CHAPTER 1

Introduction to Corporate Finance

This book is about how corporations make financial decisions. We start by explaining what these decisions are and what they are seeking to accomplish.

Corporations invest in real assets, which generate income. Some of these assets, such as plant and machinery, are tangible; others, such as brand names and patents, are intangible. Corporations finance their investments by borrowing, by retaining and reinvesting cash flow, and by selling additional shares of stock to the corporation's shareholders. Thus the corporation's financial manager faces two broad financial questions: First, what investments should the corporation make? Second, how should it pay for those investments? The investment decision involves spending money; the financing decision involves raising it.

A large corporation may have hundreds of thousands of shareholders. These shareholders differ in many ways, such as their wealth, risk tolerance, and investment horizon. Yet we shall see that they usually share the same financial objective. They want the financial manager to increase the value of the corporation and its current stock price.

Thus the secret of success in financial management is to increase value. That is easy to say, but not very helpful. Instructing the financial manager to increase value is like advising an investor in the stock market to "buy low, sell high." The problem is how to do it.

There may be a few activities in which one can read a textbook and then just "do it," but financial management is not one of them. That is why finance is worth studying. Who wants to work in a field where there is no room for judgment, experience, creativity, and a pinch of luck? Although this book cannot guarantee any of these things, it does cover the concepts that govern good financial decisions, and it shows you how to use the tools of the trade of modern finance.

This chapter begins with specific examples of recent investment and financing decisions made by well-known corporations. The chapter ends by stating the financial goal of the corporation, which is to increase, and ideally to maximize, its market value. We explain why this goal makes sense. The middle of the chapter covers what a corporation is and what its financial managers do.

Financial managers add value whenever the corporation can earn a higher return than shareholders can earn for themselves. The shareholders' investment opportunities *outside* the corporation set the standard for investments *inside* the corporation. Financial managers therefore refer to the *opportunity cost* of the capital contributed by shareholders.

Managers are of course human beings, with their own interests and circumstances; they are not always the perfect servants of shareholders. Therefore corporations must combine governance rules and procedures with appropriate incentives to make sure that all managers and employees—not just the financial managers—pull together to increase value.

Good governance and appropriate incentives also help block out temptations to increase stock price by illegal or unethical means. Thoughtful shareholders do not want the maximum possible stock price. They want the maximum honest stock price.

This chapter introduces five themes that return again and again, in various forms and circumstances, throughout the book:

1. Corporate finance is all about maximizing value.
2. The opportunity cost of capital sets the standard for investment decisions.
3. A safe dollar is worth more than a risky dollar.
4. Smart investment decisions create more value than smart financing decisions.
5. Good governance matters.

1-1 Corporate Investment and Financing Decisions

To carry on business, a corporation needs an almost endless variety of **real assets.** These do not drop free from a blue sky; they need to be paid for. The corporation pays for the real assets by selling claims on them and on the cash flow that they will generate. These claims are called **financial assets** or **securities.** Take a bank loan as an example. The bank provides the corporation with cash in exchange for a financial asset, which is the corporation's promise to repay the loan with interest. An ordinary bank loan is not a security, however, because it is held by the bank and not sold or traded in financial markets.

Take a corporate bond as a second example. The corporation sells the bond to investors in exchange for the promise to pay interest on the bond and to pay off the bond at its maturity. The bond is a financial asset, and also a security, because it can be held and traded by many investors in financial markets. Securities include bonds, shares of stock, and a dizzying variety of specialized instruments. We describe bonds in Chapter 3, stocks in Chapter 4, and other securities in later chapters.

