

The inventory level in order is received gradually & replenished as an

In the basic EOQ model:

$Q$  = total inv.

$Q/2$  = average inv.

but in this model, the max. inventory level is not simply  $Q$ , it is an amount somewhat lower  
 That

than  $Q \left[ 1 - \frac{d}{p} \right]$ , adjusted for the fact, the order quantity is depleted during the order receipt period.  $Q \left( 1 - \frac{d}{p} \right)$

$$\text{max. inventory level} = \left[ Q - \frac{Q \times d}{P} \right]$$

$$= Q \left[ 1 - \frac{d}{P} \right]$$

Since this is the max. inv. level, the avg. inv. level is determined by half of max. inv.

$$\text{avg. inv} = \frac{1}{2} \left[ Q \left( 1 - \frac{d}{p} \right) \right]$$

$$C_c = \frac{Q}{2} \times C_c = \frac{Q}{2} \left( 1 - \frac{d}{p} \right)$$

$$\text{Total carrying cost} = C_c \times \frac{Q}{2} \times \left( 1 - \frac{d}{p} \right)$$

In this case the ordering cost  $C_o$  is often the setup cost for production.  $\frac{C_o D}{Q}$

$$\underline{TC} = \frac{C_o D}{Q} + \frac{C_c Q \left( 1 - \frac{d}{p} \right)}{2}$$

Optimal value of  $Q^*$

$$\frac{C_o D}{Q} \quad \frac{C_c Q \left( 1 - \frac{d}{p} \right)}{2}$$

$$Q^* = \sqrt{\frac{2 C_o \times D}{C_c (1 - d/p)}} \quad \left( \frac{Q}{P} \right)^{x_d}$$

Q Assume, that a paint store has its own mfg. facility in which it produce Iron coat paint. The ordering cost ' $C_o$ ' is the cost of setting the production process to make paint.  $C_o = \$150$ . Also assume  $C_c = \$0.75$  per gallon of paint.  $D = 10,000$  gallon per year. The mfg. facility operates the same days the store is open only for 311 days in a year & produce 150 gallons of paint per day (Determine the optimal order size ( $Q^*$ ), total inventory cost, the length of time to receive an order, no. of order per year & max. inventory level?)

Solu  $D = 10,000$   
 $p = 150$  gallon/day  $d = \frac{D}{311} = \frac{10000}{311} = 32$  gallons per day  
 $C_o = \$150, C_c = \$0.75$  per gallon

$$2 \times C_o \times D$$

$$Q^* (\text{Optimal order size}) = \sqrt{\frac{2 \times C_o \times D}{C_c (1 - d/p)}}$$

$$= \sqrt{\frac{2 \times 150 \times 10,000}{0.75 \left(1 - \frac{32}{150}\right)}}$$

$$Q^* = 2935 \text{ gallons.}$$

optimal order size for order.

$$\frac{\text{no. of production runs}}{\text{for order}} = \frac{D}{Q^*} = \frac{10,000}{2935} = 3.4 \text{ per year} = 3 \text{ or } 4 \text{ per year.}$$

The length of time to receive an order for this type of mfg. operation is commonly called  $d$  as the length of production run.

$$\text{Production run} = \frac{Q^*}{p} = \frac{2935}{150}$$

$$10,000 \text{ production} / 3 \times 4 \text{ hr} = 19 \text{ or } 20 \text{ days per order.}$$

$$19 \times 4 = \underline{76} \text{ days.}$$

$$\text{max. inv. level} = Q^* \left(1 - \frac{d}{p}\right)$$

$$\begin{aligned}
 \underline{\underline{\text{max. Inv. level}}} &= Q^* (1 - d/P) \\
 &= 2935 \left(1 - \frac{32}{150}\right) \\
 &> 2935 - 626 \\
 &= 2309 \text{ gallons (max. Inv. level)}
 \end{aligned}$$

