

Hedging Strategies using Futures

Operation of Margins

- When 2 investors contract, one party may back out or may not have the financial resources to honor his commitments.
- Futures Exchange organizes trades so as to avoid defaults, through a system of margins.
- Different types of margins are maintained:
 - ✓ Initial Margin (IM)
 - ✓ Maintenance Margin (MM)
 - ✓ Variation Margin (VM)

Operation of Margins

- Initial Margin (IM): Good faith deposit paid by the trader at the time of entering the contract to ensure performance.
- IM may vary from contract to contract & from trader to trader.
- Typically set at 5% of the contract value.
- Trader retains title to the deposit.
- Usually equal to Maximum Daily Price fluctuation limit.
- IM is returned upon proper completion of all the obligations.
- At the end of each day, the initial margin account is adjusted to reflect investor's gain/loss.
→ daily settlement or Mark-to-market

Operation of Margins

- Maintenance Margin (MM) :(% of the Initial Margin) is the minimum amount of margin below which amount in Initial Margin account should NOT fall.
- MM is used to calculate the third margin –Variation Margin (VM).
- If the Initial Margin account falls below the MM, trader is required to replenish (*or top-up*) the Initial Margin account.
- This additional amount paid by the trader is called Variation Margin.
- Any amount in excess of the IM can be withdrawn by the investor.
- Failure to pay VM leads to the futures position being closed out.

Operation of Margins - *An Example*

- Suppose Mr. X is long on 5 Futures contract on gold at MCX. Each contract is for 100 grams. The price quoted is Rs. 15,550/-per 10 grams.
- Tick size is Re 1.
- Initial margin is 4% while maintenance margin of 90% of initial margin.

Operation of Margins

	A	B	C	D	E	F	G	H
1	Opening Price			15,550	/10 g			
2	No. of Futures Contract			5				
3	Contract size			100 g				
4	Initial Margin			4%	31,100			
5	Maintenance Margin			90%	27,990			

Case of Paul

- Let us take the case of Paul who has gone long in a futures contract expiring five days hence with Keith, at a futures price of 75. We will assume that the price at the expiration of the contract is 82.50. and that the prices at the end of each day prior to expiration are as follows.
- Paul is committed to buying 100 units of the asset, and that at the time of the trade, both the parties had to deposit 1,000 as collateral in their margin accounts. The collateral that an investor is required to deposit at the time of entering into a futures contract, is referred to as the Initial Margin.
- End of the Day Futures Prices

Day	Futures Price
0	75.00
1	78.50
2	73.50
3	71.00
4	79.50
5	82.50

Interpretations

- At the end of the first day the futures price is \$ 78.50. This means that if a trader were to enter into a contract at the end of the that day, the applicable price per unit of the underlying asset would be 78.50. If Paul were to offset the position that he had entered into earlier that day, he would obviously have to do so by agreeing to sell 100 units at 78.50 per unit. If so, he would earn a profit of 3.50 per unit, or 350 in all.
- In the process of marking Paul's position to market, the broker will behave as though he were offsetting. Thus, he would calculate his profit as 350, and would credit this amount to his margin account.
- However, remember that Paul has not actually expressed a desire to offset. Consequently, taking cognizance of this fact, the broker would act as if Paul were re-entering into a long position at the prevailing futures price of 78.50.
- At the end of the second day, the prevailing futures price is 73.50. Thus, when the contract is marked to market on this day, Paul will make a loss of \$ 500. It must be remembered, that his contract was re-written the previous day at a price of 78.50, and if the broker were to now behave as if he were offsetting at 73.50, the loss would amount to 5 per unit, or \$ 500 in all. Having marked the contract to market, the broker would once again establish a new long position for Paul, this time at a price of 73.50.

Lets take Paul's case again

- Suppose initial margin =700

Day	Futures Price	Daily Gain/Loss	Cumulative Gain/Loss	Account Balance	Margin Call
0	75.00			1,000	
1	78.50	350	350	1,350	
2	73.50	(500)	(150)	850	
3	71.00	(250)	(400)	600	400
4	79.50	850	450	1,850	
5	82.50	300	750	2,150	

Closing a Futures Position

- **Delivery**- Delivery of the goods under the contract will automatically close the position.
- **Physical Settlement**: Physical delivery of the asset at a certain location at a specified time as per the exchange rules.
- Decision regarding location of delivery is with the seller.
- If seller decides to deliver the underlying, it would issue a “Notice of intention to Deliver” to the exchange.
- The exchange will then choose the party with the long position to accept the delivery.
- Usually, the notice is passed to the party with the oldest long position outstanding.
- Parties with long positions must accept delivery.
- **Cash Settlement**: Traders make payment at expiry of contract to settle any gain or loss.

Closing a Futures Position

- **Offset:** Most Futures contracts are settled by “Offsets”, by entering into a exactly reverse trade which shall cancel the original trade.
- The trader, in order to close the contract, should enter into an exactly reverse contract in terms of:
 - (a)the underlying assets,
 - (b)No. of contracts &
 - (c)expiry date

Offset Trades – An Example

May 1	<u>Party A's Initial Position:</u> Bought 1 September Wheat Futures Contract @ Rs 2,200/-.	<u>Party B :</u> Sold 1 September Wheat Futures Contract @ Rs 2,200/-.
May 15	<u>Party A's Reversing Trade:</u> Sold 1 September Wheat Futures Contract @ Rs 2,300/-.	<u>Party C:</u> Bought 1 September Wheat Futures Contract @ Rs 2,300/-.

After the two trades, A's net position is Zero and is out of the market. B & C still have obligations to the CH.

Closing a Futures Position (Contd.)

- **Exchange for Physicals:** Buyer and Seller exchange for cash, the underlying asset outside the exchange system.
- **EFP vs. Offset:**
- Under both, the traders have completed their obligations & are now out of the market.
- Differs from Offsets:
- Traders actually exchange the physical goods.
- Futures is not closed by a transaction through the Exchange.
- Traders privately negotiate the terms, hence also called “ex-pit”.

Open Interest

- 'Open Interest' refers to the number of futures contracts outstanding (not squared off) at any point in time. It is the total no. of 'open' positions waiting to be liquidated before the contract's maturity.
- OI rises with time as more and more investors enter into new contracts.
- As maturity approaches, investors unwind their positions by entering into reverse trades. Hence, OI starts to decline.
- Today's newspaper carry yesterday's trading data and day before yesterday's Open Interest data.
 - ✓ Every trade needs a buyer & a seller
 - ✓ Any trade (long or short) initiated afresh raises OI
 - ✓ Any trade (long or short) that squares up existing position lowers OI

Basic Terms

- **Short selling:** Selling of an asset that is not owned by the Investor (seller).
- The investor borrows (thru his broker) the asset from someone who owns the asset.
- Investor has to pay to the owner of the asset, any income that is received on the asset shorted.
- Investor is required to maintain a margin account with his broker to ensure that the investor does not run away from his short position.

Determination of Forward/Futures Prices

Assumptions:

- No transaction costs.
- Uniform tax rate on all trading profits.
- All participants can borrow and lend at the same risk-free rate of interest.
- Market participants take advantage of the arbitrage opportunities which may arise.

Notations:

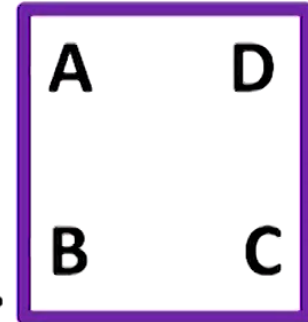
S_0 : Price of the underlying asset (in Cash Market) at $t=0$.