This suggests the following definitions:

$$\begin{aligned}\text{Investment decision} &= \text{purchase of real assets}\\ \text{Financing decision} &= \text{sale of financial assets}\end{aligned}$$

But these equations are too simple. The investment decision also involves managing assets already in place and deciding when to shut down and dispose of assets if profits decline. The corporation also has to manage and control the risks of its investments. The financing decision includes not just raising cash today but also meeting obligations to banks, bondholders, and stockholders that contributed financing in the past. For example, the corporation has to repay its debts when they become due. If it cannot do so, it ends up insolvent and bankrupt. Sooner or later the corporation will also want to pay out cash to its shareholders.[1]

Let's go to more specific examples. Table 1.1 lists nine corporations from all over the world. We have chosen very large public corporations that you are probably already familiar with. You have probably filled up at an Exxon gas station, shopped at Walmart, or used Crest toothpaste.

Company	Recent Investment Decisions	Recent Financing Decisions
Boeing (U.S.)	Delivers first Dreamliner after investing a reported $30 billion in development costs.	Reinvests $1.7 billion of profits.
Exxon Mobil (U.S.)	Spends $7 billion to develop oil sands at Fort McMurray in Alberta.	Spends $12 billion buying back shares.
GlaxoSmithKline (UK)	Spends $4 billion on research and development for new drugs.	Pays $3.2 billion as dividends.
LVMH[2] (France)	Acquires the Italian jeweler, Bulgari, for $5 billion.	Pays for the acquisition with a mixture of cash and shares.
Procter & Gamble (U.S.)	Spends $8 billion on advertising.	Raises 100 billion Japanese yen by an issue of five-year bonds.
Tata Motors (India)	Opens a new plant in India to produce the world's cheapest car, the Nano. The facility costs $400 million.	Raises $400 million by the sale of new shares.
Union Pacific (U.S.)	Invests $330 million in 100 new locomotives and 10,000 freight cars and chassis.	Repays $1.4 billion of debt.
Vale (Brazil)	Opens a huge copper mine at Salobo in Brazil. The project cost nearly $2 billion.	Maintains credit lines with its banks that allow the company to borrow at any time up to $1.6 billion.
Walmart (U.S.)	Invests $12.7 billion, primarily to open 458 new stores around the world.	Issues $5 billion of long-term bonds to repay short-term commercial paper borrowings.

TABLE 1.1 Examples of recent investment and financing decisions by major public corporations.

Investment Decisions

The second column of Table 1.1 shows an important recent investment decision for each corporation. These investment decisions are often referred to as **capital budgeting** or **capital expenditure (CAPEX)** decisions, because most large corporations prepare an annual capital budget listing the major projects approved for investment. Some of the investments in Table 1.1, such as Walmart's new stores or Union Pacific's new locomotives, involve the purchase of tangible assets—assets that you can touch and kick. However, corporations also need to invest in intangible assets, such as research and development (R&D), advertising, and marketing. For example, GlaxoSmithKline and other major pharmaceutical companies invest billions every year on R&D for new drugs. Similarly, consumer goods companies such as Procter & Gamble invest huge sums in advertising and marketing their products. These outlays are investments because they build brand recognition and reputation for the long run.

Today's capital investments generate future cash returns. Sometimes the cash inflows last for decades. For example, many U.S. nuclear power plants, which were initially licensed by the Nuclear Regulatory Commission to operate for 40 years, are now being re-licensed for 20 more years, and may be able to operate efficiently for 80 years overall.

Yet a stream of cash inflows lasting for 40-plus years may still not be enough. For example, the Southern Company has received authorization to build two new nuclear plants. The cost of the plants has been estimated (perhaps optimistically) at $14 billion. Construction will take seven years (perhaps also an optimistic estimate). Thus Southern, if it goes ahead, will have to invest at least $14 billion and wait at least seven years for any cash return. The longer it has to wait for cash to flow back in, the greater the cash inflow required to justify the investment. Thus the financial manager has to pay attention to the *timing* of cash inflows, not just to their cumulative amount.

Of course not all investments have distant payoffs. For example, Walmart spends about $40 billion each year to stock up its stores and warehouses before the holiday season. The company's return on this investment comes within months as the inventory is drawn down and the goods are sold.