Total min Annual Inv. Cost

$$\begin{aligned}
 TC(\text{inv})_{\min} &= \frac{C_o D}{Q^*} + \frac{C_c \times Q^*}{2} (1 - d/P) \\
 &= \frac{150 \times 10000}{2935} + \frac{0.75 \times 2935}{2} \left(1 - \frac{32}{150}\right) \\
 &= 511.1 + 865 \\
 &= \$1376
 \end{aligned}$$

### Carpet Manufacturer

Carpet mfg. sells in its adjoining showroom store near the Anant nag District. Estimated annual demand is 20000 Carpets. The mfg. facility  $C_o = \$2.75$  per carpet.

operates the same 365 days a year & produces 400 carpets per day. Cost of setting up the mfg. process for a production run is \$720.

1. Determine  $Q^*$  (Economic order size) → 3485 carpets
2. Total Inventory cost → \$8263
3. length of time to receive an order → 8.7 day/order
4. Total no. of orders
5. max. Inventory level.

$$d = \frac{D}{365} = 55.2$$

$$= 3485 \left(1 - \frac{55.2}{400}\right)$$

$$= 3004 \text{ carpets}$$

$$\frac{D}{Q} = \frac{20000}{3485} = 5.74 \text{ 5.74 order/year}$$

$$\text{Total Inv. Cost} = \frac{C_o D}{Q} + \frac{C_c Q}{2} \left(1 - \frac{d}{P}\right)$$

$$= \$8300$$

$$\text{max Inventory level} = Q \left(1 - \frac{d}{P}\right)$$

$$= 3004 \text{ carpets}$$

# Reorder points (R)

level of inventory at which a new order should be placed.

$$Q = \sqrt{\frac{2COI}{C_c}}$$

The reorder point for our basic EOQ model with constant demand & constant lead time to receive an order is equal to the amount demanded during the lead time

$$R = d \times L$$

$d$  = demand rate per period

(daily, weekly, monthly, yearly)

$L$  = lead time

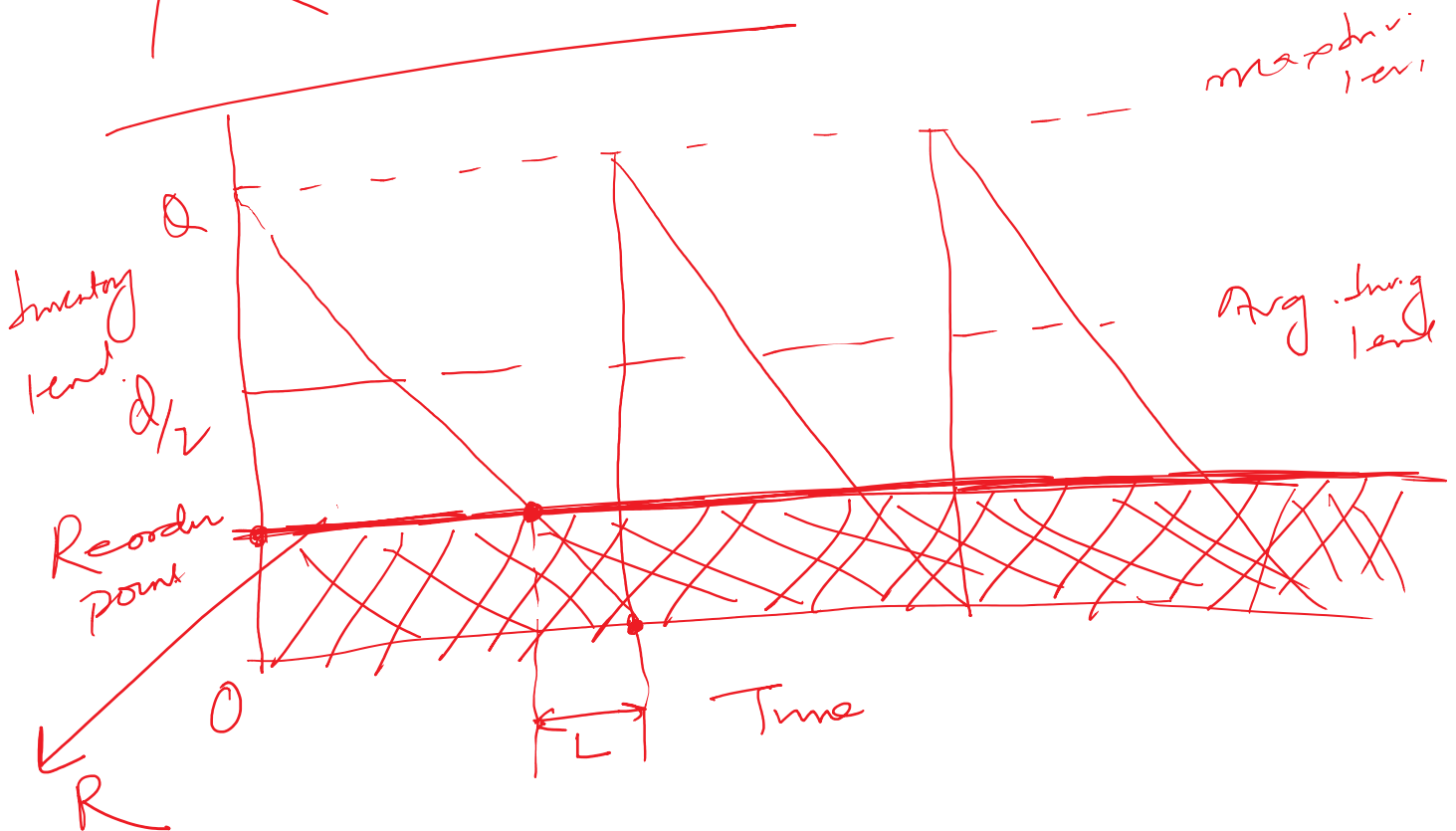
Carpet mfg is open for all 365 days. If annual demand is 20000 carpet / year & the lead time to receive an is 20 days. Determine the reorder point?

