Hedging Strategies using Futures

Hedging

A steel manufacturing co. requires 50,000 tons of iron ore in 2 months. The iron ore prices are as follows: Spot Price: Rs. 305 per ton; 2-Mth Futures: Rs.310 per ton. The company expects the iron ore prices to rise but is comfortable up to price of Rs. 310/- per ton. At the same time, the company does not want to buy iron ore today @ Rs. 305/- per ton, nor is it financially profitable as the storage costs would be more than the price difference.

What should the Steel manufacturer do?

Hedging

- Steel manufacturing company wants to buy Iron ore after 2 months faces the risk of increase in prices, hence wants to lock-in the price today itself.
- Instead of buying iron ore in the cash market now, the company goes long on 2-month Iron ore Futures @ Rs. 310/- per ton.
- After 2-mths, assume the spot price rises to Rs. 316/- per ton

Hedging

- Just prior to expiry of 2 months, the company shall:
- Futures Market
- Offset the ‘long’ futures position by going ‘short’ on Iron ore @ Rs. 316/- (due to convergence)
- =50000*316= 158 lakh
- **Effective Cost= Actual Price paid – Profit on Future**

  = 158 – 6* 50000 = 155 Lacs

Long Hedge

- Thus, a Long Hedge involves taking a Long position in the futures market i.e. buy a Futures contract at the beginning of the strategy.

Long Hedge may be used when the hedger wants to purchase the underlying asset at a later date but wants to lock-in the price at initiation, thereby reducing/ eliminating price risk.

Short Hedge

A Iron ore mining Co. wants to sell its produce of 50,000 tons Iron ore after two months. The miner is satisfied to receive Rs. 310/- per ton, 2 months hence.

What should the Mining company do?

The mining company wants to sell Iron ore at a later date but wants to lock-in the price now

Short Hedge

- The mining company decides (today) to sell 2-month Iron ore futures @ Rs. 310/- per ton.

- After two months, again assume the price of iron ore goes up to Rs. 316/- per ton.

Short Hedge

- After 2 months, the mining company would:

Cash Market	Futures Market
➢ Sell 50,000 tons of Iron ore @ Rs 316 /- per ton and receive Rs. 158 Lacs. ➢ *Notional profit:* Rs 6/- per ton (316-310) on 50,000 tons or **Rs 3 Lacs.**	➢ Offset the ‘short’ position by going ‘long’ on Iron ore Futures@ Rs. 316/-*(due to convergence)* ➢ Loss: Rs. 6/- per ton (316-310) on 50,000 tons or **Rs 3 Lacs.**

Effective price : (Actual Price received less Loss on Futures)
Rs. 158 Lacs less Rs. 3 Lacs = Rs. 155 Lacs for 50,000 tons i.e.Rs. 310/- per ton.

Short Hedge

- Thus, a Short Hedge involves taking a Short position in the futures market i.e. sell a Futures contract at the beginning of the strategy.

Short Hedge may be used when the Hedger already owns (or expects to own) the asset and wants to sell it the future but lock-in the price at initiation, thereby reducing/ eliminate

Basis Risk

- Such long & short hedges are too good to be true.
- Rarely it is possible to completely eliminate risk as :

a. Asset whose price is being hedged may not be exactly the same asset underlying the futures contract.(Commodity Basis Risk)

b. Quantity of asset to be hedged may not be exactly equal to the quantity specified under the futures contract. (Quantity Risk)

c. Horizon period over which the hedge is to be set is not clear

Basis= Spot Price of the Asset being hedged less Futures Price of the Contract used for Hedging