In addition, financial managers know (or quickly learn) that cash returns are not guaranteed. An investment could be a smashing success or a dismal failure. For example, the Iridium communications satellite system, which offered instant telephone connections worldwide, soaked up $5 billion of investment before it started operations in 1998. It needed 400,000 subscribers to break even, but attracted only a small fraction of that number. Iridium defaulted on its debt and filed for bankruptcy in 1999. The Iridium system was sold a year later for just $25 million. (Iridium has recovered and is now profitable and expanding, however.)[3]

Among the contenders for the all-time worst investment was the purchase of Chrysler by Daimler-Benz in 1998 for $40 billion. The German firm planned to turn Chrysler into a company to rival General Motors and Ford. But the new DaimlerChrysler quickly found that selling both luxury and economy-priced cars wasn't so easy. By 2003 Daimler's share price had fallen by 70% since the acquisition. *Business Week* called the merger a "colossal mistake" and reported DaimlerChrysler's biggest shareholder, Deutsche Bank, openly wanted out after losing some $15 billion in the deal. Daimler eventually sold Chrysler to Cerberus Capital Management for $6 billion in 2007, $34 billion less than it originally paid.

Financial managers do not make major investment decisions in solitary confinement. They may work as part of a team of engineers and managers from manufacturing, marketing, and other business functions. Also, do not think of the financial manager as making billion-dollar investments on a daily basis. Most investment decisions are smaller and simpler, such as the purchase of a truck, machine tool, or computer system. Corporations make thousands of these smaller investment decisions every year. The cumulative amount of small investments can be just as large as that of the occasional big investments, such as those shown in Table 1.1.

Financing Decisions

The third column of Table 1.1 lists a recent financing decision by each corporation. A corporation can raise money from lenders or from shareholders. If it borrows, the lenders contribute the cash, and the corporation promises to pay back the debt plus a fixed rate of interest. If the shareholders put up the cash, they do not get a fixed return, but they hold shares of stock and therefore get a fraction of future profits and cash flow. The shareholders are *equity investors,* who contribute *equity financing.* The choice between debt and equity financing is called the **capital structure** decision. *Capital* refers to the firm's sources of long-term financing.

The financing choices available to large corporations seem almost endless. Suppose the firm decides to borrow. Should it borrow from a bank or borrow by issuing bonds that can be traded by investors? Should it borrow for 1 year or 20 years? If it borrows for 20 years, should it reserve the right to pay off the debt early if interest rates fall? Should it borrow in Paris, receiving and promising to repay euros, or should it borrow dollars in New York? As Table 1.1 shows, Procter & Gamble borrowed Japanese yen, but it could have borrowed dollars or euros instead.

Corporations raise equity financing in two ways. First, they can issue new shares of stock. The investors who buy the new shares put up cash in exchange for a fraction of the corporation's future cash flow and profits. Second, the corporation can take the cash flow generated by its existing assets and reinvest the cash in new assets. In this case the corporation is reinvesting on behalf of existing stockholders. No new shares are issued.

What happens when a corporation does not reinvest all of the cash flow generated by its existing assets? It may hold the cash in reserve for future investment, or it may pay the cash back to its shareholders. Table 1.1 shows that in 2010 GlaxoSmithKline paid cash dividends of $3.2 billion. In the same year Exxon Mobil paid back $12 billion to its stockholders by repurchasing shares. This was in addition to $9 billion paid out as cash dividends. The decision to pay dividends or repurchase shares is called the *payout decision.* We cover payout decisions in Chapter 16.

In some ways financing decisions are less important than investment decisions. Financial managers say that "value comes mainly from the asset side of the balance sheet." In fact the most successful corporations sometimes have the simplest financing strategies. Take Microsoft as an example. It is one of the world's most valuable corporations. At the end of 2011, Microsoft shares traded for $26 each. There were about 8.4 billion shares outstanding. Therefore Microsoft's overall market value—its *market capitalization* or *market cap*—was $\$26 \times 8.4 = \$218$ billion. Where did this market value come from? It came from Microsoft's product development, from its brand name and worldwide customer base, from its research and development, and from its ability to make profitable future investments. The value did *not* come from sophisticated financing. Microsoft's financing strategy is very simple: it carries no debt to speak of and finances almost all investment by retaining and reinvesting cash flow.