$$R = d \times L$$

$$d = \frac{D}{\text{no. of days production open}}$$

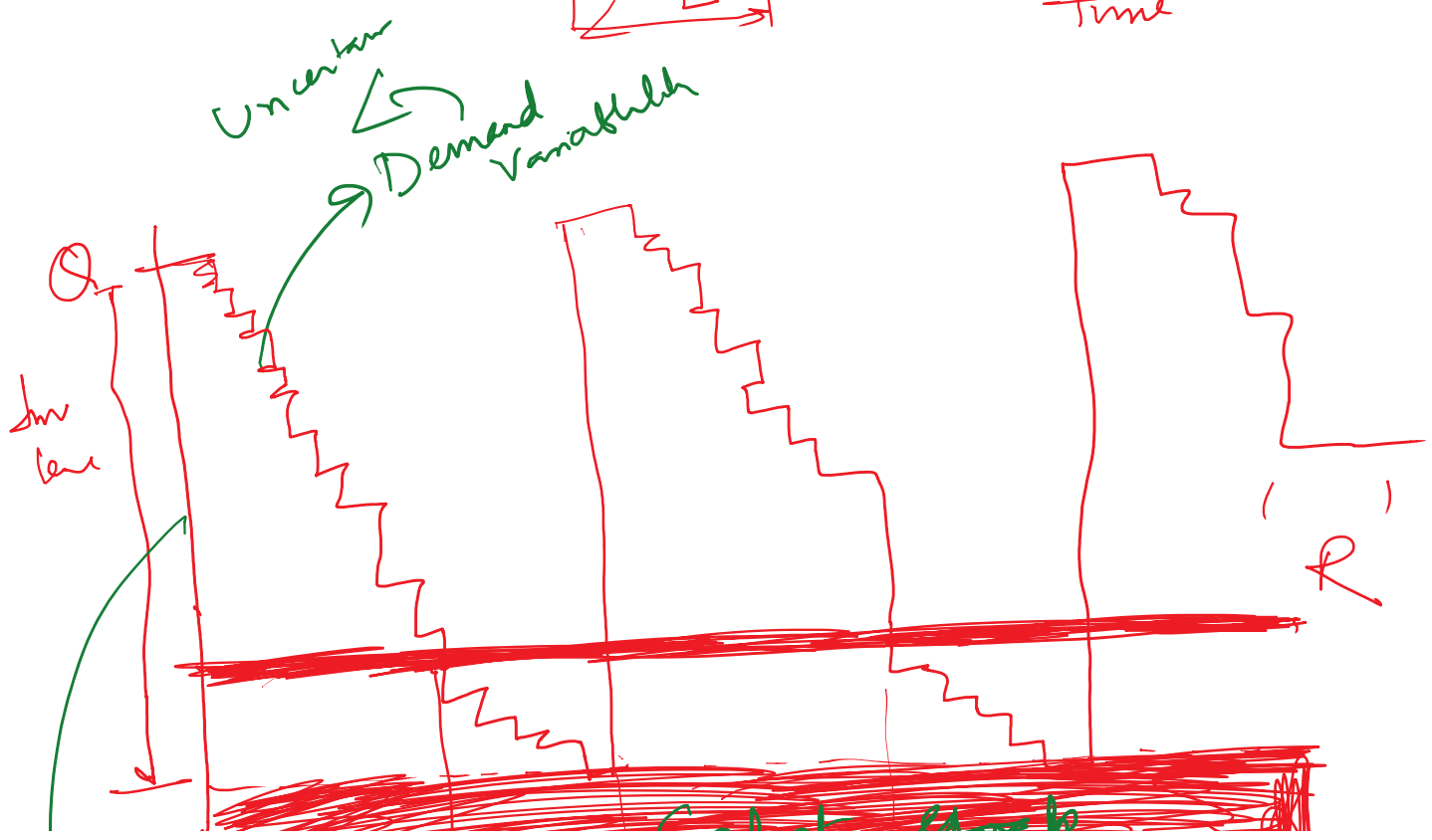