F_0 : Forward / Futures Price (in Futures Market) at $t=0$

r_f : Zero – Coupon risk-free rate pa. on continuous compounding basis.

Future Pricing

FORWARD PRICING...



- **AB**: Buy one unit of underlying in spot market for S_0 .
- **BC**: Carry the asset for delivery against forward obligation at $t=T$.
- **CD**: Deliver the asset S and receive F_0 units of M at $t=T$ under the forward contract.
- **DA**: Adjust the borrowings S_0 against the present value of the forward proceeds $F_0 \exp(-rT)$.

Underlying asset provides no income

- Thus, so long the forward price is NOT equal to Rs. 506/- there would be an opportunity for arbitrageur to adopt either of the trading strategies and earn a risk-less profits.
- For NO ABITRAGE to take place, the Forward Price (F_0) should be exactly Rs. 506/-.

$$\text{Forward Price } (F_0) = S_0 e^{r_f T}$$

- $F_0 = S_0 e^{rT} = 500 e^{(0.05)3/12} = 500 * 1.0126 = \text{Rs. } 506/-$

Underlying asset provides no income

- Suppose a non-dividend paying stock is available for Rs. 500/- (S_0) and risk-free rate of Interest (r_f) is 5% pa.
- IF the 3-month Forward price (F_0) is relatively high at Rs. 535/-.
- An arbitrageur will earn risk-less profit, if he adopts the following trading strategy:

Today:

- a) Borrows Rs. 500/- for 3 months @ 5% pa.
- b) Buys the stock for Rs.500/- in the Cash Market
- c) Sells 3-month forward contract on the stock at F_0 of Rs. 535/-.

Underlying asset provides no income

At the end of 3 months:

The arbitrageur shall:

a) Repay the amount borrowed along with Interest

= Rs. 500/- + Interest @ 5% pa for 3 months

= $500 * e^{(0.05 * 3/12)} = 500 * 1.0126 = \text{Rs. } 506/-$ (approx.)

b) Deliver the stock under the Forward contract and receive Rs. 535/- .

➤ **Cash Flows: Inflow: Rs 535/- Outflow: Rs. 506/-**

- By following this strategy, the arbitrageur has locked in a risk-less profit of Rs.29/- i.e. (535-506)

Underlying asset provides no income

- *IF* the 3-month forward price (F_0) is relatively low as Rs. 475/- (instead of Rs. 535/-)
- An arbitrageur can adopt the following strategy and make risk-less profit:

Today:

- a) Short Sell the stock @ Rs. 500/- in the cash market,
- b) Invest the proceeds for 3-months @ $r_f = 5\%$ p.a., and
- c) Buy a 3-month forward contract on the stock @ Rs. 475/-

Underlying asset provides no income

At the end of 3-months:

The arbitrageur would:

- a) Receive the loan of Rs. 500/- along with interest for 3 months = $500 * e^{(0.05)*(3/12)} = \text{Rs. } 506/-$
 - b) Receive the stock under the forward contract and pay Rs 475/- for it.
- **Cash Flows: Inflow: Rs 506/- Outflow: Rs. 475/-**
- By following this strategy, the arbitrageur has locked in a risk-less profit of Rs.31/- i.e. (506 - 475)

Underlying asset provides no income

A 6-month forward contract on a non-dividend paying stock is entered into when the stock price is Rs. 2800/-. The risk-free rate of return is 8%pa. What should be the 6-month forward price?

$$S_0 = 2,800/- ; r_f = 8\% \text{ pa} ; T = 6/12 \text{ years} = 0.5 \text{ years}$$

$$\begin{aligned} \text{Forward Price (F}_0) &= S_0 e^{rT} \\ &= 2800 e^{0.08 \times 0.5} = 2800 * 1.04081 \\ &= \text{Rs.2,914.27} \end{aligned}$$

Underlying asset provides no income

Consider the stock of Suzlon which is currently trading at Rs 85/-. The 3-month forward contract on Suzlon stock is available for Rs. 90/-. If the risk-free rate is 6% pa. (on continuous compounding basis) and the stock is not expected to pay any dividend over the next 3 months. Are there any opportunities for arbitrage?

Underlying asset provides no income

$S_0 = 85/-$; $r_f = 6\%$ pa ; $T = 3/12$ years = 0.25 years;

$F_{\text{ACTUAL}} = \text{Rs.}90/-$

$$\begin{aligned}\text{Theoretical Forward Price (F}_0\text{)} &= S_0 e^{rT} \\ &= 85e^{0.06 \times 0.25} = \text{Rs. } 86.28\end{aligned}$$

Theoretical Forward Price < Actual Forward Price, hence arbitrage opportunity exists:

Adopt Cash-and-Carry Arbitrage:

Now:

- Borrow Rs 85/- for 3 months @6%pa
- Buy stock @ Rs. 85/-
- Sell 3-month Forward @ Rs. 90/-

After 3 months:

- Repay loan with interest:
 $85e^{0.06 \times 0.25} = \text{Rs. } 86.28$
- Deliver stock under forward contract and receive Rs. 90/-
- Profit = $90 - 86.28 = \text{Rs. } 3.72$

Underlying asset provides no income

Shares of Megacorp are currently trading at Rs. 40/- and the 6-month forward on the stock is available at Rs. 38/-. If the risk-free rate is 8% pa. (cc) and the stock will not pay any dividend over the next 6 months, do you find any opportunities for arbitrage?

Underlying asset provides no income

$S_0 = 40/-$; $r_f = 8\%$ pa ; $T = 6/12$ years = 0.50 years;
 $F_{\text{ACTUAL}} = \text{Rs.} 35/-$

Theoretical Forward Price (F_0) = $S_0 e^{rT}$
 $= 40e^{0.08 \times 0.5} = \text{Rs. } 41.63 > \text{Rs. } 38/-$

Adopt Reverse Cash-and-Carry:

Now:

- Buy 6-month Forward @ Rs. 38/-
- Short Sell the stock @ Rs. 40/-
- Invest Rs 40/- for 6 months @ 8%pa

After 6 months:

- Receive the loan with interest: $40e^{0.08 \times 0.5} = \text{Rs. } 41.63$
- Buy the stock under forward contract for Rs 38/- and deliver the stock.
- Profit = $41.63 - 38 = \text{Rs. } 3.63$

At t = 0	At maturity
+40-40	+41.63-38
Net = 0	3.63

Underlying asset provides known amount of cash Income

$$\text{Forward Price } (F_0) = (S_0 - I) e^{rt}$$

where 'I' is the present value of the income 'Y' received during the forward contract. ($I = Y * e^{-rt}$.)