Financing decisions may not add much value, compared with good investment decisions, but they can destroy value if they are stupid or if they are ambushed by bad news. For example, when real estate mogul Sam Zell led a buyout of the *Chicago Tribune* in 2007 the newspaper took on about $8 billion of additional debt. This was not a stupid decision, but it did prove fatal. As advertising revenues fell away in the recession of 2008, the *Tribune* could no longer service its debt. In December 2008 it filed for bankruptcy with assets of $7.6 billion and debts of $12.9 billion.

Business is inherently risky. The financial manager needs to identify the risks and make sure they are managed properly. For example, debt has its advantages, but too much debt can land the company in bankruptcy, as the *Chicago Tribune* discovered. Companies can also be knocked off course by recessions, by changes in commodity prices, interest rates and exchange rates, or by adverse political developments. Some of these risks can be hedged or insured, however, as we explain in Chapters 26 and 27.

What Is a Corporation?

We have been referring to "corporations." Before going too far or too fast, we need to offer some basic definitions. Details follow in later chapters.

A **corporation** is a legal entity. In the view of the law, it is a legal *person* that is owned by its shareholders. As a legal person, the corporation can make contracts, carry on a business, borrow or lend money, and sue or be sued. One corporation can make a takeover bid for another and then merge the two businesses. Corporations pay taxes—but cannot vote!

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Zipcar's articles

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In the U.S., corporations are formed under state law, based on *articles of incorporation* that set out the purpose of the business and how it is to be governed and operated.[4] For example, the articles of incorporation specify the composition and role of the *board of directors*.[5] A corporation's directors are elected by the shareholders. They choose and advise top management and must sign off on some corporate actions, such as mergers and the payment of dividends to shareholders.

A corporation is owned by its shareholders but is legally distinct from them. Therefore the shareholders have **limited liability,** which means that they cannot be held personally responsible for the corporation's debts. When the U.S. financial corporation Lehman Brothers failed in 2008, no one demanded that its stockholders put up more money to cover Lehman's massive debts. Shareholders can lose their entire investment in a corporation, but no more.

When a corporation is first established, its shares may be privately held by a small group of investors, perhaps the company's managers and a few backers. In this case the shares are not publicly traded and the company is *closely held.* Eventually, when the firm grows and new shares are issued to raise additional capital, its shares are traded in public markets such as the New York or London stock exchanges. Such corporations are known as *public companies*. Most well-known corporations in the U.S. are public companies with widely dispersed shareholdings. In other countries, it is more common for large corporations to remain in private hands, and many public companies may be controlled by just a handful of investors. The latter category includes such well-known names as Fiat, Peugeot, Benetton, L'Oréal, and the Swatch Group.

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Zipcar's bylaws

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A large public corporation may have hundreds of thousands of shareholders, who own the business but cannot possibly manage or control it directly. This *separation of ownership and control* gives corporations permanence. Even if managers quit or are dismissed and replaced, the corporation survives. Today's stockholders can sell all their shares to new investors without disrupting the operations of the business. Corporations can, in principle, live forever, and in practice they may survive many human lifetimes. One of the oldest corporations is the Hudson's Bay Company, which was formed in 1670 to profit from the fur trade between northern Canada and England. The company still operates as one of Canada's leading retail chains.

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S-corporations

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The separation of ownership and control can also have a downside, for it can open the door for managers and directors to act in their own interests rather than in the stockholders' interest. We return to this problem later in the chapter.