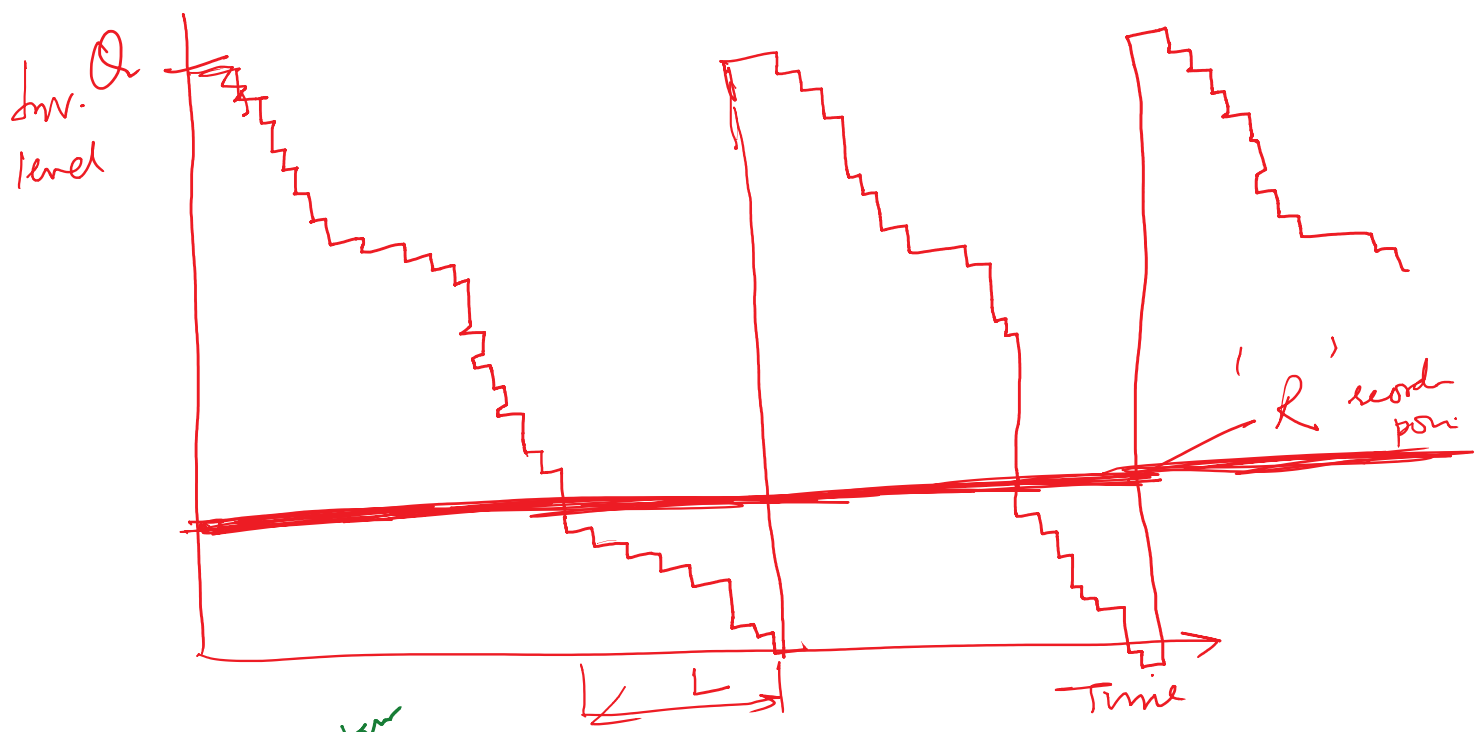
$$R = \frac{20000}{365} \times 20$$