Risks in Hedging

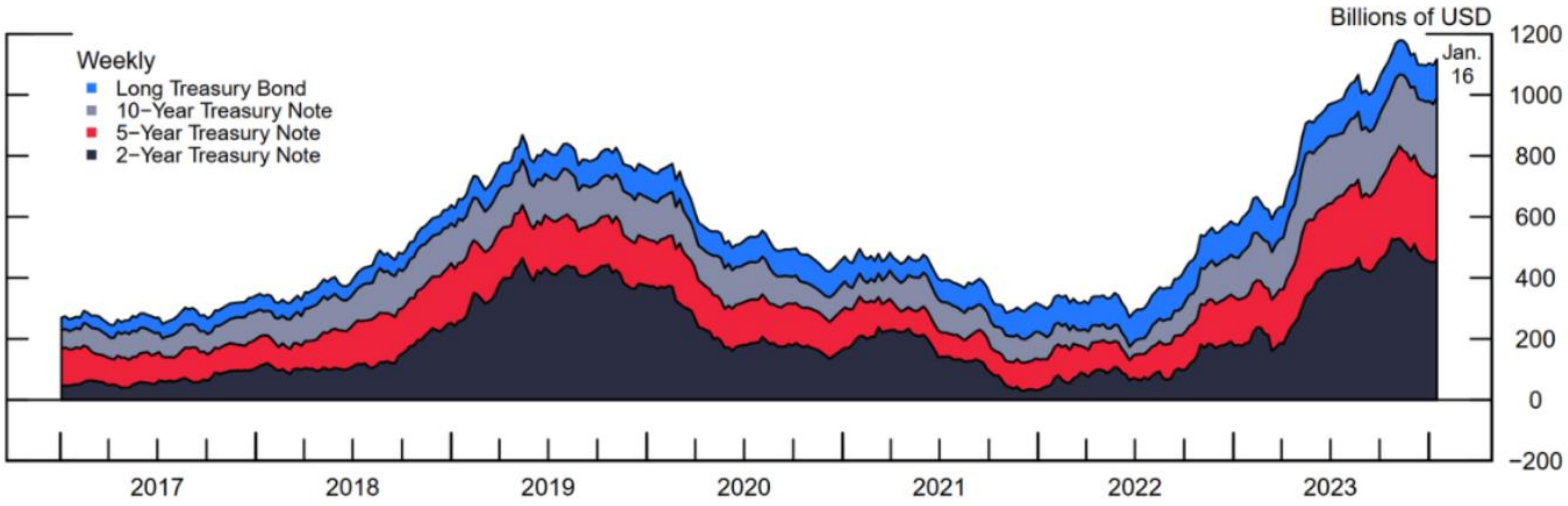
- Sometimes the asset being hedged and the asset underlying the Futures contract may not be exactly the same (Cross Hedge) and leads to Commodity basis risk.
- Futures contracts have standardized grades which may not exactly be the same as the asset being hedged.

➢Underlying asset is Plutonium which is being hedged with Futures on copper, or

➢Corporate bonds being hedged by Futures on T-Bonds

- Lets apply what we learned ……….

Figure 2. Leveraged Funds' Short Positions in Treasury Futures



Note: Key identifies in order from top to bottom. '10-Year Treasury Note' refers to both 10-year and Ultra 10-year notes. 'Long Treasury Bond' refers to both Long and Ultra Long Treasury Bonds. Notional value calculated as the number of positions multiplied by the size of the futures contract - $200,000 for 2-Year Treasury Note, $100,000 for all other Treasuries. Leveraged funds include hedge funds, registered commodity trading advisers (CTAs), and commodity pool operators (CPOs).

Source: CFTC Traders in Financial Futures.

How a Basis Trade Works

What do you do when you notice a price differential between the spot price of a commodity and the price of a futures contract? You do a basis trade (if you're an eligible counterparty, that is).

Basis = Spot Price of X - Futures Price of X

It's a pretty neat way to make money, as long as you can. Basis exists even in the market for the safest, highest liquidity asset you can possibly think of - US Treasury bonds.

