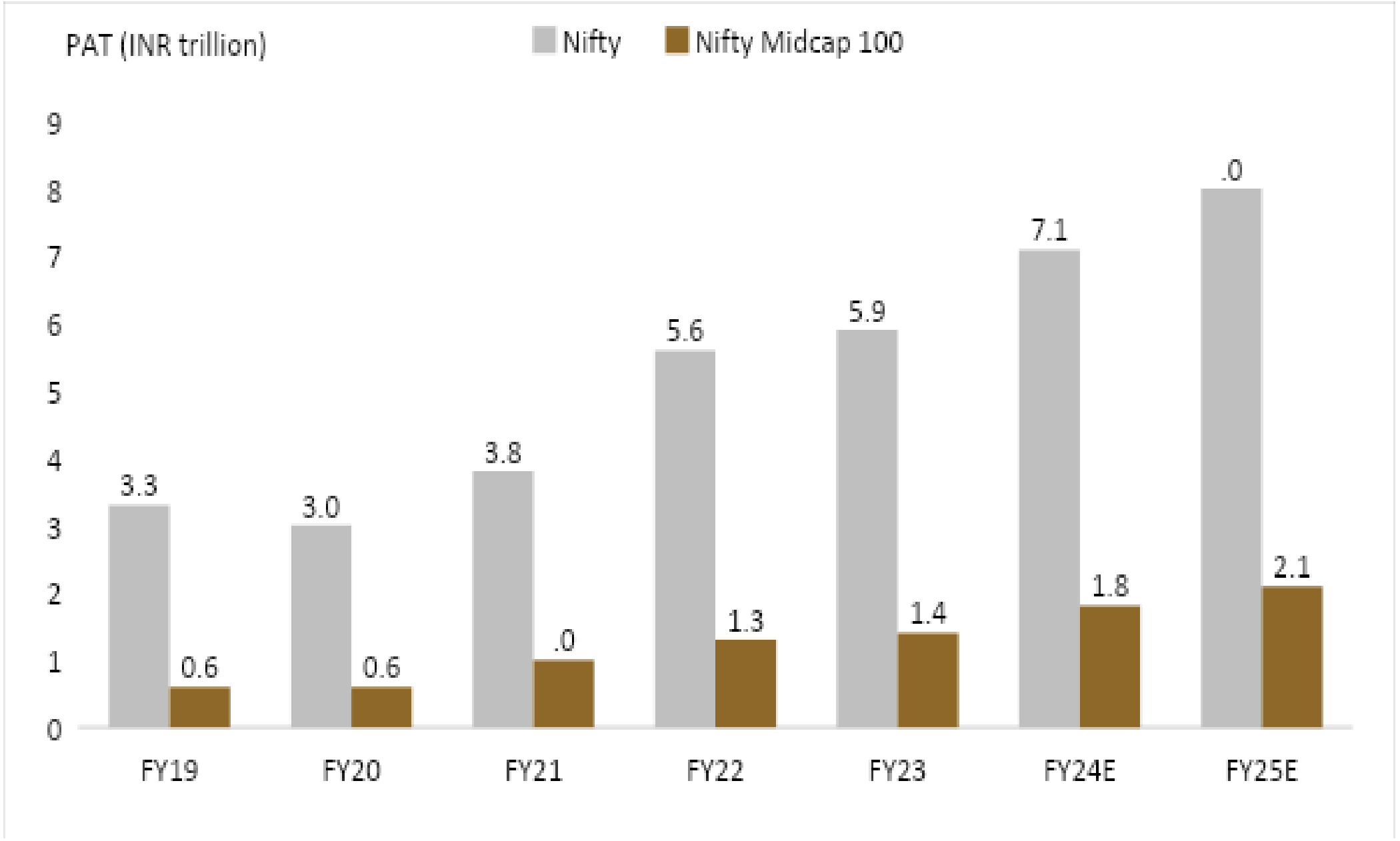
News

Stock market has rallied to all-time high valuation and the broader market is too expensive: Is that true?

Nifty is trading at 20.6x one year forward EPS – a premium of 15% over the last 10 years' average, but 10% cheaper than October'21 highs

Nifty PAT is predicted to have increased by 2.4x. The observation that markets reward greater growth is not surprising.