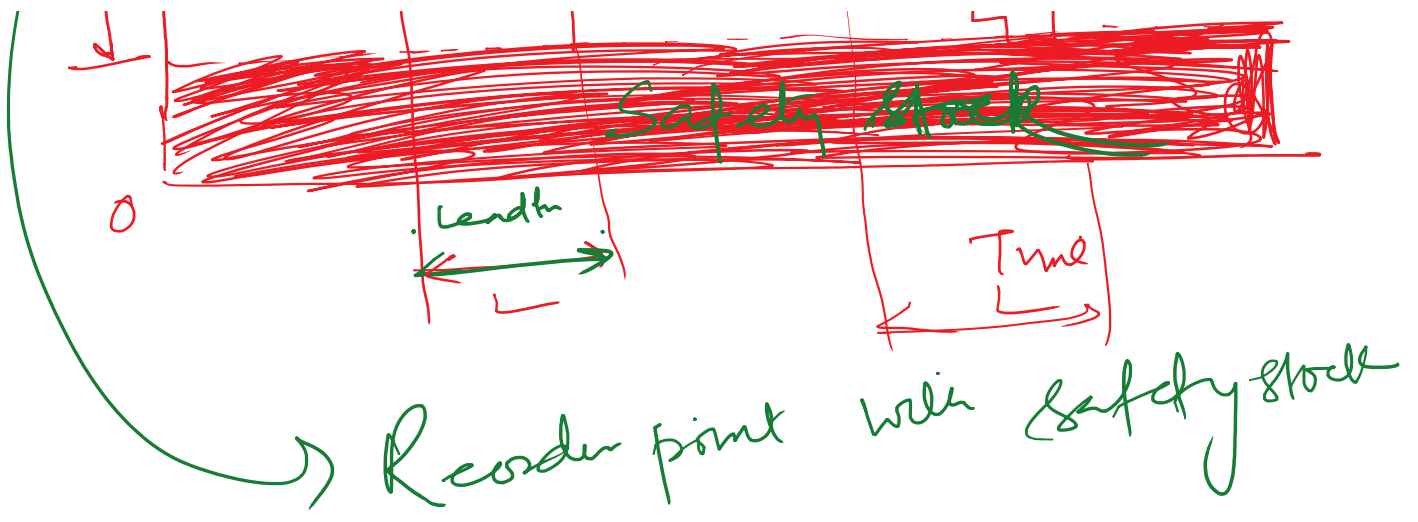
$$R = 1095 \text{ carpet}$$



# Safety stock:

It is defined as buffer added to the inventory on hand during lead time





## Reorder point with Variable / Uncertain Demand

To compute the reorder point with a safety stock that will meet specific service level.

