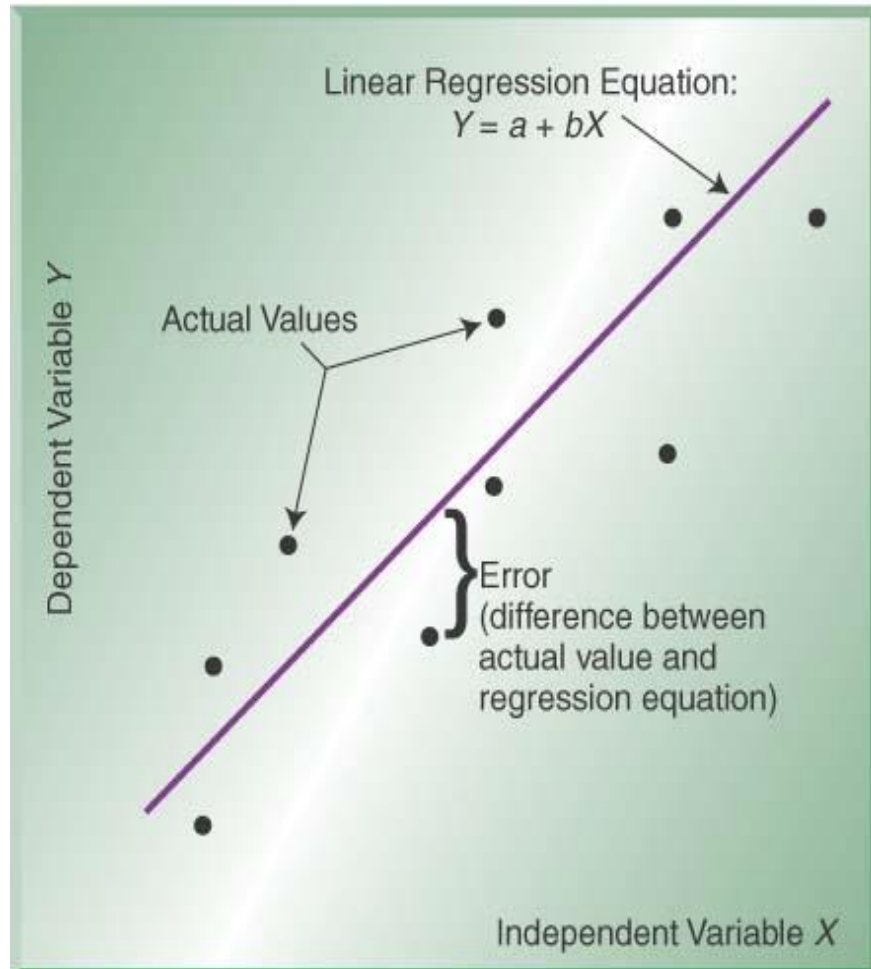


Casual Models

- Often, leading indicators hint can help predict changes in demand
- Causal models build on these cause-and-effect relationships
- A common tool of causal modeling is linear regression:

$$Y = a + bx$$

Linear Regression



- Identify dependent (**y**) and independent (**x**) variables
- Solve for the slope of the line

$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

- Solve for the y intercept

$$a = \bar{Y} - b\bar{X}$$

- Develop your equation for the trend line

$$Y = a + bX$$

Linear Regression Problem: A maker of golf shirts has been tracking the relationship between sales and advertising dollars. Use linear regression to find out what sales might be if the company invested **\$53,000 in advertising next year.**

	Sales \$ (Y)	Adv.\$ (X)	XY	X ²	Y ²
1	130	48	4240	2304	16,900
2	151	52	7852	2704	22,801
3	150	50	7500	2500	22,500
4	158	55	8690	3025	24964
5	153.85	53			
Tot	589	205	30282	10533	87165
Avg	147.25	51.25			

$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

$$b = \frac{30282 - 4(51.25)(147.25)}{10533 - 4(51.25)^2} = 3.58$$

$$a = \bar{Y} - b\bar{X} = 147.25 - 3.58(51.25)$$

$$a = -36.20$$

$$Y = a + bX = -36.20 + 3.58x$$

$$Y_5 = -36.20 + 3.58(53) = 153.54$$

How Good is the Fit?