ET 15/03/2024


PAT (INR trillion)
Nifty
Nifty Midcap 100
9
8
7
6
5
4
3
2
1
0
3.3
0.6
3.0
0.6
3.8
.0
5.6
1.3
5.9
1.4
7.1
1.8
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FY19
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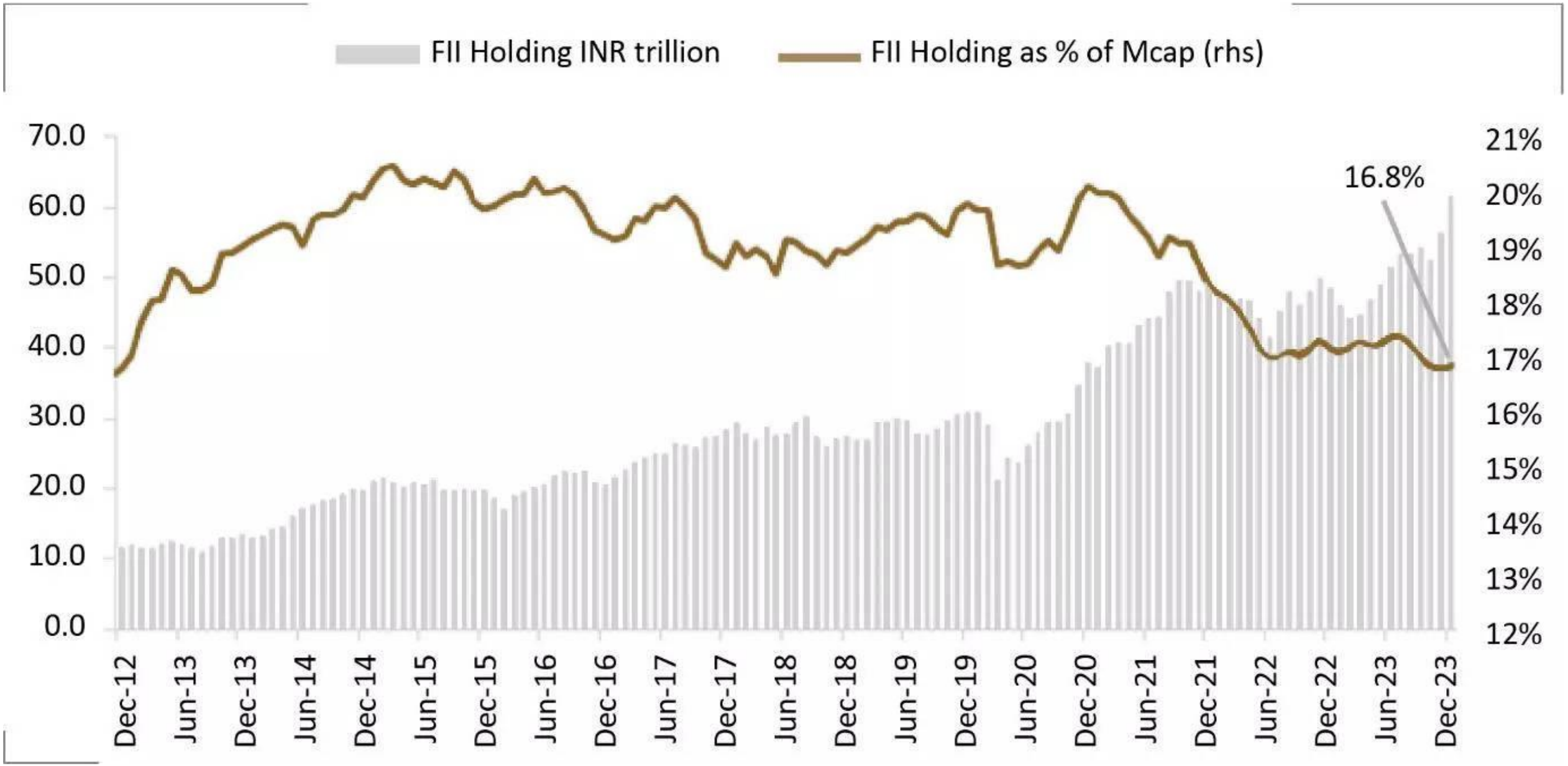

FII Holding INR trillion
FII Holding as % of Mcap (rhs)
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
21%
20%
19%
18%
17%
16%
15%
14%
13%
12%
16.8%
Dec-12
Jun-13
Dec-13
Jun-14
Dec-14
Jun-15
Dec-15
Jun-16
Dec-16
Jun-17
Dec-17
Jun-18
Dec-18
Jun-19
Dec-19
Jun-20
Dec-20
Jun-21
Dec-21
Jun-22
Dec-22
Jun-23
Dec-23

Linkage between Spot & Derivatives Market

- **Arbitrage and the Law of One Price**: Arbitrage process ensures that prices of an asset would be same across markets.
- **Storage Mechanism:** Storage is a form of investment which entails risk. Derivatives can be used to reduce that risk by providing a means of establishing today, that item's future sale price.
- **Delivery & Settlement**: At expiry, a forward / futures contract calls for immediate delivery of the underlying asset or cash payment. Thus, expiring forward /futures contract is like a Spot transaction. The price of the expiring contract should therefore be equal to spot price.
- *Hence, Derivative & Spot Markets are inextricably linked.*

Role of Derivatives Markets

- Risk Management: Derivatives are used to reduce risks.
- A wheat farmer sells his harvest (at the time of sowing itself) in the futures market there by reducing the price risk, which he faces at the time of harvest.
- Market participants whose wants to reduce their risk are called Hedgers.

- **Speculation**: Derivatives can serve as investment vehicles. Derivatives can provide away to make bets that are highly levered.
- If a Speculator has information about possible upturn in a price, he may go long on the forward market instead of the cash market.
- ▪Market participants whose wants to increase their risk are called Speculators.

Role of Derivatives Markets

- Price Discovery: Forward and futures markets are an important source of information about prices.
- Futures price is a consensus reflecting the spot price of an asset in the future.
- **Operational Advantages:**
- Derivative markets entail lower transaction costs, hence makes them easier and attractive to use in lieu of spot markets
- Derivatives Markets are usually more liquid (as less investment is required)
- Also allow for short selling of securities (not allowed in some spot markets)

- Market Efficiency: Through the use of arbitrage, the market efficiency is improved further.