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Other Forms of Business Organization

Corporations do not have to be prominent, multinational businesses such as those listed in Table 1.1. You can organize a local plumbing contractor or barber shop as a corporation if you want to take the trouble. But most corporations are larger businesses or businesses that aspire to grow. Small "mom-and-pop" businesses are usually organized as sole proprietorships.

What about the middle ground? What about businesses that grow too large for sole proprietorships but don't want to reorganize as corporations? For example, suppose you wish to pool money and expertise with some friends or business associates. The solution is to form a *partnership* and enter into a partnership agreement that sets out how decisions are to be made and how profits are to be split up. Partners, like sole proprietors, face unlimited liability. If the business runs into difficulties, each partner can be held responsible for *all* the business's debts.

Partnerships have a tax advantage. Partnerships, unlike corporations, do not have to pay income taxes. The partners simply pay personal income taxes on their shares of the profits.

Some businesses are hybrids that combine the tax advantage of a partnership with the limited liability advantage of a corporation. In a *limited partnership,* partners are classified as general or limited. General partners manage the business and have unlimited personal liability for its debts. Limited partners are liable only for the money they invest and do not participate in management.

You will also encounter *limited liability partnerships (LLPs)* or, equivalently, *limited liability companies (LLCs).* These are partnerships in which all partners have limited liability.

Another U.S. variation on the theme is the *professional corporation (PC),* which is commonly used by doctors, lawyers, and accountants. In this case, the business has limited liability, but the professionals can still be sued personally, for example, for malpractice.

Most large investment banks such as Morgan Stanley and Goldman Sachs started life as partnerships. But eventually these companies and their financing requirements grew too large for them to continue as partnerships, and they reorganized as corporations. The partnership form of organization does not work well when ownership is widespread and separation of ownership and management is essential.

There are other disadvantages to being a corporation. One is the cost, in both time and money, of managing the corporation's legal machinery. These costs are particularly burdensome for small businesses. There is also an important tax drawback to corporations in the United States. Because the corporation is a separate legal entity, it is taxed separately. So corporations pay tax on their profits, and shareholders are taxed again when they receive dividends from the company or sell their shares at a profit. By contrast, income generated by businesses that are not incorporated is taxed just once as personal income.

Almost all large and medium-sized businesses are corporations, but the nearby box describes how smaller businesses may be organized.

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The financial managers

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The Role of the Financial Manager

What is the essential role of the financial manager? Figure 1.1 gives one answer. The figure traces how money flows from investors to the corporation and back to investors again. The flow starts when cash is raised from investors (arrow 1 in the figure). The cash could come from banks or from securities sold to investors in financial markets. The cash is then used to pay for the real assets (investment projects) needed for the corporation's business (arrow 2). Later, as the business operates, the assets generate cash inflows (arrow 3). That cash is either reinvested (arrow 4*a*) or returned to the investors who furnished the money in the first place (arrow 4*b*). Of course, the choice between arrows 4*a* and 4*b* is constrained by the promises made when cash was raised at arrow 1. For example, if the firm borrows money from a bank at arrow 1, it must repay this money plus interest at arrow 4*b*.