- Correlation coefficient (**r**) measures the direction and strength of the linear relationship between two variables. The closer the r value is to 1.0 the better the regression line fits the data points.

$$r = \frac{n(\sum XY) - (\sum X)(\sum Y)}{\sqrt{n(\sum X^2) - (\sum X)^2} * \sqrt{n(\sum Y^2) - (\sum Y)^2}}$$
$$r = \frac{(4)(30,282) - (205)(589)}{\sqrt{4(10,533) - (205)^2} * \sqrt{4(87,165) - (589)^2}} = .888$$
$$r^2 = (.888)^2 = .788$$

Coefficient of determination measures the amount of variation in the dependent variable about its mean that is explained by the regression line. Values of close to 1.0 are desirable.

Forecasting Software

- Spreadsheets
 - Microsoft Excel, Quattro Pro, Lotus 1-2-3
 - Limited statistical analysis of forecast data
- Statistical packages
 - SPSS, SAS, NCSS, Minitab
 - Forecasting plus statistical and graphics
- Specialty forecasting packages
 - Forecast Master, Forecast Pro, Autobox, SCA

Inventory Analytics

Inventory Management

- Inventory management is an important function in controlling assets in the any manufacturing/service operations ([MSO](#)).
- Individuals working in the [MSO domain](#) should have at least a basic understanding of the roles, costs, and benefits of inventories.
- Inventory is often obtained from suppliers in the form of raw materials and other goods and materials through the procurement department.
- Inventory also includes work in [process and finished products](#) from manufacturing operations.

Inventory Basics

Inventory Includes:

- Raw Materials
- Work in Progress (WIP)
- Finished goods
- Merchandise
- Spare parts
- Other operating supplies

Inventories may be found in:

- Factories
- Warehouses
- Retail Stores
- Other type of storage facilities
- Home/office

Inventory Basics

- Ideally, an organization would have sufficient inventory to satisfy customer demands for products without losing any revenue due to insufficient stock.
- An organization does not want to have too much inventory on hand, because it costs too much money to both acquire and hold inventory.



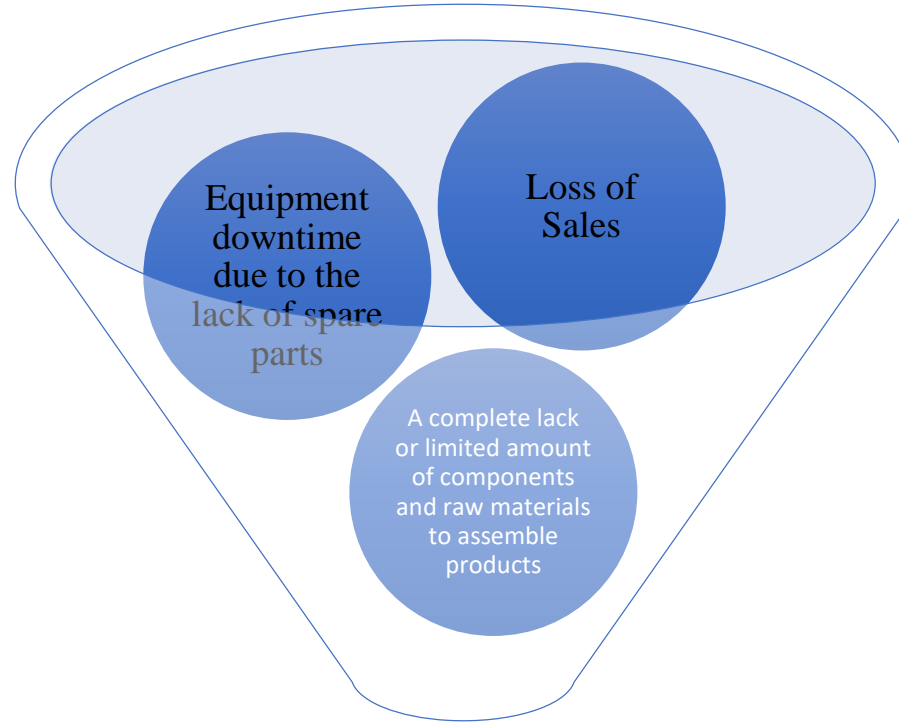
Greatest Challenges
for managing
inventories