High Differential in Basis Risk

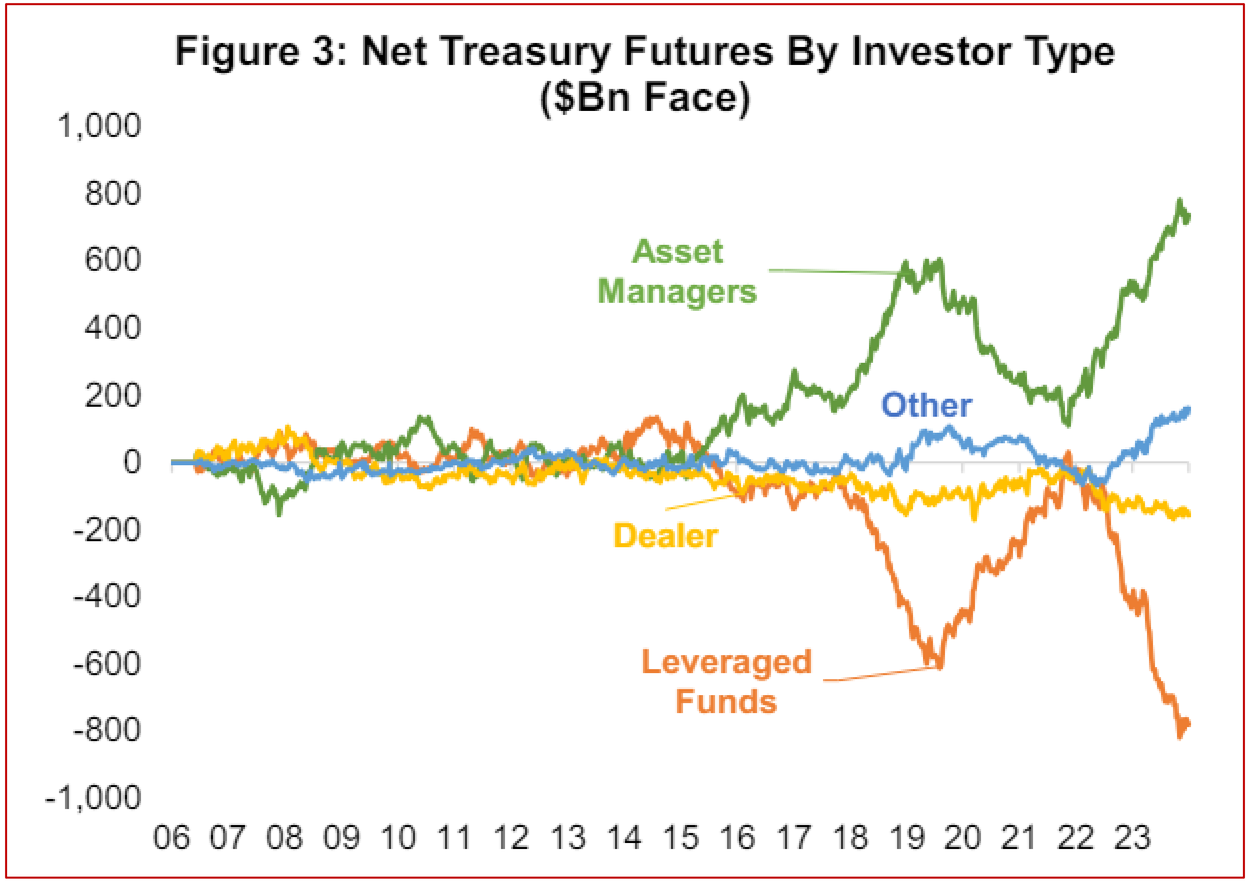
- *Lots of treasury securities being issued* - when the Fed is trying to ‘normalize’ its balance sheet. Higher supply >> Lower price for treasuries (compared to its futures contracts.
- *Huge demand for Treasury futures coming from long term asset managers.* Why? Because it gives them leverage >> they can either achieve leverage through repos (in which case they’ll have to report interest expenses in their total expense ratios - making them look more expensive) OR through buying futures (which looks like the more promising deal, expense-wise)

Hedge Fund Strategy

- These guys buy a lot of Treasury bonds, with huge sums of borrowed money (thanks to repo markets).

- On top of that, they sell Treasury futures - with very little of their own money as margin. Think 50x leverage - they'll put down $10 of their own money as initial margin when initiating a futures position for $500.

- As of Sep 2023 : Hedge funds had purchased, on net, $626 billion in Treasury securities since Sep 2017, with $478 billion by funds that were classified as likely basis traders.



Source: TBAC Report

Risks in Hedging

1. Sometimes, the hedger does not know the exact quantity of the underlying asset to be hedged (Quantity Risk)

- A farmer wanting to lock-in the price of his produce that is yet to be harvested Quantity risk is more in agri-product.

2. Hedger may not know the exact date on which to buy(sell) the underlying asset. When horizon date is not certain, it would be difficult to align with the expiration date of futures contract.