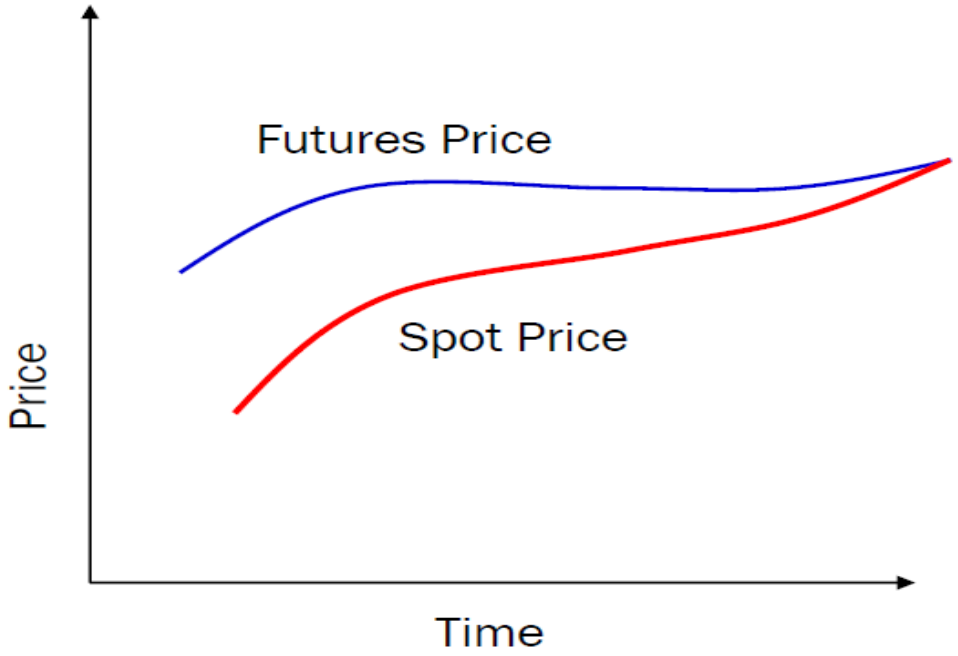
Equity Futures on NSE

Parameter	Index Futures	Futures on Individual Securities
Underlying	• Nifty 50 • Nifty IT • Nifty Bank • Nifty Midcap 50 • Nifty Infrastructure • Nifty PSE • Nifty CPSE • India VIX • DJIA • S&P 500 • FTSE 100	209 securities
Instrument	FUTIDX	FUTSTK
Trading Cycle	3 month trading cycle - the near month (one), the next month (two) and the far month (three)	
Expiry Day	Last Thursday of the expiry month. If the last Thursday is a trading holiday, then the expiry day is the previous trading day.	
Permitted Lot Size	Underlying specific	Underlying specific
Price Steps	Rs.0.05	Rs.0.05