Underlying asset provides known amount of cash Income

A stock will pay a dividend of Re 1 per share in 2 months and again in 5 months from now. The stock is currently selling for Rs. 50/- and the risk-free interest rate is 6%pa. What should be the price of a 6-month forward contract on the stock?

$$S_0 = 50/- ; r_f = 6\% \text{ pa} ; T = 6/12 \text{ yrs} ; D_2 = \text{Re } 1 ; D_5 = \text{Re } 1$$

$$\text{Forward Price } (F_0) = (S_0 - I)e^{rT}$$

$$I = D_2e^{-rt_1} + D_5e^{-rt_2} = 1e^{-0.06*2/12} + 1e^{-0.06*5/12}$$

$$= 0.9900 + 0.9753 = 1.9653$$

$$F_0 = (50 - 1.9653)e^{0.06*6/12} = 48.0347 * 1.0305$$

$$= \text{Rs. } 49.4976 = \text{Rs. } 49.50$$

Value of a Forward Contract

- At inception, the value of Forward contract is Zero.
- Buyer(Seller) are indifferent between buying/(selling) the underlying asset now (at $t=0$) for spot price or buying (selling) at maturity for Forward price.
- As time passes, the price of the underlying asset changes, which makes the existing forward contract to become an asset or a liability.
- For **Long position** on Forward Contracts:

Value of FC on Maturity = Forward Price_{New} - Forward Price_{Old}

Value of FC today = PV of $(F_N - F_0) = (F_N - F_0) e^{-rt^*}$

- For **Short position** on Forward Contracts:

Value of FC on Maturity = Forward Price_{Old} - Forward Price_{New} Value of FC today

= PV of $(F_0 - F_N) = (F_0 - F_N) e^{-rt^*}$

Value of a Forward Contract

One month ago the stock of ABC was trading at Rs. 114/-. Now, the stock price has declined to Rs. 109/-. What would be the impact on the value of the 6-month forward contract that was contracted a month ago (*when the stock price was Rs. 114/-*)? Assume the risk-free rate of return as 7% pa. (cc).

$S_{-1} = 114/-$; $S_0 = 109/-$; $r_f = 7\%$ pa; $T = 6$ months.

- Forward Price:

At $t = -1$: $F_{\text{OLD}} = 114 e^{(0.07)*6/12} = 114 * 1.03562 = 118/-$

At $t = 0$: $F_{\text{NEW}} = 109 e^{(0.07)*5/12} = 109 * 1.02960 = 112/-$

Value of Forward Contract (old) on **Maturity** = $F_N - F_0 = 112 - 118 = -6$

Value of Forward Contract (old) **today**

$$= \text{PV} (F_N - F_0) = -6 e^{-(0.07)*5/12} = -6(0.97125) = -\text{Rs } 5.83$$

Value of a Forward Contract

A long forward contract on a non-dividend paying stock was entered into some time back at a forward price of Rs. 24/-. The forward contract now has 6 months to maturity. The risk-free rate of return is 10% pa (cc), the stock is trading at Rs 25/-. What would be the impact on the value of the original forward contract now?

$F_0 = 24/-$; $S^*_0 = 25/-$; $r = 10\%$ pa; $T = 6$ months.

Forward Price of New contract $F_N = S^*_0 e^{rT}$

At $t = 0$: $F_{OLD} = 24/-$

At $t = 1$: $F_{NEW} = 25 e^{(0.10)*6/12} = 25 * 1.05127 = 26.28$

Value of Forward Contract on Maturity = $F_N - F_0 = 26.28 - 24 = 2.28$

Value of Forward Contract today = $PV (F_N - F_0)$
 $= 2.28 e^{-(0.10)*6/12} = 2.28 (0.9512) = \text{Rs. } 2.17$

Value of a Forward Contract

2 months back a US investor sold forward £ 2 million at a forward price of \$ 1.61 per pound. After one month, the forward price for delivery in one month is \$ 1.585 per pound. Suppose the one-month rate of interest is 6% (cc), what is the value of the investor's position?

$F_0 = \$1.61$; $F_N = \$1.585$; $r_f = 6\%$ pa; $T = 1$ month.

For **Short** Futures Position:

Value of Forward Contract on Maturity= $F_0 - F_N = 1.61 - 1.585$

Value of Forward Contract today= $PV(F_0 - F_N)$

$$= 0.025e^{-(0.06)*1/12} = 0.025(0.9952) = \$ 0.024875/ \text{£}$$

Investor with **short position** would **receive**

$\text{£ } 2,000,000 * 0.024875 = \$49,750/-$ on unwinding of the position.

Value of a Forward Contract

Consider the current spot price of ABC shares is Rs. 800/. Mr. A has bought one 6 month forward contract at Rs.900/. After one month, Mr. B offers to buy a 5-month forward contract on ABC shares at 925/- . If the risk-free interest rate is 9%, what is the value of Mr. A's contract?

- After one-month , if Mr A goes short on 5-month forward, he would receive Rs, 925/- (F_N) and would be required to deliver the stock of ABC, which he would get under the 6-month forward on which he is long for Rs 900/- (F_0).
- So his profit at $T=6$ or Value of Forward Contract on Maturity= $F_N - F_0 = 925 - 900 = \text{Rs. } 25/-$.
- Value of Forward Contract **today** (when the 2nd forward is entered) = $PV (F_N - F_0)$
 $= (925 - 900)e^{-0.09*5/12} = 25*0.96319 = 24.07986$

Forward Price vs. Futures Price

- When short term risk-free interest rate is constant, the **forward price** on a certain delivery date is same as the **futures price** for a contract with the same delivery date.
- However, when interest rates vary, the Forward price *may not* be same as the Futures price.
- *Suppose the price of the underlying asset is **strongly and positively** correlated with the interest rate.*
 - In case of a Futures contract, when the **price** of the u/l asset **increases**, the investor with a long position would gain (due to daily settlement)
 - As price increases, the interest rates would also increase.
 - Hence, the gain will be invested at a higher interest rate.

Forward Price vs. Futures Price

- Similarly, when the price of the u/l asset decreases, the investor will incur an immediate loss.
- This loss will be financed at a lower interest rate.
- An investor holding a **Forward** contract (instead of a Futures contract) is **not affected** in this way by interest rate movements.
- So, a **Long Futures** contract will be **slightly more attractive** than a similar long forward contract.
- Futures price tend to be slightly more than the forward price, if price of u/l asset and interest rates are ***strongly positively correlated***.

Forward Price vs. Futures Price

- *Suppose the price of the underlying asset is **strongly and negatively** correlated with the interest rate.*
 - In case of a Futures contract, when the **price** of the u/l asset **increases**, the investor with a long position would gain (due to daily settlement)
 - As price increases, the interest rates would now decrease.
 - Hence, the gain will be invested at a lower interest rate.
 - Similarly, when the price of the u/l asset decreases, the investor will incur an immediate loss.
 - This loss will be financed at a higher interest rate.
 - An investor holding a Forward contract (instead of a Futures contract) is not affected in this way by interest rate movements.

Forward Price vs. Futures Price

- So a Long Futures contract will be slightly less attractive than a similar long forward contract.
 - **Futures price tend to be slightly less than the forward price, if price of underlying asset and interest rates are strongly positively correlated.**
- *The theoretical difference between Forward price and Futures price for short term contracts (less than a few months) usually small and hence ignored.*
- ***For most purposes, Forward and Futures price is considered to be same.***

Forward/Futures prices of Currencies

Direct Quote: Units of HC per FC

100 units of USD
at t=0

Spot: INR 70 = 1USD

Forward: INR 74.50 = 1USD

$r_{f(FC)} = 1\%$ pa (cc)

$r_{f(HC)} = 6.5\%$ pa (cc)