- we will assume the demand during the lead time is uncertain, independent & can be described by normal distribution.

- Avg. demand for the lead time is the sum of avg. daily demand for the days of the lead time which is also the product of avg. daily demand multiplied by lead time.

- why the variation of the dispersion  
 sum of daily variances for the no. of days in the  
 lead time.

Reorder point

$$R = \bar{d} \times L + Z \sigma_d \sqrt{L}$$

$\bar{d}$  = average daily demand  
 $L$  = lead time

$\sigma_d$  = standard deviation of daily demand  
 $Z$  = no. of standard deviation corresponding to  
 the service level probability

$$\text{Safety stock} = Z \sigma_d \sqrt{L}$$

$\sigma_d \sqrt{L}$  in the safety stock formula  
 for the reorder point is the square root of  
 the sum of the daily variances during lead time

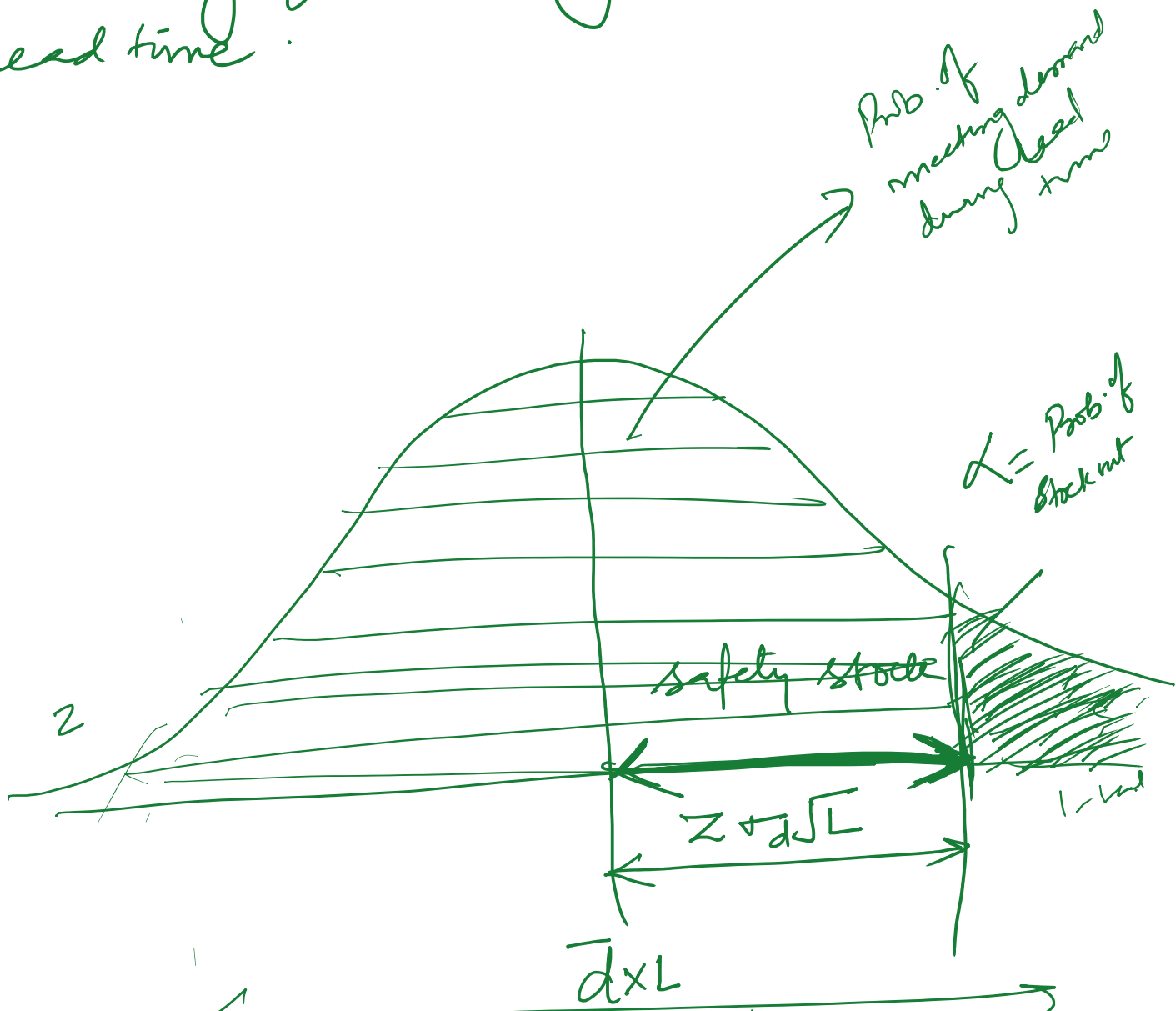
$$\text{Variance} = \text{daily variance} \times \text{no. of days during lead time}$$