Presence of Basis Risk implies that the Cashflows cannot be completely risk-less by hedging (as in Perfect Hedge)

Which Futures Contracts to use ?

- It is important to select a futures contract on an asset that is highly correlated with the asset being hedged.
- In most cases the choice is obvious, but not in others.
- ❑ An investor wanting to hedge a highly diversified portfolio of mid-cap stocks has the following choices:

o Futures on Mid-cap Index (Not actively traded & portfolio does not match with the Index)

- Futures on Large-cap Index (actively traded)
- Futures on Small-cap Index (actively traded)
- Hedger may like to use the large or small-cap index, or better still – a forward contract customized to match his portfolio

Which expiration month?

- The hedger should, therefore, choose an expiration as close as possible to but after the month in which the time horizon end.

Whether to be long or short ?

- Taking a wrong position in the futures market would increase the risk.
- If the hedger goes long (or short) when he was required to take a short (or long) position, he would increase his risk.

Current Spot Position Method:

Determine current position in Spot market

➢ If you own the asset: Current Spot position is Long

➢ If you sell/Short on the asset: Current Spot position is Short

➢ If you intend to buy the asset in the future: Current Spot position is Short.

2. Take a futures position opposite to current Spot position

Number of Futures contracts ?

- Hedge ratio is the ratio of no. of Futures contracts (H) to the exposure in the spot market (Q) or no. of futures positions taken per unit of spot exposure.

- Hedge ratio = $\dfrac{\text{Size of position taken in Futures Contracts(H)}}{\text{Size of the Exposure in Cash Market}(Q)}$

- Size of Exposure= 8,000 tons; Futures Contract = 4,000 tons; HR = 0.5
- When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a HR of 1.0.
- However, when cross hedging is used, adopting HR of 1.0 may not be optimal as Futures & Spot prices do not change in the same proportion.
- Hedger chooses When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a HR of 1.0

• However, when cross hedging is used, adopting HR of 1.0 may not be optimal as Futures & Spot prices do not change in the same proportion.

• Hedger chooses a HR such that it minimizes the variance of the value of the Hedged position

Basis risk

- S_0 : Spot price today
- F_0 : Futures price today
- S_t : Spot price at t prior to expiration
- F_t : Futures price at t prior to expiration
- S_T : Spot price at expiration
- F_T : Futures price at expiration

Cash flow from a Hedged Position

	Cash Flow in		Total Cash Flow
	Spot Market	Futures Market	
Short Hedge (Selling Price)	QS_t	$H(F_0 - F_t)$ SP CP	$QS_t + H(F_0 - F_t)$ $= QS_t - H(F_t - F_0)$ = 1480 – (1490 - 1650) =1640
Long Hedge (Cost price)	$- QS_t$	$H(F_t - F_0)$	$-QS_t + H(F_t - F_0)$ $= - [QS_t - H(F_t - F_0]$ = - [1480 – (1490 -1650)] = -[-10+1650] = -[1640]

- In both the cases, 'H' should be such that it minimizes the variance of the Net Cash Flow.

In case of No Basis Risk

- If Basis Risk is zero, then $S_T = F_T$, and the net cash flow from a Long Hedge is :

$$= QS_T - H(F_T - F_O) = QS_T - H(S_T - F_O)$$
$$= (Q - H)\,S_T + HF_O$$

- At t=O, Q, H, & F_O is known, while S_T is unknown.
- If we set H = Q, then Cash Flow reduces to HF_O (or QF_O), a *known* quantity, the variance of which is Zero.