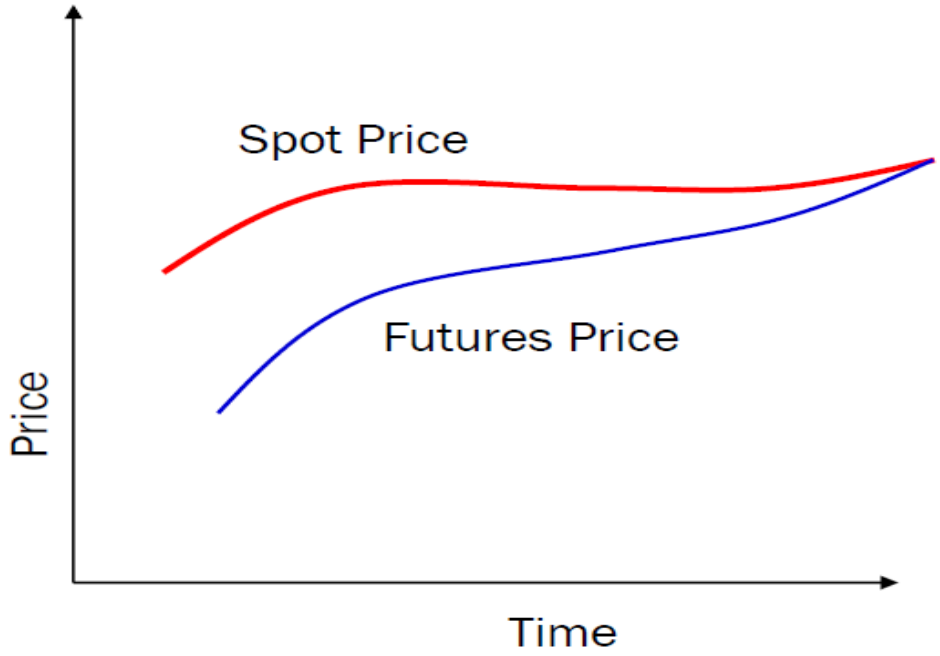
Currency Futures on NSE

Symbol	USDINR	EURINR	GBPINR	JPYINR
Unit of trading	1 - 1 unit denotes 1000 USD.	1 - 1 unit denotes 1000 EURO.	1 - 1 unit denotes 1000 POUND STERLING.	1 - 1 unit denotes 100000 JAPANESE YEN.
Underlying / Order Quotation	The exchange rate in Indian Rupees for US Dollars	The exchange rate in Indian Rupees for Euro.	The exchange rate in Indian Rupees for Pound Sterling.	The exchange rate in Indian Rupees for 100 Japanese Yen.
Tick size	0.25 paise or INR 0.0025			
Trading hours	Monday to Friday **9:00 a.m. to 5:00 p.m.**			
Contract trading cycle	12 month trading cycle.			
Last trading day	Two working days prior to the last business day of the expiry month at 12 noon.			
Final settlement day	Last working day (excluding Saturdays) of the expiry month. The last working day will be the same as that for Interbank Settlements in Mumbai.			
Initial margin	SPAN Based Margin			

Convergence of Futures to Spot Price



Contango
(Futures > Spot)



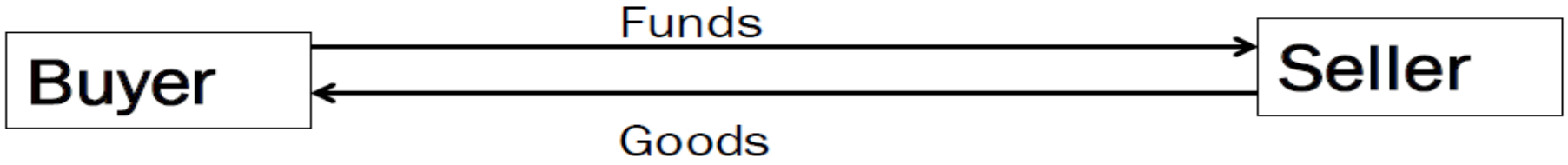
Backwardation
(Spot > Futures)

Clearing House

- To ensure smooth functioning, each Futures Exchange has a Clearing House (CH) associated with it.
- CH “guarantees” ALL trades on the Exchange.
- This is achieved by CH “adopting” the position of a Buyer for every Seller & “Seller” for every “Buyer”.
- Each trader has obligations only to the CH & hopes that CH will execute its side of the trade as well.
- CH substitutes its own credibility for the promise of each trader.
- CH, however, does not take ACTIVE position but “interposes” itself between all parties to every transactions.

- NSE : National Securities Clearing Corporation Ltd (NSCCL)
- BSE : Indian Securities Clearing Corporation Ltd (ISCCL)
- MSEI: MCX-SX Clearing Corporation Limited (MCCL)

Obligations without Clearing House



- Without CH, both parties would deal with each other-direct obligation to each other.

Obligations with Clearing House

- With CH, each party has obligation to the CH which ensures that both parties perform, the two parties need not trust or know each other.
- They need to be concerned about the reliability of the CH. Hence, CH is a large, well-capitalised Institution.

Operation of Margins

- When 2 investors contract, one party may back out or may not have the financial resources to honour his commitments.
- Futures Exchange organizes trades so as to avoid defaults, through a system of margins.
- Different types of margins are maintained:

✓Initial Margin (IM)

✓Maintenance Margin (MM)

✓Variation Margin (VM)

Operation of Margins

- Initial Margin (IM):Good faith deposit paid by the trader at the time of entering the contract to ensure performance.
- IM may vary from contract to contract & from trader to trader.
- Typically set at 5% of the contract value.
- Trader retains title to the deposit.
- Usually equal to Maximum Daily Price fluctuation limit.
- IM is returned upon proper completion of all the obligations.
- At the end of each day, the initial margin account is adjusted to reflect investor's gain/loss. →daily settlement or Mark–to –market

Operation of Margins

- **Maintenance Margin (MM)** :(% of the Initial Margin) is the minimum amount of margin below which amount in Initial Margin account should NOT fall.
- MM is used to calculate the third margin –Variation Margin (VM).
- If the Initial Margin account falls below the MM, trader is required to replenish (*or top-up*) the Initial Margin account.
- This additional amount paid by the trader is called Variation Margin.
- Any amount in excess of the IM can be withdrawn by the investor.
- Failure to pay VM leads to the futures position being closed out.

Operation of Margins -*An Example*

- Suppose Mr X is long on 5 Futures contract on gold at MCX. Each contract is for 100 grams. The price quoted is Rs. 15,550/-per 10 grams.
- Tick size is Re 1.
- Initial margin is 4% while maintenance margin of 90% of initial margin.

Operation of Margins

	A	B	C	D	E	F	G	H
1	Opening Price			15,550	/10 g			
2	No. of Futures Contract			5				
3	Contract size			100	g			
4	Initial Margin			4%	31,100			
5	Maintenance Margin			90%	27,990			

Operation of Margins

	B	C	D	E	F	G	H
23	**Without Mark-to-Market:**						
24	Profit on **Long** Position: (Closing Price - Opening Price)*100/10*5						
25	= (15870 -15550)*100/10*5 = **Rs. 16,000/-**						
26	Profit on **Short** Position: (Opening Price - Closing Price)*100/10*5						
27	= (15550-15870)*100/10*5 = **(Rs. 16,000/-)**						
28	**With Mark-to-Market:**						
29	Profit/Loss = Final Margin - Initial Margin - Margin Call Paid						
30	**Long** Position = 63600-31100-16500= **Rs.16,000/-**						
31	**Short** Position = 31100 - 31100 -16000 =**(Rs.16,000/-)**						
32							

Closing a Futures Position

- Delivery: Delivery of the goods under the contract will automatically close the position.
- Physical Settlement: Physical delivery of the asset at a certain location at a specified time as per the exchange rules.
- Decision regarding location of delivery is with the seller.
- If seller decides to deliver the underlying, it would issue a “Notice of intention to Deliver” to the exchange.
- The exchange will then choose the party with the long position to accept the delivery.
- Usually, the notice is passed to the party with the oldest long position outstanding.
- Parties with long positions must accept delivery.
- Cash Settlement: Traders make payment at expiry of contract to settle any gain or loss.