Time = 1 year

Invest FC @ $r_{f(FC)}$ & Convert FC
in HC @ F_0 today

Convert FC in HC @ S_0 &
invest HC @ $r_{f(HC)}$ today

Invest FC @ $r_{f(FC)}$ for 1 year

$$100e^{0.01 \cdot 1} = 101.00502 \text{ USD}$$

Convert FC into HC @ S_0 today

$$100 \cdot 70 = 7000 \text{ INR}$$

Convert FC into HC @ F_0 today

$$101.00502 \text{ USD} \cdot 74.50 = 7524.87374 \text{ INR}$$

Invest HC @ $r_{f(HC)}$ for 1 year

$$7000e^{0.065 \cdot 1} = 7470.11317 \text{ INR}$$

$100e^{r_{f(FC)} T} F_0$ should be equal to $100S_0e^{r_{f(HC)} T}$

$$F_0 = S_0 e^{(r_{HC} - r_{FC}) T} \rightarrow \text{Interest Rate Parity}$$

$$\text{Theoretical } F_0 = 70e^{(0.065 - 0.01) \cdot 1} = 70(1.05654) = 73.95784$$

Forward/Futures prices of Currencies

2-month interest rates in Switzerland and India are 3% & 6% pa (cc). Spot price of Swiss franc is Rs. 33.778. What should be the 2-month forward of Swiss franc?

$$S_0 = 33.778; r_{HC} = 6\% \text{ pa}; r_{FC} = 3\% \text{ pa}; T = 2/12 \text{ years}$$

$$F_0 = S_0 e^{(r_{HC} - r_{FC})T}$$

$$= 33.778 e^{(0.06 - 0.03)2/12}$$

$$= 33.778 e^{0.005} = 33.778 (1.00501) = \text{Rs. } 33.9473$$

Currency with higher interest rate should depreciate.

Forward/Futures prices of Currencies

Suppose the spot price of AUD is 0.7500USD and the 2-year interest rate in Australia and USA are 3% & 1% respectively. What should be the arbitrage-free forward price of AUD. If the forward rate actually turns out to be (a) 0.7000 USD; (b) 0.7600 USD, instead, what arbitrage opportunities will arise. (Direct Quote)

1 AUD = 0.7500 USD; $r_{HC(USD)} = 1\%pa$; $r_{FC(AUD)} = 1\% pa$;
 $T = 2$ years

$$F_0 = S_0 e^{(r_{HC} - r_{FC})T}$$
$$= 0.7500 e^{(0.01 - 0.03)*2}$$

$$= 0.7500 * 0.9608 = 0.7206 \text{ USD}$$

Currency with higher interest rate should depreciate.

(USD has appreciated against AUD in the forward market.)

Forward/Futures prices of Currencies

- If $F_{\text{ACTUAL}} = 0.7000$ USD/AUD: *(In the future, less USD are required to get 1 AUD, so 'x' units of USD would fetch more AUD, hence we need to have USD in the future or AUD now.)*

1.	Borrow 1000 AUD @ 3%pa for 2 years	1000.00 AUD
2.	Convert 1000 AUD in USD @ 0.7500 USD/AUD	750.00 USD
3.	Invest 750 USD @ 1% pa for 2 years to yield ($750e^{0.01*2} = 750*1.02020 = 765.15101$ USD)	765.15 USD
4.	Amount required to repay the AUD borrowings ($1000e^{0.03*2} = 1000*1.06184 = 1061.8366$)	1061.84 AUD
5.	Out of 765.15 USD, convert Amount Equivalent to the amount required in AUD ($1061.8366*0.7000=743.2856$)	743.29 USD
6.	Risk-free Profit (3 – 5)	21.86 USD

Forward/Futures prices of Currencies

- If $F_{\text{ACTUAL}} = 0.7600$ USD/AUD: *(In the future more USD are required to get 1 AUD, so 'x' units of AUD would fetch more USD, hence need to have AUD in the future i.e USD now)*

1.	Borrow 1000 USD @ 1%pa for 2 years	1000.00 USD
2.	Convert 1000 USD to AUD @ 0.7500 USD (1000USD/0.7500)	1333.33 AUD
3.	Invest 1333.33 AUD @ 3% pa for 2 years to yield ($1333.33e^{0.03*2} = 1333.33 * 1.06184$)	1415.78 AUD
4.	Convert AUD in USD @ F_A today ($1415.78 * 0.7600$)	1075.99 USD
5.	Amount required to repay the USD borrowings ($1000e^{0.01*2} = 1000 * 1.0202$)	1020.20 USD
6.	Risk-free Profit (4 - 5)	55.79 USD

Hedging

A steel manufacturing co. requires 50,000 tons of iron ore in 2 months. The iron ore prices are as follows: Spot Price: Rs. 305 per ton; 2-Mth Futures: Rs.310 per ton. The company expects the iron ore prices to rise but is comfortable up to price of Rs. 310/- per ton. At the same time, the company does not want to buy iron ore today @ Rs. 305/- per ton, nor is it financially profitable as the storage costs would be more than the price difference.

What should the Steel manufacturer do?

Hedging

- Steel manufacturing company wants to buy Iron ore after 2 months faces the risk of increase in prices, hence wants to lock-in the price today itself.
- Instead of buying iron ore in the cash market now, the company goes long on 2-month Iron ore Futures @ Rs. 310/- per ton.
- After 2-mths, assume the spot price rises to Rs. 316/- per ton

Hedging

- Just prior to expiry of 2 months, the company shall:
- Futures Market
- Offset the 'long' futures position by going 'short' on Iron ore @ Rs. 316/- (due to convergence)
- $= 50000 * 316 = 158$ lakh
- **Effective Cost = Actual Price paid – Profit on Future**
 $= 158 - 6 * 50000 = 155$ Lacs

Long Hedge

- Thus, a Long Hedge involves taking a Long position in the futures market i.e. buy a Futures contract at the beginning of the strategy.

Long Hedge may be used when the hedger wants to purchase the underlying asset at a later date but wants to lock-in the price at initiation, thereby reducing/eliminating price risk.

Short Hedge

A Iron ore mining Co. wants to sell its produce of 50,000 tons Iron ore after two months. The miner is satisfied to receive Rs. 310/- per ton, 2 months hence.

What should the Mining company do?

The mining company wants to sell Iron ore at a later date but wants to lock-in the price now

Short Hedge

- The mining company decides (today) to sell 2-month Iron ore futures @ Rs. 310/- per ton.
- After two months, again assume the price of iron ore goes up to Rs. 316/- per ton.

Short Hedge

- After 2 months, the mining company would:

Cash Market

- Sell 50,000 tons of Iron ore @ Rs 316 /- per ton and receive Rs. 158 Lacs.
- *Notional profit:* Rs 6/- per ton (316-310) on 50,000 tons or **Rs 3 Lacs.**

Futures Market

- Offset the 'short' position by going 'long' on Iron ore Futures@ Rs. 316/-(*due to convergence*)
- Loss: Rs. 6/- per ton (316-310) on 50,000 tons or **Rs 3 Lacs.**

Effective price : (Actual Price received less Loss on Futures)
Rs. 158 Lacs less Rs. 3 Lacs = Rs. 155 Lacs for 50,000 tons
i.e.Rs. 310/- per ton.

Short Hedge

- Thus, a Short Hedge involves taking a Short position in the futures market i.e. sell a Futures contract at the beginning of the strategy.