Thus, in case of **Zero Basis Risk, it is Optimal to Hedge completely i.e. have a Hedge Ratio of 1.0**

When Basis Risk is present

- If Basis Risk is NOT zero, then the net cash flow from a Long Hedge $[QS_T - H(F_T - F_O)]$ in terms of change in prices, is:

$= QS_T - \mathbf{QS_O} + \mathbf{QS_O} - H(F_T - F_O)$

$= Q(S_T - S_O) - H(F_T - F_O) + QS_O$

$= Q\Delta_S - H\Delta_F + QS_O$ *Where,* $\Delta_S = S_T - S_O$ & $\Delta_F = F_T - F_O$

$= Q\Delta_S - hQ\Delta_F + QS_O$ *Where,* Hedge ratio (h) = H/Q or H = hQ

$= \boxed{Q(\Delta_S - h\Delta_F) + QS_O}$

- Thus, **'h' should be such that it minimizes the variance of the Net Cash Flow.**

In case Basis Risk is present

- Cash Flow : $Q\,(\Delta_S - h\Delta_F) + QS_0$
- As QS_0 is known at t=0, the variance in cash flow is due to:
  - ✓ Variance of change in Spot prices = $\sigma^2_{\Delta S}$
  - ✓ Variance of change in Futures prices = $\sigma^2_{\Delta F}$
  - ✓ Covariance between the Δ_S & Δ_F = $Cov(\Delta_S, \Delta_F)$
- Variance of $Q\,(\Delta_S - h\Delta_F) = Q^2\, Var(\Delta_S - h\Delta_F)$

$$= Q^2\,[\sigma^2_{\Delta S} + h^2\sigma^2_{\Delta F} - 2h\, Cov(\Delta_S, \Delta_F)]$$

- **To minimize the variance, equate the first derivative (wrt 'h') to zero,**

In case Basis Risk is present

$$= 2h\sigma^2_{\Delta F} - 2\,Cov(\Delta_S,\Delta_F) = 0$$

$$h\sigma^2_{\Delta F} = Cov(\Delta_S\,,\,\Delta_F)$$

$[Cov(\Delta_S\,,\,\Delta_F) = \rho\sigma_{\Delta S}\sigma_{\Delta F}]$

ρ = correlation co-efficient between ΔS & ΔF

$\sigma_{\Delta S}$ = Std. Deviation in change in Spot Prices

$\sigma_{\Delta F}$ = Std. Deviation in change in Futures Prices

$$h^* = \frac{Cov(\Delta_S,\Delta_F)}{\sigma^2_{\Delta F}} = \rho\frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$$

h* is the Minimum Variance Hedge Ratio

Minimum Variance Hedge Ratio

- Consider that you have a position in 200 shares of InfoTech, a technology stock with a standard deviation of change in stock price of 30. You want to hedge this position with a technology stock index futures which has a standard deviation of 20. The correlation between the two is 0.80. What should be the Optimal Hedge ratio?

$$\sigma_{\Delta S} = 30;\ \sigma_{\Delta F} = 20;\ \rho_{\Delta S\ \Delta F} = 0.80$$

$$h^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.8 \frac{30}{20} = 1.20$$

Minimum Variance Hedge Ratio

- Standard deviation of changes in spot price of wheat is 0.00278, standard deviation of changes in futures price of wheat is 0.00259 and the coefficient of correlation between the two is 0.98031. What is the optimal hedge ratio ? What if the coefficient of correlation is: 0.3; 0.8; 1.0; and -0.6?

$$\sigma_{\Delta S} = 0.00278; \ \sigma_{\Delta F} = 0.00259; \ \rho_{\Delta S\, \Delta F} = 0.98031$$

$$h^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.98031 \frac{0.00278}{0.00259} = 1.052$$

The size of the Futures position should be 1.052 times the size of the exposure of wheat position.

ρ	h*
0.3	0.32
0.8	0.86
1.0	1.07
-0.6	-0.64

Cash Flow Variance at MVHR

- Variance of Cash Flow = $Q^2 (\sigma^2_{\Delta S} + h^2\sigma^2_{\Delta F} - 2h\,Cov(\Delta_S, \Delta_F))$
- Optimal Hedge Ratio (MVHR or h*) $= \rho \dfrac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$
- Cash Flow variance at h* =