Closing a Futures Position

- Offset: Most Futures contracts are settled by “Offsets”, by entering into a exactly reverse trade which shall cancel the original trade.
- •The trader, in order to close the contract, should enter into an exactly reverse contract in terms of:

  (a)the underlying assets,

  (b)No. of contracts &

  (c)expiry date

Offset Trades – *An Example*

May 1	**Party A's Initial Position**: Bought 1 September Wheat Futures Contract @ Rs 2,200/-.	**Party B** : Sold 1 September Wheat Futures Contract @ Rs 2,200/-.
May 15	**Party A's Reversing Trade**: Sold 1 September Wheat Futures Contract @ Rs 2,300/-.	**Party C**: Bought 1 September Wheat Futures Contract @ Rs 2,300/-.

After the two trades, A's net position is Zero and is out of the market. B & C still have obligations to the CH.

Closing a Futures Position (Contd.)

- Exchange for Physicals: Buyer and Seller exchange for cash, the underlying asset outside the exchange system.
- EFP vs. Offset:
- • Under both, the traders have completed their obligations & are now out of the market.
- • Differs from Offsets:
- • Traders actually exchange the physical goods.
- • Futures is not closed by a transaction through the Exchange.
- • Traders privately negotiate the terms, hence also called “ex-pit”

Open Interest

- ‘Open Interest’ refers to the number of futures contracts outstanding (not squared off) at any point in time. It is the total no. of ‘open’ positions waiting to be liquidated before the contract’s maturity.
- OI rises with time as more and more investors enter into new contracts.
- As maturity approaches, investors unwind their positions by entering into reverse trades. Hence, OI starts to decline.
- Today’s newspaper carry yesterday’s trading data and day before yesterday’s Open Interest data.
- ✓Every trade needs a buyer & a seller
- ✓Any trade (long or short) initiated afresh raises OI
- ✓Any trade (long or short) that squares up existing position lowers OI

Basic Terms

- **Short selling:** Selling of an asset that is not owned by the Investor (seller).
- The investor borrows (thru his broker) the asset from someone who owns the asset.
- Investor has to pay to the owner of the asset, any income that is received on the asset shorted.
- Investor is required to maintain a margin account with his broker to ensure that the investor does not run away from his short position.

Determination of Forward/Futures Prices

Assumptions:

- No transaction costs.
- Uniform tax rate on all trading profits.
- All participants can borrow and lend at the same risk-free rate of interest.
- Market participants take advantage of the arbitrage opportunities which may arise.

Notations:

S_0 : Price of the underlying asset (in Cash Market) at t=0.

F_0 : Forward / Futures Price (in Futures Market) at t=0

r_f : Zero – Coupon risk-free rate pa. on continuous compounding basis.

Underlying asset provides no income

- Suppose a non-dividend paying stock is available for Rs. 500/-(S_O) and risk-free rate of Interest (r_f) is 5% pa.
- ***IF*** the 3-month Forward price (F_O) is relatively high at Rs. 535/-.
- An arbitrageur will earn risk-less profit, if he adopts the following trading strategy:

Today:

a) Borrows Rs. 500/- for 3 months @ 5% pa.

b) Buys the stock for Rs.500/- in the Cash Market

c) Sells 3-month forward contract on the stock at F_O of Rs. 535/-.

Underlying asset provides no income

At the end of 3 months:

The arbitrageur shall:

a) Repay the amount borrowed along with Interest

= Rs. 500/- + Interest @ 5% pa for 3 months

= $500*e^{(0.05*3/12)}$ = 500*1.0126= Rs. 506/- (approx.)

b) Deliver the stock under the Forward contract and receive Rs. 535/- .

➢ Cash Flows: **Inflow: Rs 535/- Outflow: Rs. 506/-**

- By following this strategy, the arbitrageur has locked in a risk-less profit of Rs.29/- i.e. (535-506)

Underlying asset provides no income

- *IF* the 3-month forward price (F_0)is relatively low as Rs. 475/- (instead of Rs. 535/-)
- An arbitrageur can adopt the following strategy and make risk-less profit:

Today:

a) Short Sell the stock @ Rs. 500/- in the cash market,

b) Invest the proceeds for 3-months @ r_f =5%p.a., and

c) Buy a 3-month forward contract on the stock @ Rs. 475/-

Underlying asset provides no income

At the end of 3-months:

The arbitrageur would:

a) Receive the loan of Rs. 500/- along with interest for 3 months = $500 * e^{(0.05)*(3/12)}$ = Rs. 506/-

b) Receive the stock under the forward contract and pay Rs 475/- for it.