Short Hedge may be used when the Hedger already owns (or expects to own) the asset and wants to sell it the future but lock-in the price at initiation, thereby reducing/ eliminate

Basis Risk

- Such long & short hedges are too good to be true.
- Rarely it is possible to completely eliminate risk as :
 - a. Asset whose price is being hedged may not be exactly the same asset underlying the futures contract.(Commodity Basis Risk)
 - b. Quantity of asset to be hedged may not be exactly equal to the quantity specified under the futures contract. (Quantity Risk)
 - c. Horizon period over which the hedge is to be set is not clear

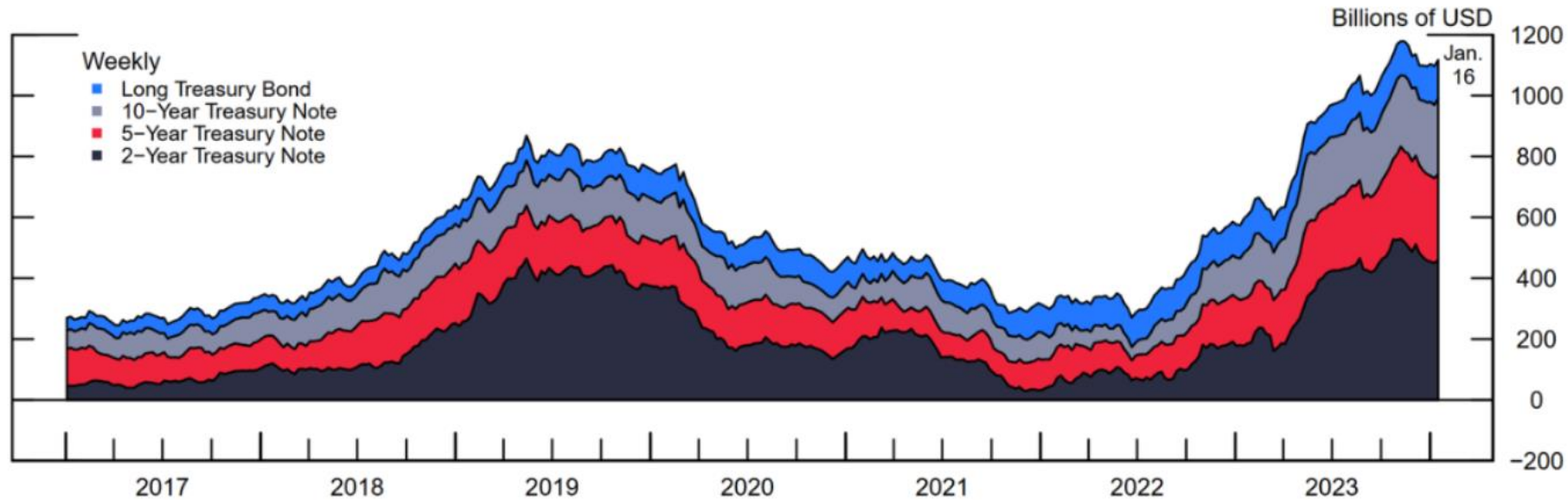
Basis= Spot Price of the Asset being hedged **less** Futures Price of the Contract used for Hedging

Risks in Hedging

- Sometimes the asset being hedged and the asset underlying the Futures contract may not be exactly the same (Cross Hedge) and leads to Commodity basis risk.
- Futures contracts have standardized grades which may not exactly be the same as the asset being hedged.
 - Underlying asset is Plutonium which is being hedged with Futures on copper, or
 - Corporate bonds being hedged by Futures on T-Bonds.

- Lets apply what we learned

Figure 2. Leveraged Funds' Short Positions in Treasury Futures



Note: Key identifies in order from top to bottom. '10-Year Treasury Note' refers to both 10-year and Ultra 10-year notes. 'Long Treasury Bond' refers to both Long and Ultra Long Treasury Bonds. Notional value calculated as the number of positions multiplied by the size of the futures contract - \$200,000 for 2-Year Treasury Note, \$100,000 for all other Treasuries. Leveraged funds include hedge funds, registered commodity trading advisers (CTAs), and commodity pool operators (CPOs).

Source: CFTC Traders in Financial Futures.

How a Basis Trade Works

What do you do when you notice a price differential between the spot price of a commodity and the price of a futures contract? You do a basis trade (if you're an eligible counterparty, that is).

Basis = Spot Price of X - Futures Price of X

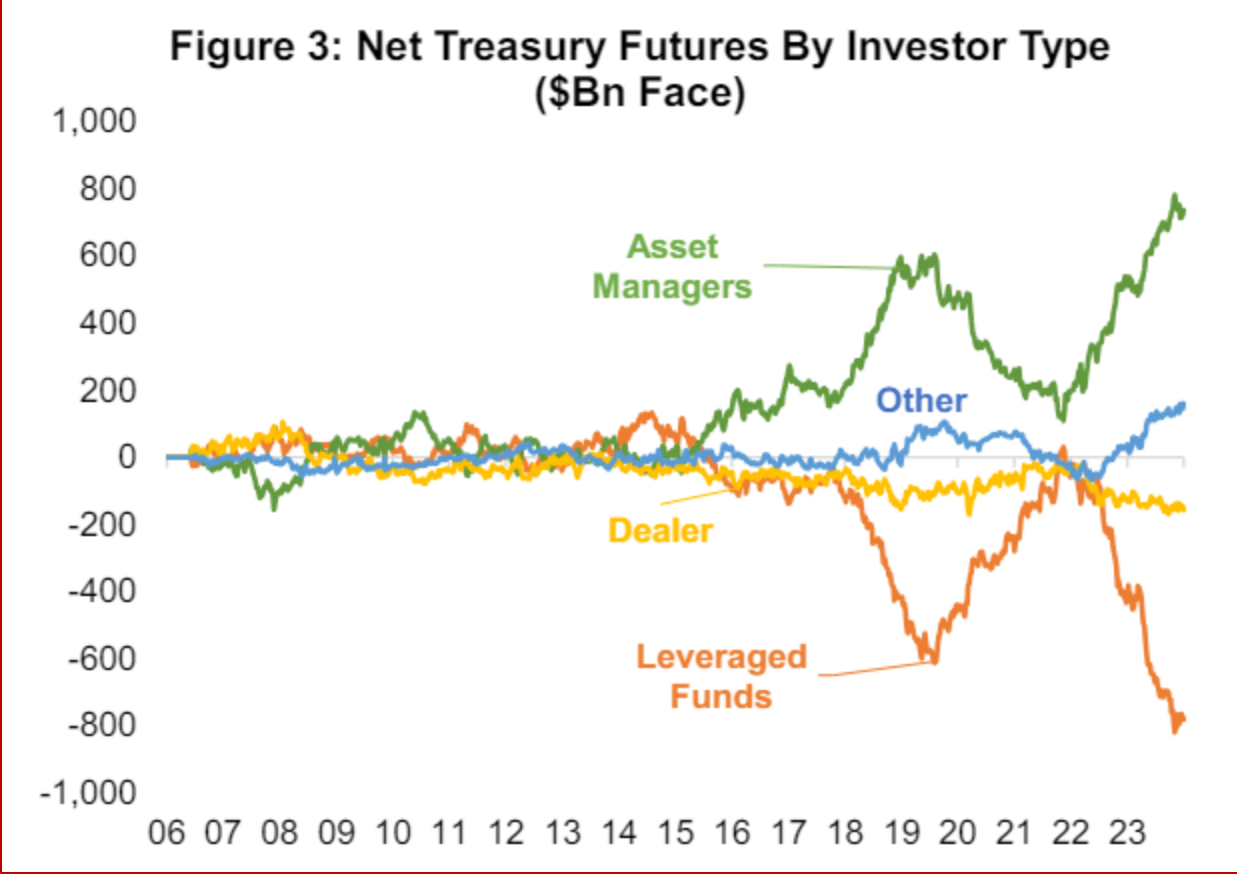
It's a pretty neat way to make money, as long as you can. Basis exists even in the market for the safest, highest liquidity asset you can possibly think of - US Treasury bonds.