Balancing Supply
and Demand

Inventory Basics

- Inventory management involves striking a balance between three classes of costs:
 - **Acquisition costs** are incurred during purchase order (PO) preparation and processing and during receiving and inspecting purchase items
 - **Carrying costs** are incurred in maintaining a stock of goods in storage
 - **Stockout costs** (also called shortage costs) are incurred when an item is out of stock
 -

Inventory basics



Insufficient Inventory

Why inventory analytics in industry 4.0



<https://www.youtube.com/watch?v=Nf-P-qNej3c>

Inventory types

Inventory form	Inventory function	Driver of demand
Finished goods (FG) for sale (new and refurbished)	Cycle stock; safety stock; in-transit inventory; pre-build inventory to support market launches	Customer and consumer demand
Service parts (spare parts)	Service level based on criticality	Mean time between failure (MTBF)
Work in progress (WIP)	Sub-assemblies awaiting further production. Postponement; assemble to order (ATO)	FG inventory; production capacity; customer orders
Raw materials (RM)	Dependent demand items. Component parts; packaging materials	Purchased items required for production
Maintenance, repair and overhaul (MRO)	Service level based on criticality	Mean time between failure (MTBF)
Consumables	Support production. Two bin replacement system	As required based on production
General supplies	Office and technical items. Two bin replacement	As required by the enterprise

Critical analysis (VBL)

<https://www.youtube.com/watch?v=tn0OCaf3O1Y>

- Why is excess inventory a negative factor for companies?
- Why cut prices to move excess inventory?
- What can retailers do to prevent excess inventory?

<https://www.youtube.com/watch?v=tZAQi6jSfec>

Henry Ford's famous proclamation

Customers can have any color they want, as long as it is black.



Practical challenges

- Customer do not easily forgive **shortage of delivery delays**
- Inventory management critical to a **firm's strategic viability**
- Success stores in retailing (Wal-Mart), auto (Toyota), computer (Dell) are founded on operational capabilities that among other things keep inventories lean
- Amazon.com
 - operation without huge inventory
 - innovation in inventory management enabled by technology

Need of inventory planning

- The average manufacturing company spends over one-half of its sales revenue on inventory.

How much inventory is enough?

Marketing department wants large inventory, it does not like stockouts.

Finance department likes low inventory and high turnover to minimize funds tied up in inventories; opportunity cost of capital.

Production department likes to keep production costs low. It likes uniform production and long uninterrupted runs of a small number of products.

Inventory system

Types of inventory systems

- Order point planning (OPP)
- Fixed-Quantity system
- Fixed-Interval system -
- Minimum-Maximum System, (s,S)-system
- Material requirement planning (MRP)
- Just-in-time system

- Information technology allows us to easily keep and update information
- Simple inventory system can include:
 - forecasting module
 - determination of order points and order quantities
 - monitoring of inventory levels

Costs

- holding costs including opportunity costs
- ordering or setup costs
- shortage costs or service levels

Capacity Constraints

- demand distribution
- lead times

Inventory Costs type

- Item purchase Cost
- Ordering Cost
- Shortage Costs
- Risk costs
- Storage costs

Method for Order Quantities

Lot-for-lot

Order exactly what is needed

Fixed-order quantity

Specifies the number of units to order whenever an order is placed

Min-max system

Places a replenishment order when the on-hand inventory falls below the predetermined minimum level.