➢ Cash Flows: **Inflow: Rs 506/- Outflow: Rs. 475/-**

- By following this strategy, the arbitrageur has locked in a risk-less profit of Rs.31/- i.e. (506 - 475)

Underlying asset provides no income

- Thus, so long the forward price is NOT equal to Rs. 506/- there would be an opportunity for arbitrageur to adopt either of the trading strategies and earn a risk-less profits.
- For NO ABITRAGE to take place, the Forward Price (F_0) should be exactly Rs. 506/-.

$$\text{Forward Price } (F_0) = S_0 e^{r_f T}$$

- $F_0 = S_0 e^{rT} = 500e^{(0.05)3/12} = 500 * 1.0126 =$ **Rs. 506/-**

Underlying asset provides no income

A 6-month forward contract on a non-dividend paying stock is entered into when the stock price is Rs. 2800/-. The risk-free rate of return is 8%pa.What should be the 6-month forward price?

S_0 = 2,800/- ; r_f = 8% pa ; T = 6/12 years = 0.5 years

Forward Price (F_0) $= S_0 e^{rT}$

$= 2800e^{0.08*0.5} = 2800*1.04081$

= Rs.2,914.27

Underlying asset provides no income

Consider the stock of Suzlon which is currently trading at Rs 85/-. The 3-month forward contract on Suzlon stock is available for Rs. 90/-. If the risk-free rate is 6% pa. (on continuous compounding basis) and the stock is not expected to pay any dividend over the next 3 months. Are there any opportunities for arbitrage?

Underlying asset provides no income

S_0 = 85/- ; r_f = 6% pa ; T = 3/12 years = 0.25 years;

F_{ACTUAL} = Rs.90/-

Theoretical Forward Price (F_0) = $S_0 e^{rT}$

$= 85e^{0.06*0.25}$ = Rs. 86.28

Theoretical Forward Price < Actual Forward Price, hence arbitrage opportunity exists:

Adopt Cash-and-Carry Arbitrage:

Now:

- Borrow Rs 85/- for 3 months @6%pa
- Buy stock @ Rs. 85/-
- Sell 3-month Forward @ Rs. 90/-

After 3 months:

- Repay loan with interest: $85e^{0.06*0.25}$= Rs. 86.28
- Deliver stock under forward contract and receive Rs. 90/-
- Profit = 90 – 86.28 = **Rs. 3.72**

Underlying asset provides no income

Shares of Megacorp are currently trading at Rs. 40/- and the 6-month forward on the stock is available at Rs. 38/-. If the risk-free rate is 8% pa. (cc) and the stock will not pay any dividend over the next 6 months, do you find any opportunities for arbitrage?

Underlying asset provides no income

S_0 = 40/- ; r_f = 8% pa ; T = 6/12 years = 0.50 years;
F_{ACTUAL}=Rs.35/-

Theoretical Forward Price (F_0)=S_0e^{rT}

$=40e^{0.08*0.5}$= Rs. 41.63 > Rs. 38/-

Adopt Reverse Cash-and-Carry:

Now:

- Buy 6-month Forward @ Rs. 38/-
- Short Sell the stock @ Rs. 40/-
- Invest Rs 40/- for 6 months @ 8%pa

After 6 months:

- Receive the loan with interest: $40e^{0.08*0.5}$= Rs. 41.63
- Buy the stock under forward contract for Rs 38/- and deliver the stock.
- Profit = 41.63 - 38 = **Rs. 3.63**

At t = 0	At maturity
+40-40	+41.63-38
Net = 0	3.63

Underlying asset provides known amount of cash Income

Forward Price (F_0) = (S_0 - I) e^{rt}

where 'I' is the present value of the income 'Y' received during the forward contract. ($I = Y * e^{-rt.}$)

Underlying asset provides known amount of cash Income

A stock will pay a dividend of Re 1 per share in 2 months and again in 5 months from now. The stock is currently selling for Rs. 50/- and the risk-free interest rate is 6%pa.What should be the price of a 6-month forward contract on the stock?

S_0 = 50/- ; r_f = 6% pa ; T= 6/12 yrs; D_2 = Re 1; D_5 = Re 1

Forward Price (F_0) = $(S_0 - I)e^{rT}$

$I = D_2e^{-rt_1} + D_5e^{-rt_2} = 1e^{-0.06*2/12} + 1e^{-0.06*5/12}$

$= 0.9900 + 0.9753 = 1.9653$

$F_0 = (50 - 1.9653)e^{0.06*6/12} = 48.0347*1.0305$

= Rs.49.4976 = Rs. 49.50

Value of a Forward Contract

- At inception, the value of Forward contract is Zero.
- Buyer(Seller) are indifferent between buying/(selling) the underlying asset now (at t=0) for spot price or buying (selling) at maturity for Forward price.
- As time passes, the price of the underlying asset changes, which makes the existing forward contract to become an asset or a liability.
- For **Long position** on Forward Contracts:

Value of FC on Maturity = **Forward Price** $_{New}$ – **Forward Price** $_{Old}$

Value of FC today = **PV of** $(F_N - F_0) = (F_N - F_0)\, e^{-rt^*}$

- For **Short position** on Forward Contracts:

Value of FC on Maturity = **Forward Price** $_{Old}$ – **Forward Price** $_{New}$ Value of FC today

= **PV of** $(F_0 - F_N) = (F_0 - F_N)e^{-rt^*}$

Value of a Forward Contract

One month ago the stock of ABC was trading at Rs. 114/. Now, the stock price has declined to Rs. 109/-. What would be the impact on the value of the 6-month forward contract that was contracted a month ago *(when the stock price was Rs. 114/-)*? Assume the risk-free rate of return as 7% pa. (cc).