High Differential in Basis Risk

- *Lots of treasury securities being issued* - when the Fed is trying to 'normalize' its balance sheet. Higher supply >> Lower price for treasuries (compared to its futures contracts).
- *Huge demand for Treasury futures coming from long term asset managers.* Why? Because it gives them leverage >> they can either achieve leverage through repos (in which case they'll have to report interest expenses in their total expense ratios - making them look more expensive) OR through buying futures (which looks like the more promising deal, expense-wise)

Hedge Fund Strategy

- These guys buy a lot of Treasury bonds, with huge sums of borrowed money (thanks to repo markets).
- On top of that, they sell Treasury futures - with very little of their own money as margin. Think 50x leverage - they'll put down \$10 of their own money as initial margin when initiating a futures position for \$500.
- As of Sep 2023 : Hedge funds had purchased, on net, \$626 billion in Treasury securities since Sep 2017, with \$478 billion by funds that were classified as likely basis traders.



Source: TBAC Report

Risks in Hedging

1. Sometimes, the hedger does not know the exact quantity of the underlying asset to be hedged (Quantity Risk)

- A farmer wanting to lock-in the price of his produce that is yet to be harvested Quantity risk is more in agri-product.

2. Hedger may not know the exact date on which to buy(sell) the underlying asset. When horizon date is not certain, it would be difficult to align with the expiration date of futures contract.

Presence of Basis Risk implies that the Cashflows cannot be completely risk-less by hedging (as in Perfect Hedge)

Which Futures Contracts to use ?

- It is important to select a **futures contract** on an asset that is highly **correlated** with the asset being hedged.
- In most cases the choice is obvious, but not in others.
- An investor wanting to hedge a highly diversified portfolio of mid-cap stocks has the following choices:
 - o Futures on Mid-cap Index (Not actively traded & portfolio does not match with the Index)
 - Futures on Large-cap Index (actively traded)
 - Futures on Small-cap Index (actively traded)
 - Hedger may like to use the large or small-cap index, or better still – a forward contract customized to match his portfolio

Which expiration month?

- The hedger should, therefore, choose an expiration as close as possible to but after the month in which the time horizon end.

Whether to be long or short ?

- Taking a wrong position in the futures market would increase the risk.
- If the hedger goes long (or short) when he was required to take a short (or long) position, he would increase his risk.

Current Spot Position Method:

Determine current position in Spot market

- If you own the asset: Current Spot position is Long
 - If you sell/Short on the asset: Current Spot position is Short
 - If you intend to buy the asset in the future: Current Spot position is Short.
2. Take a futures position opposite to current Spot position

Number of Futures contracts ?

- Hedge ratio is the ratio of no. of Futures contracts (H) to the exposure in the spot market (Q) or no. of futures positions taken per unit of spot exposure.
- Hedge ratio = $\frac{\text{Size of position taken in Futures Contracts}(H)}{\text{Size of the Exposure in Cash Market}(Q)}$
- Size of Exposure= 8,000 tons; Futures Contract = 4,000 tons; HR = 0.5
- When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a HR of 1.0.
- However, when cross hedging is used, adopting HR of 1.0 may not be optimal as Futures & Spot prices do not change in the same proportion.
- Hedger chooses When the asset underlying the futures contract is the same as the asset being hedged, it is natural to use a HR of 1.0
- However, when cross hedging is used, adopting HR of 1.0 may not be optimal as Futures & Spot prices do not change in the same proportion.
- Hedger chooses a HR such that it minimizes the variance of the value of the Hedged position.

Basis risk

- S_0 : Spot price today
- F_0 : Futures price today
- S_t : Spot price at t prior to expiration
- F_t : Futures price at t prior to expiration
- S_T : Spot price at expiration
- F_T : Futures price at expiration

Cash flow from a Hedged Position

	Cash Flow in		Total Cash Flow
	Spot Market	Futures Market	
Short Hedge (Selling Price)	QS_t	$H(F_0 - F_t)$ SP CP	$QS_t + H(F_0 - F_t)$ $= QS_t - H(F_t - F_0)$ $= 1480 - (1490 - 1650) = 1640$
Long Hedge (Cost price)	$-QS_t$	$H(F_t - F_0)$	$-QS_t + H(F_t - F_0)$ $= -[QS_t - H(F_t - F_0)]$ $= -[1480 - (1490 - 1650)]$ $= -[-10 + 1650] = -[1640]$

- In both the cases, 'H' should be such that it minimizes the variance of the Net Cash Flow.

In case of No Basis Risk

- If Basis Risk is zero, then $S_T = F_T$, and the net cash flow from a Long Hedge is :
$$= QS_T - H(F_T - F_0) = QS_T - H(S_T - F_0)$$
$$= (Q - H) S_T + HF_0$$
- At $t=0$, Q , H , & F_0 is known, while S_T is unknown.
- If we set $H = Q$, then Cash Flow reduces to HF_0 (or QF_0), a *known* quantity, the variance of which is Zero.

Thus, in case of Zero Basis Risk, it is Optimal to Hedge completely i.e. have a Hedge Ratio of 1.0

When Basis Risk is present

- If Basis Risk is NOT zero, then the net cash flow from a Long Hedge $[QS_T - H(F_T - F_0)]$ in terms of change in prices, is:

$$= QS_T - QS_0 + QS_0 - H(F_T - F_0)$$

$$= Q(S_T - S_0) - H(F_T - F_0) + QS_0$$

$$= Q\Delta_S - H\Delta_F + QS_0$$

$$= Q\Delta_S - hQ\Delta_F + QS_0$$

$$= Q(\Delta_S - h\Delta_F) + QS_0$$

Where, $\Delta_S = S_T - S_0$ & $\Delta_F = F_T - F_0$

Where, Hedge ratio (h) = H/Q or $H = hQ$

- Thus, 'h' should be such that it minimizes the variance of the Net Cash Flow.

In case Basis Risk is present

- Cash Flow : $Q (\Delta_S - h\Delta_F) + QS_0$
- As QS_0 is known at $t=0$, the variance in cash flow is due to:
 - ✓ Variance of change in Spot prices = $\sigma^2_{\Delta_S}$
 - ✓ Variance of change in Futures prices = $\sigma^2_{\Delta_F}$
 - ✓ Covariance between the Δ_S & $\Delta_F = \text{Cov}(\Delta_S, \Delta_F)$
- Variance of $Q (\Delta_S - h\Delta_F) = Q^2 \text{Var}(\Delta_S - h\Delta_F)$
$$= Q^2 [\sigma^2_{\Delta_S} + h^2\sigma^2_{\Delta_F} - 2h \text{Cov}(\Delta_S, \Delta_F)]$$
- To minimize the variance, equate the first derivative (wrt 'h') to zero,

In case basis risk is present

$$= 2h\sigma_{\Delta F}^2 - 2 \text{Cov}(\Delta_S, \Delta_F) = 0$$

$$h\sigma_{\Delta F}^2 = \text{Cov}(\Delta_S, \Delta_F)$$

$$h^* = \frac{\text{Cov}(\Delta_S, \Delta_F)}{\sigma_{\Delta F}^2} = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$$

$$[\text{Cov}(\Delta_S, \Delta_F) = \rho\sigma_{\Delta S}\sigma_{\Delta F}]$$

ρ = correlation co-efficient between ΔS & ΔF

$\sigma_{\Delta S}$ = Std. Deviation in change in Spot Prices

$\sigma_{\Delta F}$ = Std. Deviation in change in Futures

Prices

h^* is the Minimum Variance Hedge Ratio

Minimum Variance Hedge Ratio

- Consider that you have a position in 200 shares of InfoTech, a technology stock with a standard deviation of change in stock price of 30. You want to hedge this position with a technology stock index futures which has a standard deviation of 20. The correlation between the two is 0.80. What should be the Optimal Hedge ratio?