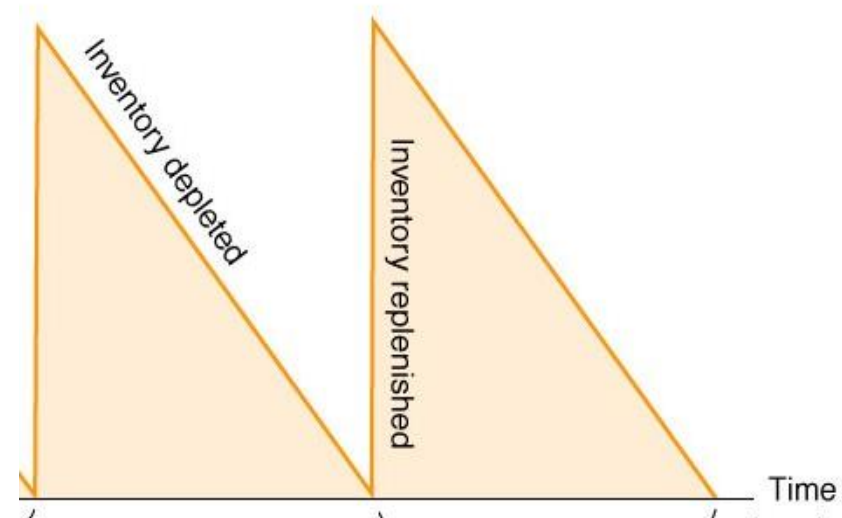
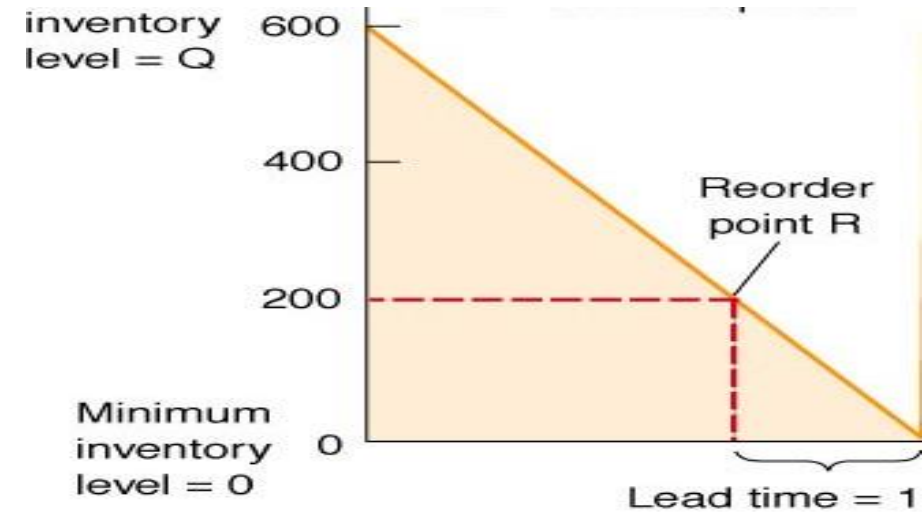
Mathematical Models for Determining Order Quantity/ inventory control system

- Economic Order Quantity (EOQ or Q System)
 - An optimizing method used for determining order quantity and reorder points
 - Part of **continuous review system** which tracks on-hand inventory each time a withdrawal is made
 - **Periodic review system** is also called as fixed time system
- Economic Production Quantity (EPQ)
 - A model that allows for **incremental product delivery**
- Quantity Discount Model
 - Modifies the EOQ process to consider cases where quantity discounts are available

Economic Order Quantity

Assumptions:

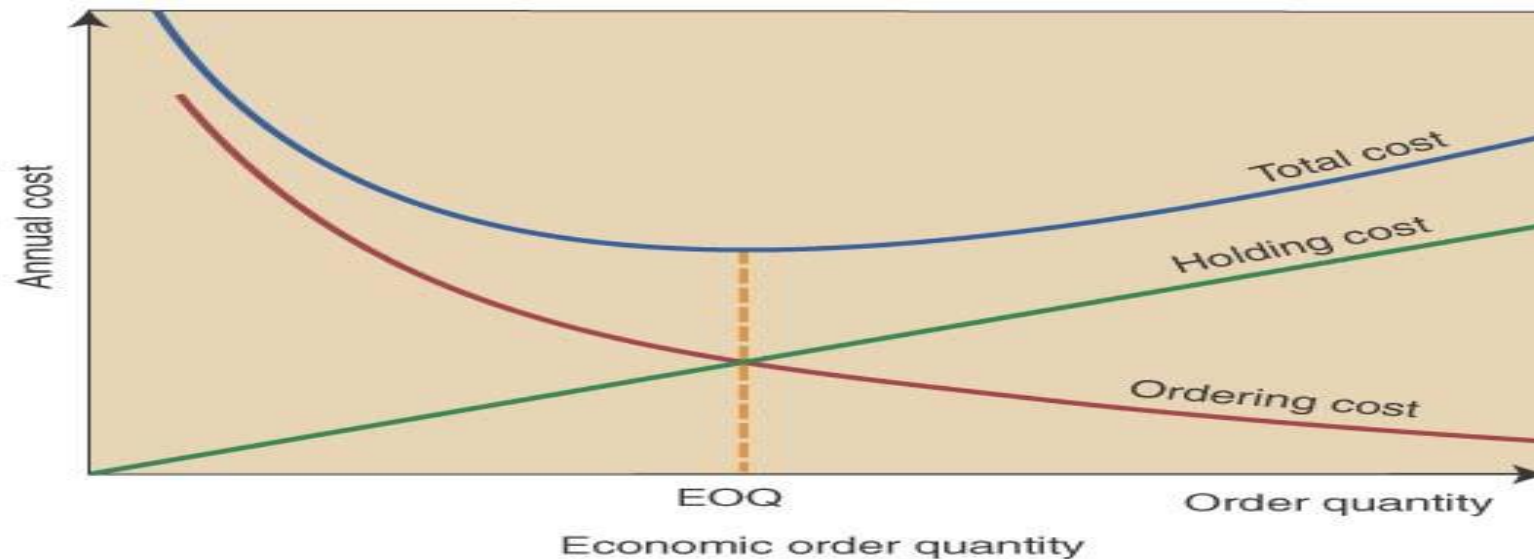
- Demand is known & constant - no safety stock is required
- Lead time is known & constant
- No quantity discounts are available
- Ordering (or setup) costs are constant
- All demand is satisfied (no shortages)
- The order quantity arrives in a single shipment



Total AIC with EOQ Model

- Total annual cost= annual ordering cost + annual holding costs

$$TC_Q = \left(\frac{D}{Q}\right)S + \left(\frac{Q}{2}\right)H; \text{ and } Q = \sqrt{\frac{2DS}{H}}$$



Continuous (Q) Review System Example: A computer company has annual demand of 10,000. They want to determine EOQ for circuit boards which have an annual holding cost (H) of \$6 per unit, and an ordering cost (S) of \$75. They want to calculate TC and the reorder point (R) if the purchasing lead time is 5 days.

- **EOQ (Q)**

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 * 10,000 * \$75}{\$6}} = 500 \text{ units}$$

- **Reorder Point (R)**

$$R = \text{Daily Demand} \times \text{Lead Time} = \frac{10,000}{250 \text{ days}} * 5 \text{ days} = 200 \text{ units}$$

- **Total Inventory Cost (TC)**

$$TC = \left(\frac{10,000}{500}\right) \$75 + \left(\frac{500}{2}\right) \$6 = \$1500 + \$1500 = \$3000$$

Reorder point computation 604

- EOQ models answer the question of **how much to order**, but not the question of **when to order**.
- The reorder point occurs when the **quantity on hand drops to a predetermined amount**.
- That amount generally includes expected demand during lead time and perhaps an extra cushion of stock, which serves to reduce the probability of experiencing a stockout during lead time.
- There are four determinants of the reorder point quantity:
 1. The rate of demand (usually based on a forecast)
 2. The lead time
 3. The extent of demand and/or lead time variability
 4. The degree of stockout risk acceptable to management
- If demand and lead time are both constant, the reorder point is simply

$$ROP = d \times LT$$

$$d =$$

Class exercise

The e-paint store stocks paint in its warehouse and sells it online on its website. The store stocks several brands of paint; however, its biggest seller is Sherman-Wilson iron coat paint. The company wants to determine the optimal order size and total inventory cost for iron coat paint given an estimated annual demand of 10000 gallons of paint, an annual carrying cost of \$ 0.75 per gallon, and an ordering cost of \$150 per order. It would also like to know the number of orders that will be made annually and the time between orders. Assume company operates 300 days annually.