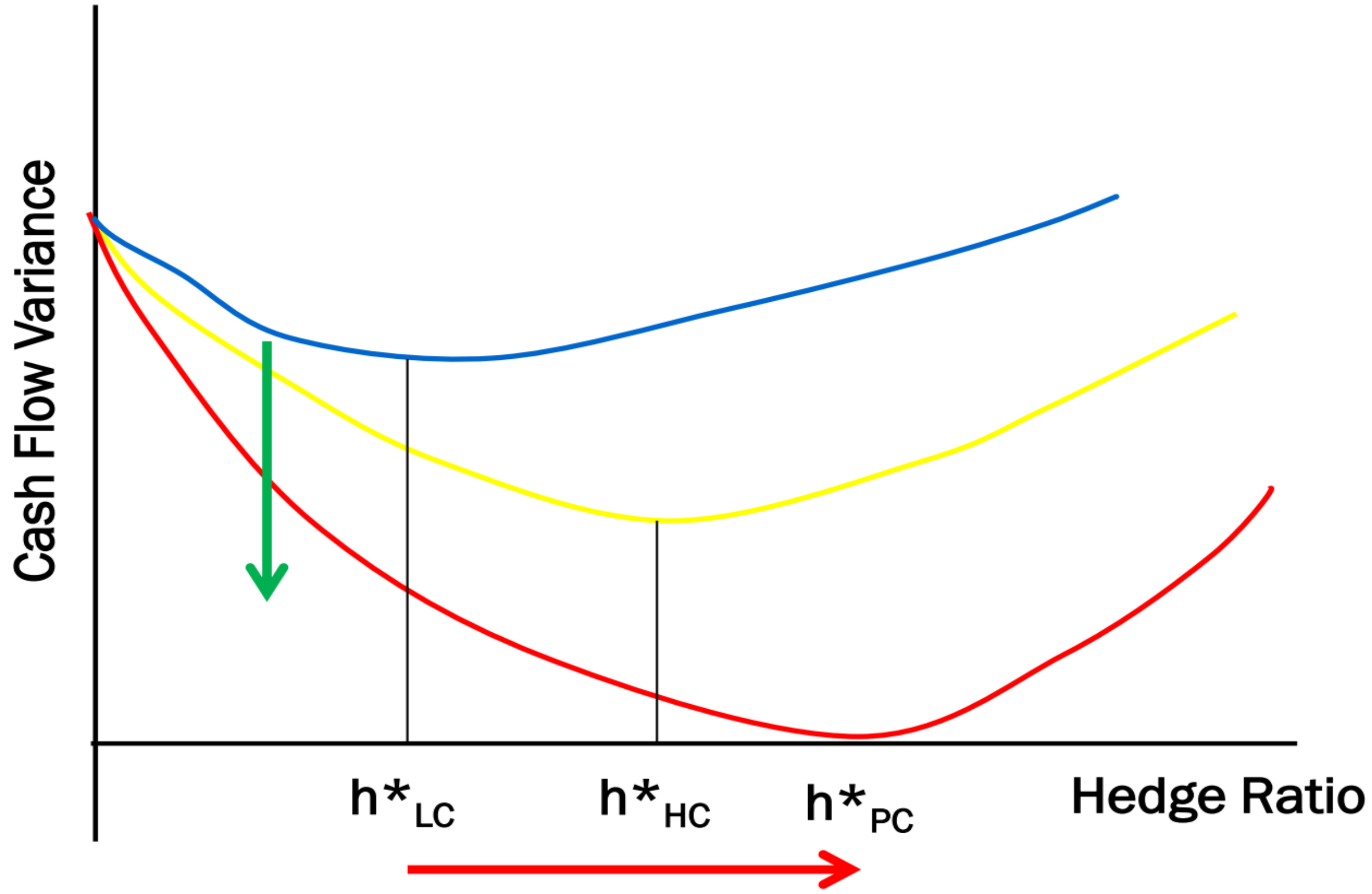
$$Q^2\left[\sigma^2_{\Delta S} + \rho^2 \frac{\sigma^2_{\Delta S}}{\sigma^2_{\Delta F}}\sigma^2_{\Delta F} - 2\rho\frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}\rho\sigma_{\Delta S}\sigma_{\Delta F}\right]$$

$$= Q^2\sigma^2_{\Delta S}\,(1-\rho^2)$$

Minimized Cash Flow Variance

- This minimized variance is zero when ρ = +/-1 i.e. when futures and spot prices are perfectly correlated (+ve or –ve) i.e. when basis risk is zero.
- **If Basis risk is present, there will always be some residual uncertainty even after hedging.**

Minimum Variance Hedge Ratio



- ✓ MVHR increases as correlation increases

ρ	h*
0.3	0.32
0.8	0.86
0.98	1.052
1.0	1.07
-0.6	-0.64

- ✓ Variance of Cash flows is lower as correlation is higher- i.e. higher correlation gives better protection.