S_{-1} = 114/- ; S_0 = 109/-; r_f = 7% pa; T= 6 months.

- Forward Price:

At t = -1 : $F_{OLD} = 114\ e^{(0.07)*6/12} = 114*1.03562 = 118/-$

At t = 0 : $F_{NEW} = 109\ e^{(0.07)*5/12} = 109*1.02960 = 112/-$

Value of Forward Contract (old) on **Maturity** = $F_N - F_0 = 112-118 = -6$

Value of Forward Contract (old) **today**

$= PV(F_N - F_0) = -6\ e^{-(0.07)*5/12} = -6(0.97125) = -$ Rs 5.83

Value of a Forward Contract

A long forward contract on a non-dividend paying stock was entered into some time back at a forward price of Rs. 24/-. The forward contract now has 6 months to maturity. The risk-free rate of return is 10% pa (cc), the stock is trading at Rs 25/-. What would be the impact on the value of the original forward contract now?

F_O = 24/-; S^*_O = 25/- ; r = 10% pa; T= 6 months.

Forward Price of New contract $F_N = S^*_O e^{rT}$

At t = O : F_{OLD} = **24/-**

At t = 1 : $F_{NEW} = 25\ e^{(0.10)*6/12} = 25*1.05127$ = **26.28**

Value of Forward Contract on Maturity= $F_N - F_O = 26.28 - 24 = 2.28$

Value of Forward Contract today = **PV** $(F_N - F_O)$

= **2.28** $e^{-(0.10)*6/12}$ = **2.28** (0.9512) = **Rs. 2.17**

Value of a Forward Contract

2 months back a US investor sold forward £ 2 million at a forward price of $ 1.61 per pound. After one month, the forward price for delivery in one month is $ 1.585 per pound. Suppose the one-month rate of interest is 6% (cc), what is the value of the investor's position?

$F_0 = \$1.61$; $F_N = \$1.585$; $r_f = 6\%$ pa; T= 1 month.

For **Short** Futures Position:

Value of Forward Contract on Maturity= $F_0 - F_N = 1.61 - 1.585$

Value of Forward Contract today= $PV(F_0 - F_N)$

$= 0.025e^{-(0.06)*1/12} = 0.025(0.9952) = \$\ 0.024875/\ £$

Investor with **short position** would receive

£ 2,000,000*0.024875 = $49,750/- on unwinding of the position.

Value of a Forward Contract

Consider the current spot price of ABC shares is Rs. 800/. Mr. A has bought one 6 month forward contract at Rs.900/. After one month, Mr. B offers to buy a 5-month forward contract on ABC shares at 925/- . If the risk-free interest rate is 9%, what is the value of Mr. A's contract?

- After one-month , if Mr A goes short on 5-month forward, he would receive Rs, 925/- (F_N) and would be required to deliver the stock of ABC, which he would get under the 6-month forward on which he is long for Rs 900/- (F_0).
- So his profit at T=6 or Value of Forward Contract on Maturity= $F_N - F_0 = 925 - 900 =$ Rs. 25/-.
- Value of Forward Contract **today** (when the 2nd forward is entered) = $PV(F_N - F_0)$

$$= (925 - 900)e^{-0.09*5/12} = 25*0.96319 = 24.07986$$

Forward Price vs. Futures Price

- When short term risk-free interest rate is constant, the forward price on a certain delivery date is same as the futures price for a contract with the same delivery date.
- However, when interest rates vary, the Forward price ***may not*** be same as the Futures price.

❑ *Suppose the price of the underlying asset is* ***strongly and positively*** *correlated with the interest rate.*

- In case of a Futures contract, when the price of the u/l asset increases, the investor with a long position would gain (due to daily settlement)
- As price increases, the interest rates would also increase.
- Hence, the gain will be invested at a higher interest rate.

Forward Price vs. Futures Price

- Similarly, when the price of the u/l asset decreases, the investor will incur an immediate loss.
- This loss will be financed at a lower interest rate.
- An investor holding a Forward contract (instead of a Futures contract) is not affected in this way by interest rate movements.
- So, a Long Futures contract will be slightly more attractive than a similar long forward contract.
- **Futures price tend to be slightly more than the forward price, if price of u/l asset and interest rates are *strongly positively correlated.***

Forward Price vs. Futures Price

- *Suppose the price of the underlying asset is **strongly and negatively** correlated with the interest rate.*

- In case of a Futures contract, when the price of the u/l asset increases, the investor with a long position would gain (due to daily settlement)
- As price increases, the interest rates would now decrease.
- Hence, the gain will be invested at a lower interest rate.
- Similarly, when the price of the u/l asset decreases, the investor will incur an immediate loss.
- This loss will be financed at a higher interest rate.
- An investor holding a Forward contract (instead of a Futures contract) is not affected in this way by interest rate movements.