$$\sigma_{\Delta S} = 30; \quad \sigma_{\Delta F} = 20; \quad \rho_{\Delta S \Delta F} = 0.80$$

$$h^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.8 \frac{30}{20} = 1.20$$

Minimum Variance Hedge Ratio

- Standard deviation of changes in spot price of wheat is 0.00278, standard deviation of changes in futures price of wheat is 0.00259 and the coefficient of correlation between the two is 0.98031. What is the optimal hedge ratio? What if the coefficient of correlation is: 0.3; 0.8; 1.0; and -0.6?

$$\sigma_{\Delta S} = 0.00278; \sigma_{\Delta F} = 0.00259; \rho_{\Delta S \Delta F} = 0.98031$$

$$h^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.98031 \frac{0.00278}{0.00259} = 1.052$$

The size of the Futures position should be 1.052 times the size of the exposure of wheat position.

ρ	h^*
0.3	0.32
0.8	0.86
1.0	1.07
-0.6	-0.64

Cash Flow Variance at MVHR

- Variance of Cash Flow = $Q^2 (\sigma_{\Delta S}^2 + h^2 \sigma_{\Delta F}^2 - 2h \text{Cov}(\Delta_S, \Delta_F))$
- Optimal Hedge Ratio (MVHR or h^*) = $\rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}}$
- Cash Flow variance at h^* =

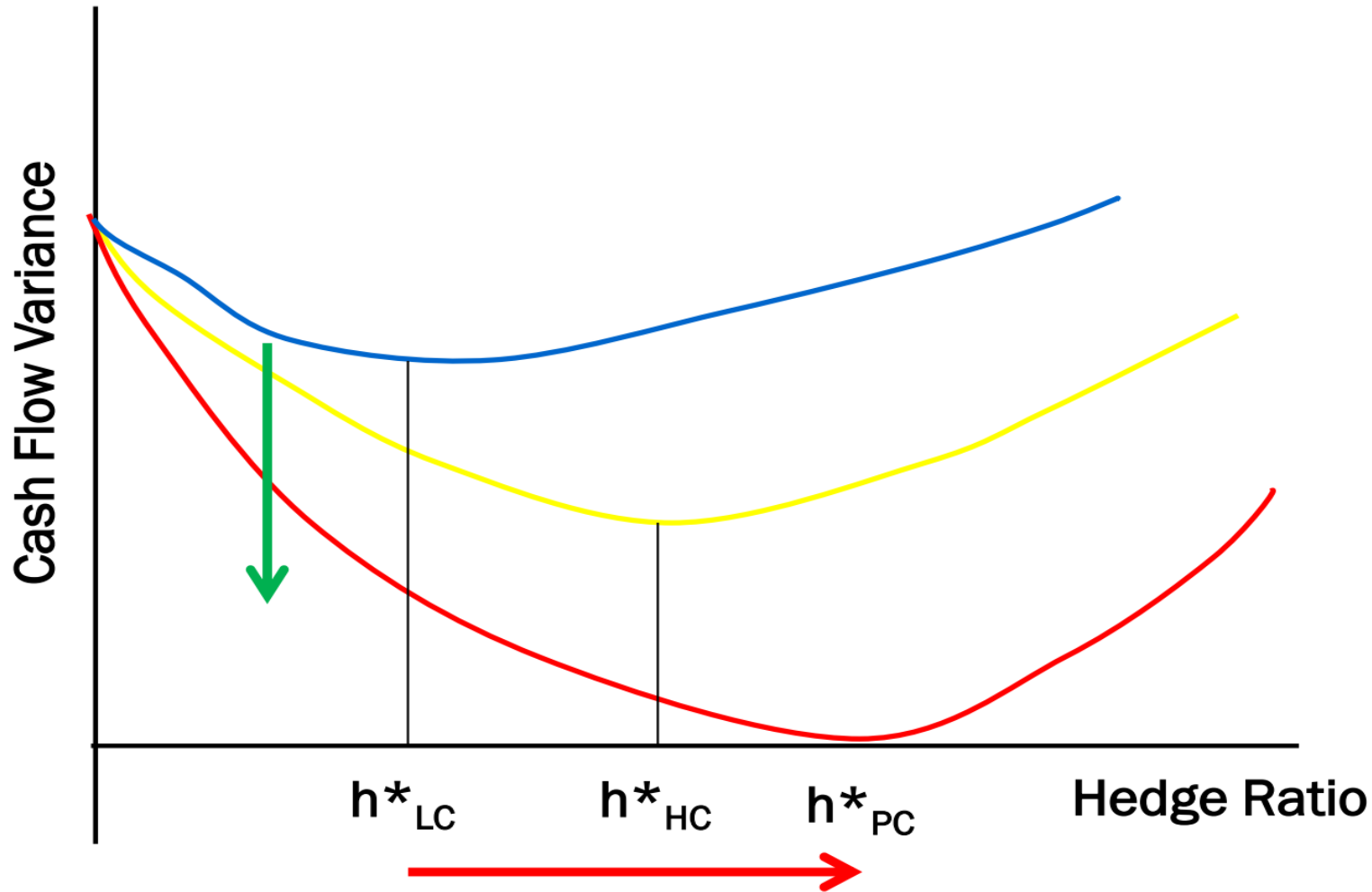
$$Q^2 \left[\sigma_{\Delta S}^2 + \rho^2 \frac{\sigma_{\Delta S}^2}{\sigma_{\Delta F}^2} \sigma_{\Delta F}^2 - 2\rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} \rho \sigma_{\Delta S} \sigma_{\Delta F} \right]$$

$$= Q^2 \sigma_{\Delta S}^2 (1 - \rho^2)$$

Minimized Cash Flow Variance

- This minimized variance is zero when $\rho = +/-1$ i.e. when futures and spot prices are perfectly correlated (+ve or -ve) i.e. when basis risk is zero.
- **If Basis risk is present, there will always be some residual uncertainty even after hedging.**

Minimum Variance Hedge Ratio



- ✓ MVHR increases as correlation increases

ρ	h^*
0.3	0.32
0.8	0.86
0.98	1.052
1.0	1.07
-0.6	-0.64

- ✓ Variance of Cash flows is lower as correlation is higher- i.e. higher correlation gives better protection.

Minimum Variance Hedge Ratio

- Alternatively, the slope of the regression line of Δ_S on Δ_F , is h^* .