Minimum Variance Hedge Ratio

- Alternatively, the slope of the regression line of Δ_S on Δ_F, is h^*.

Week	Change in F_t	Change in S_t	
1	0.0032	0.0038	
2	0.0040	0.0034	
3	0.0025	0.0030	
4	0.0035	0.0035	
5	0.0037	0.0048	
6	-0.0015	-0.0012	
7	-0.0005	-0.0005	
8	-0.0001	0.0005	
9	-0.0025	-0.0022	
10	-0.0020	-0.0028	
Std Dev.	**0.00259**	**0.00278**	<-- =STDEV(C4:C13)
Correlation	**0.98031**		<-- =CORREL(C4:C14,B4:B14)
h^*	**1.052**		<-- =(C14*B15)/B14
Slope	**1.052**		<-- =SLOPE(C4:C13,B4:B13)
$Corr^2$	0.96		<-- =B15^2

Going Long or Short in Futures Market

	Position in Spot Market at t=0	Position to be taken in Futures Market at t=0
If ρ > 0	**Short** (i.e. want to buy in the future) (*will lose on increase in Cash price*)	**Go Long on Futures now** (Futures price will also increase and hence long position would lead to gains in Futures market)
	Long (i.e. have /will have the asset, and want to sell in the future) (*will lose by fall in Cash price*)	**Go short on Futures now** (Futures price will also decrease in the future and hence short position would lead to gains in Futures market)
If ρ < 0	**Short** (i.e. want to buy in futures) (*will lose on increase in Cash price*)	**Go Short on Futures now** (Futures price will decrease hence short position would lead to gains in Futures market)
	Long (i.e. have /will have the asset, and want to sell in the future) (*will lose by fall in Cash price*)	**Go Long on Futures now** (Futures price will increase, and hence long position would lead to gains in Futures market)

Which Futures Contract to choose?

You have a long position on 500 tons of Chana. Assuming futures on Chana are not available while futures on Rice (F_1) and Jerra (F_2)are available. Using these futures, you are asked to hedge your position in Chana. Correlations between change in Spot price of Chana and change in Futures prices are : between S & F_1: 0.68 , between S & F_2: -0.98, and standard deviations of the price changes: $\sigma_{\Delta S} = 0.30$; $\sigma_{\Delta F1} = 0.25$; $\sigma_{\Delta F2} = 0.15$.

Find the Minimum-Variance Hedge Ratio for S using futures contracts F_1 and F_2. Using which futures contract is more effective?

Which Futures Contract to choose?

$Q = 500$ tons of Chana $\sigma_{\Delta S} = 0.30$; $\sigma_{\Delta F1} = 0.25$;
$\sigma_{\Delta F2} = 0.15$; $\rho_{\Delta S\, \Delta F1} = 0.68$; $\rho_{\Delta S\, \Delta F2} = -0.98$

- **Using F_1:**

$$h_1^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F1}} = 0.68 \frac{0.30}{0.25} = 0.816$$

$$\left[Q^2 \sigma^2_{\Delta S}\ (1-\rho^2)\right] = 500^2 (0.30)^2 (1-0.68^2) = 12,096$$

- **Using F_2:**

$$h_2^* = -0.98 \frac{0.30}{0.15} = -1.96$$

$$\left[Q^2 \sigma^2_{\Delta S}\ (1-\rho^2)\right] = 500^2 (0.30)^2 (1-(-0.98^2) = 891$$

Optimal No. of Futures Contracts

- Based on the Optimal hedge ratio (h*), the optimal no. of futures contracts (N*) required to hedge an exposure of QA units, given the size of one futures contract as QF is given as follows:

$$N^* = h^* \frac{Q_A}{Q_F}$$

Optimal No. of Futures Contracts

The standard deviation of monthly changes in spot price of copper is 1.2%. The standard deviation of changes in futures price of copper is 1.4%. The coefficient of correlation between the two is 0.7. A copper wire manufacturer is committed to purchase 200,000 kg. of copper on 15-Nov. The producer wants to use the Dec futures to hedge his risk. Each futures contract is of 40,000 kg. of copper. What strategy should the wire manufacturer follow?