Answer

$$D=10000$$

$$S=\$150$$

$$H=\$0.75$$

$$EOQ = 2000 \text{ gallons}$$

$$T_{cmin} = \text{ordering cost} + \text{holding cost} = 750 + 750 = 1500 \text{ USD}$$

$$\text{No. of orders per year} = D/EOQ = 10000/2000 = 5 \text{ orders per year}$$

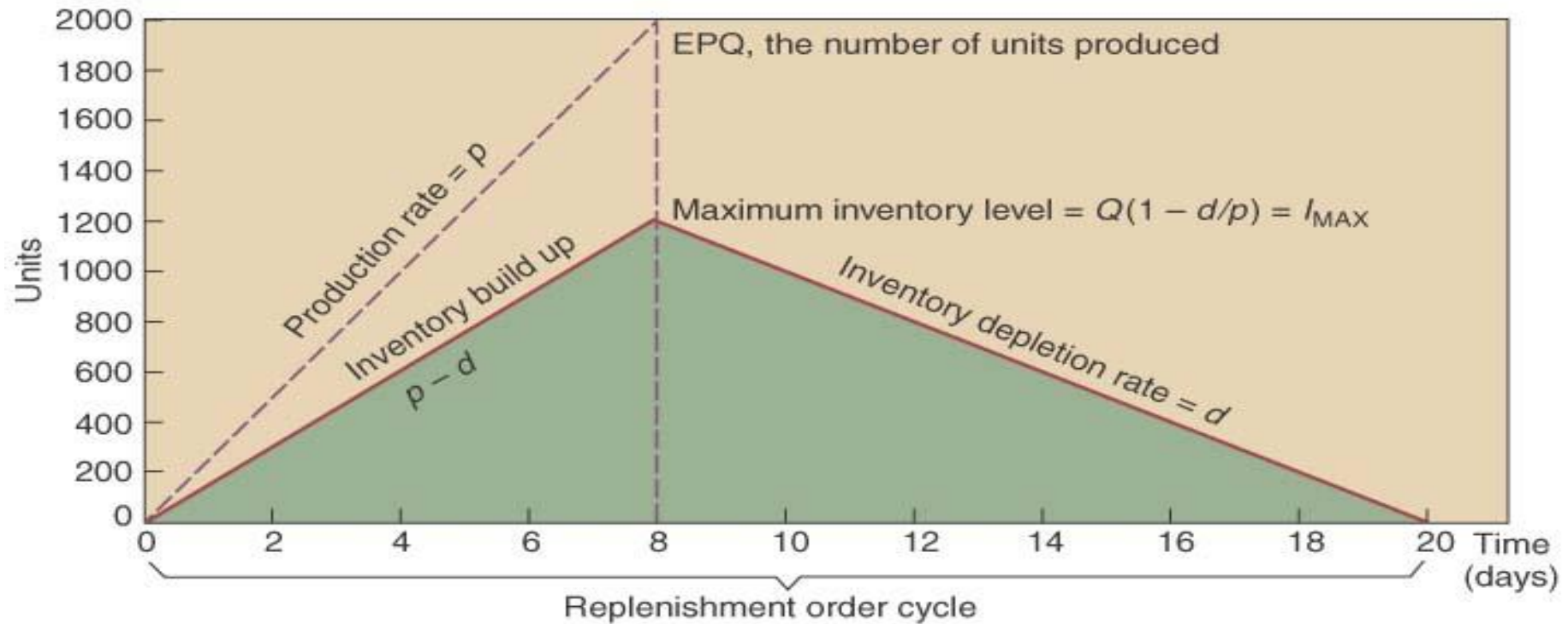
$$\text{Order cycle time} = \text{working time} / \text{no. of orders} = 300/5 = 75 \text{ days}$$

Extension 14.5

If store open 31 days per year and demand 10000 gallons and lead time to receive orders is 10 days then reorder point = $dL = (10000/31) * 10 = 321.54$ gallons

Economic Production Quantity (EPQ)

- Same assumptions as the EOQ except: inventory arrives in increments & is drawn down as it arrives



Order quantity 2000 units
Daily demand (d) = 100 units
Daily production (p) = 250 units