Week	Change in F_t	Change in S_t	
1	0.0032	0.0038	
2	0.0040	0.0034	
3	0.0025	0.0030	
4	0.0035	0.0035	
5	0.0037	0.0048	
6	-0.0015	-0.0012	
7	-0.0005	-0.0005	
8	-0.0001	0.0005	
9	-0.0025	-0.0022	
10	-0.0020	-0.0028	
Std Dev.	0.00259	0.00278	<-- =STDEV(C4:C13)
Correlation	0.98031		<-- =CORREL(C4:C14,B4:B14)
h^*	1.052		<-- =(C14*B15)/B14
Slope	1.052		<-- =SLOPE(C4:C13,B4:B13)
Corr²	0.96		<-- =B15^2

Going Long or Short in Futures Market

	Position in Spot Market at $t=0$	Position to be taken in Futures Market at $t=0$
If $p > 0$	Short (i.e. want to buy in the future) <i>(will lose on increase in Cash price)</i>	Go Long on Futures now (Futures price will also increase and hence long position would lead to gains in Futures market)
	Long (i.e. have /will have the asset, and want to sell in the future) <i>(will lose by fall in Cash price)</i>	Go short on Futures now (Futures price will also decrease in the future and hence short position would lead to gains in Futures market)
If $p < 0$	Short (i.e. want to buy in futures) <i>(will lose on increase in Cash price)</i>	Go Short on Futures now (Futures price will decrease hence short position would lead to gains in Futures market)
	Long (i.e. have /will have the asset, and want to sell in the future) <i>(will lose by fall in Cash price)</i>	Go Long on Futures now (Futures price will increase, and hence long position would lead to gains in Futures market)

Which Futures Contract to choose?

You have a long position on 500 tons of Chana. Assuming futures on Chana are not available while futures on Rice (F_1) and Jerra (F_2) are available. Using these futures, you are asked to hedge your position in Chana. Correlations between change in Spot price of Chana and change in Futures prices are : between S & F_1 : 0.68 , between S & F_2 : -0.98, and standard deviations of the price changes: $\sigma_{\Delta S} = 0.30$; $\sigma_{\Delta F_1} = 0.25$; $\sigma_{\Delta F_2} = 0.15$.

Find the Minimum-Variance Hedge Ratio for S using futures contracts F_1 and F_2 . Using which futures contract is more effective?

Which Futures Contract to choose?

$Q = 500$ tons of Chana $\sigma_{\Delta S} = 0.30$; $\sigma_{\Delta F1} = 0.25$;

$\sigma_{\Delta F2} = 0.15$; $\rho_{\Delta S \Delta F1} = 0.68$; $\rho_{\Delta S \Delta F2} = -0.98$

- **Using F_1 :**

$$h_1^* = \rho \frac{\sigma_{\Delta S}}{\sigma_{\Delta F1}} = 0.68 \frac{0.30}{0.25} = 0.816$$

$$\left[Q^2 \sigma_{\Delta S}^2 (1 - \rho^2) \right] = 500^2 (0.30)^2 (1 - 0.68^2) = 12,096$$

- **Using F_2 :**

$$h_2^* = -0.98 \frac{0.30}{0.15} = -1.96$$

$$\left[Q^2 \sigma_{\Delta S}^2 (1 - \rho^2) \right] = 500^2 (0.30)^2 (1 - (-0.98)^2) = 891$$

Optimal No. of Futures Contracts

- Based on the Optimal hedge ratio (h^*), the optimal no. of futures contracts (N^*) required to hedge an exposure of Q_A units, given the size of one futures contract as Q_F is given as follows:

$$N^* = h^* \frac{Q_A}{Q_F}$$

Optimal No. of Futures Contracts

The standard deviation of monthly changes in spot price of copper is 1.2%. The standard deviation of changes in futures price of copper is 1.4%. The coefficient of correlation between the two is 0.7. A copper wire manufacturer is committed to purchase 200,000 kg. of copper on 15-Nov. The producer wants to use the Dec futures to hedge his risk. Each futures contract is of 40,000 kg. of copper. What strategy should the wire manufacturer follow?

Optimal No. of Futures Contracts

$$\sigma_{\Delta S} = 1.2\%; \quad \sigma_{\Delta F} = 1.4\%; \quad \rho_{\Delta S \Delta F} = 0.70;$$

$$Q_A = 200,000 \text{ kg}; \quad Q_F = 40,000 \text{ kg}$$

$$h^* = \rho_{\Delta S \Delta F} \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = 0.7 \frac{1.2}{1.4} = 0.60$$

$$N^* = \frac{h^* Q_A}{Q_F} = \frac{0.6 * 200,000}{40,000} = 3$$

- The copper wire manufacturer is currently short on copper. He loses, if spot price of copper increases.
- If cash price increases, futures price will also increase (as ρ is +ve). So, in order to compensate for loss in cash market, he needs to gain in futures market, which can be achieved by going long on Copper Futures.
- Hence, wire manufacturer should **go long on 3 Copper futures expiring in Dec.**

Optimal No. of Futures Contracts

Now consider that the hedge is set using futures on gold *instead of copper*. The standard deviation of changes in futures price of gold is 1.8% & coefficient of correlation between the two is -0.5. Each futures contract is of 40,000 kg. of gold. What strategy should the wire manufacturer follow now?

Optimal No. of Futures Contracts

$\sigma_{\Delta S} = 1.2\%$; $\sigma_{\Delta F} = 1.8\%$; $\rho_{\Delta S \Delta F} = -0.50$; $Q_A = 200,000$ kg; $Q_F = 40,000$ kg

$$h^* = \rho_{\Delta S \Delta F} \frac{\sigma_{\Delta S}}{\sigma_{\Delta F}} = -0.5 \frac{1.2}{1.8} = -0.33$$

$$N^* = \frac{h^* Q_A}{Q_F} = \frac{-0.33 * 200,000}{40,000} = -1.65 \approx -2$$

- The copper wire manufacturer is currently short on copper and would lose, if spot price of copper increases.
- If Copper cash price increases, futures price of Gold will decrease (as ρ is -ve).
- In order to compensate for loss in cash market, he needs to gain in futures market, which can be achieved by going short on Gold Futures.
- Hence, wire manufacturer should **go Short on 2 Gold futures expiring in Dec.**

Hedging an Equity Portfolio using Stock Index Futures

- Stock Index Futures may be used to hedge a well diversified portfolio of stocks.
- During times of unusual volatility in the market, stock index futures may be used to change or eliminate systematic risk.
- The no. of stock Index futures contracts to be used for hedging:

$$N_F^* = \beta_P \frac{V_A}{V_F}$$

where, V_A = Value of the Portfolio

V_F = Value of One Futures contract (Futures price * contract size)

Hedging an Equity Portfolio using Futures on Stock Index

- Suppose, the current value of your portfolio is Rs. 5 Lacs, with a beta of 1.2, which you want to protect from loss of value from decline in the market over the next 3 months. The Index value today is 1000 and the 4-month Futures on the Index is at 1010. Each futures contract is for 20x. How many futures contracts should be bought or sold? Assume risk-free interest as 4% pa and dividend yield of 1%pa (both on cc).

Hedging an Equity Portfolio using Futures on Stock Index

	A	B	C	D	E	F	G
2	Value of Portfolio today (V_A)	500,000	(Long Position in Cash Market) -> Hence the risk				
3	Index Value today (t=0)	1,000	is fall in Value of Portfolio.				
4	4-month Index Futures today	1,010					
5	Beta of Portfolio	1.2					
6	Risk-free rate (%pa on cc)	4%	1.00% Over 3 months				
7	Dividend Yield(%pa on cc)	1%	0.25% Over 3 months				
8	1 Futures Contracts	20 x					
9							
10	Value of 1 Futures Contract (V_F)	20,200	← =B4*B8				
11							
12	No. of Futures to Sell	29.7030	← =(B5*B2)/B10 (Go Short on Index Futures)				
13	$N^* = (\beta \cdot V_A) / V_F$	30	← =ROUND(B12,0)				