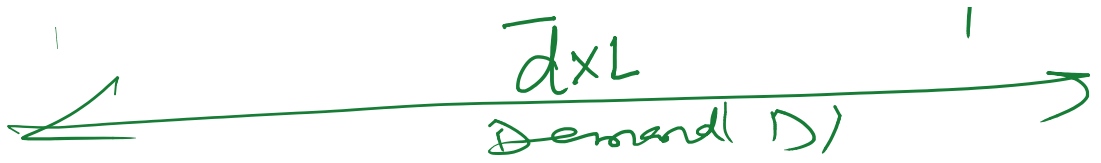
$$= \sigma_d^2 \times L$$

$$\text{Stand deviation} = \sqrt{\sigma_d^2 \times L}$$

$$= \sigma_d \sqrt{L}$$

Service level : It is defined as the probability of meeting demand during the lead time.





Q. For the e-paint store, we will assume that daily demand for Iron Coat paint is normally distributed with an average daily demand of 30 gallons & a standard deviation of 5 gallon of paint per day. The lead time for receiving a new order of paint is 10 days.

1. Determine the reorder point
2. Safety stock of the store want a service level of 95%. (Assume the prob. of stockout is 5%)

at service level 95% = 0.95

from normal distribution table

The value of 'Z' against 0.95  
= 1.65

$$\bar{d} = 30 \text{ gallon per day}$$

$$L = 10 \text{ days.}$$

$$\sigma_d = 5 \text{ gallons per day}$$

$$\begin{aligned} \text{Safety stock} &= Z \sigma_d \sqrt{L} \\ &= 1.65 \times 5 \times \sqrt{10} \\ &\rightarrow \underline{\underline{26 \text{ gallons}}} \end{aligned}$$

Reorder point

$$\begin{aligned} R &= \bar{d}L + Z \sigma_d \sqrt{L} \\ &= 30 \times 10 + 1.65 \times 5 \times \sqrt{10} \\ &= \underline{\underline{326.07 \text{ gallons}}} \end{aligned}$$

Case 1

Airone mfg store stocks of Honda Jazz  
" distributed

& the daily demand is normally ...  
 with a mean size of 1.6 Honda Jazz  
 & a standard deviation of 0.4 Honda  
 Jazz per day. The lead time to  
 receive an order from the manufacturer is  
 15 days. Determine the safety stock to achieve  
 a service level of 98% & reorder point?