Model formulation

- Total cost:

$$TC_{EPQ} = \left(\frac{D}{Q} S \right) + \left(\frac{I_{MAX}}{2} H \right)$$

- Maximum inventory:

- d=avg. daily demand rate
- p=daily production rate

$$I_{MAX} = Q \left(1 - \frac{d}{p} \right)$$

- Calculating EPQ

$$EPQ = \sqrt{\frac{2DS}{H \left(1 - \frac{d}{p} \right)}}$$

EPQ Problem: HP Ltd. Produces its premium plant food in 50# bags. Demand is 100,000 lbs. per week and they operate 50 wks. each year and HP can produce 250,000 lbs. per week. The setup cost is \$200 and the annual holding cost rate is \$.55 per bag. Calculate the EPQ. Determine the maximum inventory level. Calculate the total cost of using the EPQ policy.

$$EPQ = \sqrt{\frac{2DS}{H\left(1 - \frac{d}{p}\right)}}$$

$$I_{MAX} = Q\left(1 - \frac{d}{p}\right)$$

$$TC_{EPQ} = \left(\frac{D}{Q}S\right) + \left(\frac{I_{MAX}}{2}H\right)$$

$$EPQ = \sqrt{\frac{2(50)(100,000)(200)}{.55\left(1 - \frac{100,000}{250,000}\right)}} = 77,850 \text{ Bags}$$

$$I_{MAX} = 77,850\left(1 - \frac{100,000}{250,000}\right) = 46,710 \text{ bags}$$

$$TC = \left(\frac{5,000,000}{77,850}\right)(200) + \left(\frac{46,710}{2}\right)(.55) = \$25,690$$

Class exercise

The Production Quantity Model

Assume that the ePaint Store has its own manufacturing facility in which it produces Iron-coat paint. The ordering cost, C_o , is the cost of setting up the production process to make paint. $C_o = \$150$. Recall that $C_c = \$0.75$ per gallon and $D = 10,000$ gallons per year. The manufacturing facility operates the same days the store is open (i.e., 311 days) and produces 150 gallons of paint per day. Determine the optimal order size, total inventory cost, the length of time to receive an order, the number of orders per year, and the maximum inventory level.

$$C_o = \$150$$

$$C_c = \$0.75 \text{ per gallons}$$

$$D = 10,000 \text{ gallons}$$

$$d = \frac{10,000}{311} = 32.2 \text{ gallons per day}$$

$$p = 150 \text{ gallons per day}$$

The optimal order size is determined as follows:

$$Q_{\text{opt}} = \sqrt{\frac{2C_o D}{C_c \left(1 - \frac{d}{p}\right)}}$$

$$= \sqrt{\frac{2(150)(10,000)}{0.75 \left(1 - \frac{32.2}{150}\right)}} = 2256.8 \text{ gallons}$$

Although an order of 2256.8 gallons should be rounded to 2257, we will use the 2256.8 to compute total cost. This value is substituted into the following formula to determine total minimum annual inventory cost:

$$\begin{aligned} TC_{\min} &= \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{p}\right) \\ &= \frac{(150)(10,000)}{2256.8} + \frac{(0.75)(2256.8)}{2} \left(1 - \frac{32.2}{150}\right) \\ &= \$1329 \end{aligned}$$

The length of time to receive an order for this type of manufacturing operation is commonly called the length of the production run.

$$\begin{aligned} \text{Production run} &= \frac{Q}{p} \\ &= \frac{2256.8}{150} \\ &= 15.05 \text{ days per order} \end{aligned}$$

The number of orders per year is actually the number of production runs that will be made:

$$\begin{aligned} \text{Number of production runs (from orders)} &= \frac{D}{Q} \\ &= \frac{10,000}{2256.8} \\ &= 4.43 \text{ runs per year} \end{aligned}$$

Finally, the maximum inventory level is

$$\begin{aligned} \text{Maximum inventory level} &= Q \left(1 - \frac{d}{p}\right) \\ &= 2256.8 \left(1 - \frac{32.2}{150}\right) \\ &= 1772 \text{ gallons} \end{aligned}$$

Thus, ePaint will need to set aside storage space sufficient to accommodate these 1772 gallons of paint.