Optimal No. of Futures Contracts

$\sigma_{\Delta S} = 1.2\%$; $\sigma_{\Delta F} = 1.4\%$; $\rho_{\Delta S\, \Delta F} = 0.70$;

$Q_A = 200{,}000$ kg; $Q_F = 40{,}000$ kg

$$h^* = \rho_{\Delta S \Delta F} \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.7 \frac{1.2}{1.4} = 0.60$$

$$N^* = \frac{h^* \; Q_A}{Q_F} = \frac{0.6 * 200{,}000}{40{,}000} = 3$$

- The copper wire manufacturer is currently short on copper. He loses, if spot price of copper increases.
- If cash price increases, futures price will also increase (as ρ is +ve). So, in order to compensate for loss in cash market, he needs to gain in futures market, which can be achieved by going long on Copper Futures.
- Hence, wire manufacturer should **go long** on **3 Copper futures expiring in Dec.**

Optimal No. of Futures Contracts

Now consider that the hedge is set using futures on gold ***instead of copper***. The standard deviation of changes in futures price of gold is 1.8% & coefficient of correlation between the two is -0.5. Each futures contract is of 40,000 kg. of gold. What strategy should the wire manufacturer follow now?

Optimal No. of Futures Contracts

$\sigma_{\Delta S}$ = 1.2%; $\sigma_{\Delta F}$ = 1.8% ; $\rho_{\Delta S\, \Delta F}$ = -0.50; Q_A = 200,000 kg; Q_F = 40,000 kg

$$h^* = \rho_{\Delta S \Delta F} \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = -0.5 \frac{1.2}{1.8} = -0.33$$

$$N^* = \frac{h^* \; Q_A}{Q_F} = \frac{-0.33 * 200{,}000}{40{,}000} = -1.65 \approx -2$$

- The copper wire manufacturer is currently short on copper and would lose, if spot price of copper increases.
- If Copper cash price increases, futures price of Gold will decrease (as ρ is - ve).
- In order to compensate for loss in cash market, he needs to gain in futures market, which can be achieved by going short on Gold Futures.
- Hence, wire manufacturer should **go Short** on **2 Gold futures expiring in Dec.**

Hedging an Equity Portfolio using Stock Index Futures

- Stock Index Futures may be used to hedge a welldiversified portfolio of stocks.
- During times of unusual volatility in the market, stock index futures may used to change or eliminate systematic risk.
- The no. of stock Index futures contracts to be used for hedging:

$$N_F^* = \beta_P \frac{V_A}{V_F}$$

where, VA = Value of the Portfolio
VF = Value of One Futures contract (Futures price * contract size)

Hedging an Equity Portfolio using Futures on Stock Index

- Suppose, the current value of your portfolio is Rs. 5 Lacs, with a beta of 1.2, which you want to protect from loss of value from decline in the market over the next 3 months. The Index value today is 1000 and the 4-month Futures on the Index is at 1010. Each futures contract is for 20x. How many futures contracts should be bought or sold? Assume risk-free interest as 4% pa and dividend yield of 1%pa (both on cc).

Hedging an Equity Portfolio using Futures on Stock Index

	A	B	C	D	E	F	G
2	Value of Portfolio today (V_A)	500,000	(Long Position in Cash Market) -->		Hence the risk		
3	Index Value today (t=0)	1,000			is fall in Value of Portfolio.		
4	4-month Index Futures today	1,010					
5	Beta of Portfolio	1.2					
6	Risk-free rate (%pa on cc)	4%	1.00%	Over 3 months			
7	Dividend Yield(%pa on cc)	1%	0.25%	Over 3 months			
8	1 Futures Contracts	20	x				
9							
10	Value of 1 Futures Contract (V_F)	20,200	<-- **=B4*B8**				
11							
12	No. of Futures to Sell	29.7030	<-- **=(B5*B2)/B10**		(Go Short on Index Futures)		
13	$N^* = (\beta^* V_A)/V_F$	30	<-- **=ROUND(B12,0)**				