$$\bar{d} = 1.6 \text{ Honda Jazz per day}$$

$$L = 15 \text{ days}$$

$$\sigma_d = 0.4 \text{ Honda Jazz per day}$$

$$Z = 2.05 \text{ at } 98\% \text{ prob}$$

$$\text{Safety stock} = Z \sigma_d \sqrt{L}$$

$$= 2.05 \times 0.4 \times \sqrt{15}$$

$$= 3.17 \text{ Honda Jazz}$$

$$\text{Reorder point} = \bar{d} \times L + \text{Safety stock}$$

$$= 1.6 \times 15 + 3.1$$

$$= 27.17 \text{ hours per}$$

## Order Quantity with Uncertain Demand

If the demand rate & lead time are constant, then the fixed period model will have fixed order quantity that will be made at specified time interval, which is the same as the fixed quantity (EOQ) model under similar conditions.

When the demand is uncertain, then ~~fixed order quantity~~ fixed period model react differently than fixed order model.

The order size for fixed period model given uncertain daily demand that is normally distributed by

$$Q = \bar{r}(t_b + L) + Z \sigma_r \sqrt{t_b + L} - I$$

normally known

$$Q = \bar{d}(t_b + L) + Z \sigma_d \sqrt{t_b + L} + I$$

↓ Safety stock

$\bar{d}$  = avg. daily demand

$L$  = lead time

$t_b$  = fixed time b/w order

$\sigma_d$  = standard deviation of demand

$I$  = Inventory in stock

## Case

The Sun Pharma stock a popular brand of ~~over~~ Hepatitis & Typhoid medicine. The average demand for the medicine is 6 boxes per day with a standard deviation of 1.2 boxes per day. A supplier of Sun Pharma checks that Sun Pharma stock every 6 days. During one visit the ~~store~~ store had 8 to receive

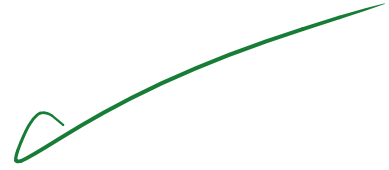
box in stock. The lead time  
an order is 5 days.

99%

Determine ~~the~~

7. Safety Stock

8. Reorder point (order size)



$$\bar{d} = 6 \text{ box per day}$$

$$\sigma_d = 0.2 \text{ box}$$

$$t_b = 60 \text{ days}$$

$$L = 5 \text{ days}$$

$$Z = ? \quad \text{at } 99\% \text{ prob}$$

$$I = 8 \text{ box}$$

$$Q = ?$$

$$SS = ?$$

$$SS = 2.33 \times 1.2 \sqrt{60 + 5}$$

$$= 22.4$$

$$Q = \bar{d}(t_b + L) + \overset{SS}{22.4} - 8$$

$$= \overset{\text{I}}{\cancel{391.4}} + 22.4 - 8$$

$$= 6(60 + 5) + 22.4 - 8$$

$$= 6 \times 65 + 22.4 - 8$$

$$= \underline{\underline{391.4}} \text{ box}$$

Basic EOQ model

$$TC = \frac{C_o D}{Q^*} + \frac{C_c Q^*}{2}$$

$$Q^* = \sqrt{\frac{2 C_o D}{C_c}}$$

$$Q^* = \sqrt{\frac{C_o D}{C_c}}$$

$\Sigma OQ$  with non instantaneous receipt

Production quantity model

$$Q^* = \sqrt{\frac{2 C_o D}{C_c (1 - d/p)}}$$

$$TC = \frac{C_o D}{Q} + \frac{C_c Q}{2} (1 - d/p)$$

Reorder point

$$R = d \times L$$

Reorder point with uncertain demand

$$R = \bar{d} \times L + Z \sigma_d \sqrt{L}$$

fixed ~~order~~ <sup>time</sup> point order quantity with uncertain demand

$$Q - \bar{d}(t_b + L) + Z \sigma_d \sqrt{t_b + L} - I$$

$$Q = \overline{F}(t_b + L) + Z \sigma_d \sqrt{t_b + L}$$