Quantity Discount Model

Whenever the price per unit is not fixed but varies based on the size of your order, the total annual cost formula for any inventory policy used must include the cost of material.

- **Same as the EOQ model, except:**
 - Unit price depends upon the quantity ordered
- **The total cost equation becomes:**

$$TC_{QD} = \left(\frac{D}{Q} S \right) + \left(\frac{Q}{2} H \right) + CD$$

D annual demand in units
Q order quantity in units
S ordering or setup cost
H annual holding cost
C unit price

Quantity Discount Procedure

- Calculate the EOQ at the lowest price
- Determine whether the EOQ is feasible at that price
 - Will the vendor sell that quantity at that price?
- If yes, stop – if no, continue
- Check the feasibility of EOQ at the next higher price

QD Procedure

- Continue until you identify a feasible EOQ
- Calculate the total costs (including total item cost) for the feasible EOQ model
- Calculate the total costs of buying at the minimum quantity required for each of the cheaper unit prices
- Compare the total cost of each option & choose the lowest cost alternative
- Any other issues to consider?

Quantity Discount Example: Collin's Sport store is considering going to a different hat supplier. The present supplier charges \$10 each and requires minimum quantities of 490 hats. The annual demand is 12,000 hats, the ordering cost is \$20, and the inventory carrying cost is 20% of the hat cost, a new supplier is offering hats at \$9 in lots of 4000. Who should he buy from?

- **EOQ at lowest price \$9. Is it feasible?**

$$\text{EOQ}_{\$9} = \sqrt{\frac{2(12,000)(20)}{\$1.80}} = 516 \text{ hats}$$

- **Since the EOQ of 516 is not feasible, calculate the total cost (C) for each price to make the decision**

$$C_{\$10} = \frac{12,000}{490}(\$20) + \frac{490}{2}(\$2) + \$10(12,000) = \$120,980$$

$$C_{\$9} = \frac{12,000}{4000}(\$20) + \frac{4000}{2}(\$1.80) + \$9(12,000) = \$101,660$$

- **4000 hats at \$9 each saves \$19,320 annually. Space?**

Determining the Optimal Order Quantity When There Are Quantity Discounts and Carrying Costs are Constant

- The maintenance department of a large hospital uses about 816 cases of liquid cleanser annually. Ordering costs are \$12, carrying costs are \$4 per case a year, and the new price schedule indicates that orders of less than 50 cases will cost \$20 per case, 50 to 79 cases will cost \$18 per case, 80 to 99 cases will cost \$17 per case, and larger orders will cost \$16 per case. Determine the optimal order quantity and the total cost.

$D = 816$ cases per year

$S = \$12$

$H = \$4$ per case per year

Range	Price
1 to 49	\$20
50 to 79	18
80 to 99	17
100 or more	16

1. Compute the common minimum quantity $Q: = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(816)12}{4}} = 69.97 \approx 70$ cases.

The 70 cases can be bought at \$18 per case because 70 falls in the range of 50 to 79 cases. The total cost to purchase 816 cases a year, at the rate of 70 cases per order, will be

$$\begin{aligned} TC_{70} &= \text{Carrying cost} + \text{Order cost} + \text{Purchase cost} \\ &= (Q/2)H + (D/Q)S + PD \\ &= (70/2)4 + (816/70)12 + 18(816) = \$14,968 \end{aligned}$$

$$TC_{80} = (80/2)4 + (816/80)12 + 17(816) = \$14,154$$

$$TC_{100} = (100/2)4 + (816/100)12 + 16(816) = \$13,354$$

Example 14.4

EXAMPLE 14.4

A Quantity Discount with Constant Carrying Cost

Avtek, a distributor of audio and video equipment, wants to reduce a large stock of televisions. It has offered a local chain of stores a quantity discount pricing schedule, as follows:

Quantity	Price
1–49	\$1400
50–89	1100
90+	900

The annual carrying cost for the stores for a TV is \$190, the ordering cost is \$2500, and annual demand for this particular model TV is estimated to be 200 units. The chain wants to determine if it should take advantage of this discount or order the basic EOQ order size.