Forward Price vs. Futures Price

- So a Long Futures contract will be slightly less attractive than a similar long forward contract.
- **Futures price tend to be slightly less than the forward price, if price of underlying asset and interest rates are strongly positively correlated.**

➢ *The theoretical difference between Forward price and Futures price for short term contracts (less than a few months) usually small and hence ignored.*

➢ ***For most purposes, Forward and Futures price is considered to be same.***

Forward/Futures prices of Currencies

Direct Quote: Units of HC per FC

Spot: INR 70 = 1USD
Forward: INR 74.50= 1USD
$r_{f\,(FC)}$ = 1% pa (cc)
$r_{f\,(HC)}$ = 6.5%pa (cc)
Time = 1 year

100 units of USD at t=0

Invest FC @ $r_{f\,(FC)}$ & Convert FC in HC @ F_0 today

Convert FC in HC @ S_0 & invest HC @ $r_{f\,(HC)}$ today

Invest FC @ $r_{f\,(FC)}$ for 1 year

$100e^{0.01*1} = 101.00502$ USD

Convert FC into HC @ So today

$100*70 = 7000$ INR

Convert FC into HC @ Fo today

101.00502 USD*74.50 =7524.87374 INR

Invest HC @ $r_{f\,(HC)}$ for 1 year

$7000e^{0.065*1} = 7470.11317$ INR

$100e^{r_{f\,(FC)}T} F_0$ should be equal to $100S_0e^{r_{f\,(HC)}T}$

$$F_0 = S_0e^{(r_{HC} - r_{FC})T}$$ → Interest Rate Parity

Theoretical $F_0 = 70e^{(0.065 - 0.01)\,1} = 70(1.05654) = 73.95784$

Forward/Futures prices of Currencies

2-month interest rates in Switzerland and India are 3% & 6% pa (cc). Spot price of Swiss franc is Rs. 33.778. What should be the 2-month forward of Swiss franc?

$S_0 = 33.778$; $r_{HC} = 6\%$pa ; $r_{FC} = 3\%$ pa; $T = 2/12$ years

$$F_0 = S_0 e^{(r_{HC} - r_{FC})T}$$

$$= 33.778\, e^{(0.06 - 0.03)2/12}$$

$$= 33.778\, e^{0.005} = 33.778\,(1.00501) = \textbf{Rs. 33.9473}$$

Currency with higher interest rate should depreciate.

Forward/Futures prices of Currencies

Suppose the spot price of AUD is 0.7500USD and the 2-year interest rate in Australia and USA are 3% & 1% respectively. What should be the arbitrage–free forward price of AUD. If the forward rate actually turns out to be (a) 0.7000 USD; (b)0.7600 USD, instead, what arbitrage opportunities will arise. (Direct Quote)

1 AUD = 0.7500 USD; $r_{HC\ (USD)}$ = 1%pa ; $r_{FC(AUD)}$ = 1% pa; T = 2 years

$$F_0 = S_0 e^{(r_{HC} - r_{FC})T}$$

$$= 0.7500\ e^{(0.01 - 0.03)*2}$$

$$= 0.7500*0.9608 = 0.7206 \text{ USD}$$

Currency with higher interest rate should depreciate.

(USD has appreciated against AUD in the forward market.)

Forward/Futures prices of Currencies

- If F_{ACTUAL} = 0.7000 USD/AUD: *(In the future, less USD are required to get 1 AUD, so 'x' units of USD would fetch more AUD, hence we need to have USD in the future or AUD now.)*

1.	Borrow 1000 AUD @ 3%pa for 2 years	1000.00 AUD
2.	Convert 1000 AUD in USD @ 0.7500 USD/AUD	750.00 USD
3.	Invest 750 USD @ 1% pa for 2 years to yield ($750e^{0.01*2}$ = 750*1.02020 = 765.15101 USD)	765.15 USD
4.	Amount required to repay the AUD borrowings ($1000e^{0.03*2}$ = 1000*1.06184 =1061.8366)	1061.84 AUD
5.	Out of 765.15 USD, convert Amount Equivalent to the amount required in AUD (1061.8366* 0.7000=743.2856)	743.29 USD
6.	**Risk-free Profit (3 – 5)**	**21.86 USD**

Forward/Futures prices of Currencies

- If F_{ACTUAL} = 0.7600 USD/AUD: *(In the future more USD are required to get 1 AUD, so 'x' units of AUD would fetch more USD, hence need to have AUD in the future i.e USD now)*

1.	Borrow 1000 USD @ 1%pa for 2 years	1000.00 USD
2.	Convert 1000 USD to AUD @ 0.7500 USD (1000USD/0.7500)	1333.33 AUD
3.	Invest 1333.33 AUD @ 3% pa for 2 years to yield ($1333.33e^{0.03*2}$ = 1333.33*1.06184)	1415.78 AUD
4.	Convert AUD in USD @ F_A today (1415.78 *0.7600)	1075.99 USD
5.	Amount required to repay the USD borrowings ($1000e^{0.01*2}$ = 1000*1.0202)	1020.20 USD
6.	**Risk-free Profit (4 – 5)**	**55.79 USD**