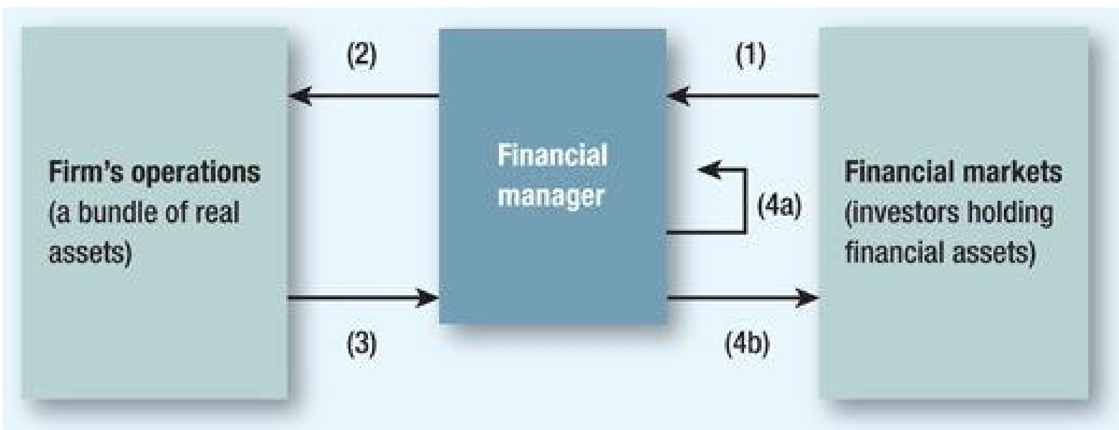

FIGURE 1.1 Flow of cash between financial markets and the firm's operations. Key: (1) Cash raised by selling financial assets to investors; (2) cash invested in the firm's operations and used to purchase real assets; (3) cash generated by the firm's operations; (4*a*) cash reinvested; (4*b*) cash returned to investors.

You can see examples of arrows 4*a* and 4*b* in Table 1.1. Walmart financed its investment in new stores by reinvesting earnings (arrow 4*a*). Exxon Mobil decided to return cash to shareholders by buying back its stock (arrow 4*b*). It could have chosen instead to pay the money out as additional cash dividends.

Notice how the financial manager stands between the firm and outside investors. On the one hand, the financial manager helps manage the firm's operations, particularly by helping to make good investment decisions. On the other hand, the financial manager deals with investors—not just with shareholders but also with financial institutions such as banks and with financial markets such as the New York Stock Exchange.

1-2 The Financial Goal of the Corporation

Shareholders Want Managers to Maximize Market Value

Walmart has nearly 300,000 shareholders. There is no way that these shareholders can be actively involved in management; it would be like trying to run New York City by town meetings. Authority has to be delegated to professional managers. But how can Walmart's managers make decisions that satisfy all the shareholders? No two shareholders are exactly the same. They differ in age, tastes, wealth, time horizon, risk tolerance, and investment strategy. Delegating the operation of the firm to professional managers can work only if the shareholders have a common objective. Fortunately there is a natural financial objective on which almost all shareholders agree: Maximize the current market value of shareholders' investment in the firm.

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B-corporations

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A smart and effective manager makes decisions that increase the current value of the company's shares and the wealth of its stockholders. This increased wealth can then be put to whatever purposes the shareholders want. They can give their money to charity or spend it in glitzy nightclubs; they can save it or spend it now. Whatever their personal tastes or objectives, they can all do more when their shares are worth more.

Maximizing shareholder wealth is a sensible goal when the shareholders have access to well-functioning financial markets.[6] Financial markets allow them to share risks and transport savings across time. Financial markets give them the flexibility to manage their own savings and investment plans, leaving the corporation's financial managers with only one task: to increase market value.

A corporation's roster of shareholders usually includes both risk-averse and risk-tolerant investors. You might expect the risk-averse to say, "Sure, maximize value, but don't touch too many high-risk projects." Instead, they say, "Risky projects are OK, *provided* that expected profits are more than enough to offset the risks. If this firm ends up too risky for my taste, I'll adjust my investment portfolio to make it safer." For example, the risk-averse shareholders can shift more of their portfolios to safer assets, such as U.S. government bonds. They can also just say good-bye, selling shares of the risky firm and buying shares in a safer one. If the risky investments increase market value, the departing shareholders are better off than if the risky investments were turned down.

A Fundamental Result

The goal of maximizing shareholder value is widely accepted in both theory and practice. It's important to understand why. Let's walk through the argument step by step, assuming that the financial manager should act in the interests of the firm's owners, its stockholders.

1. Each stockholder wants three things:
   a. To be as rich as possible, that is, to maximize his or her current wealth.
   b. To transform that wealth into the most desirable time pattern of consumption either by borrowing to spend now or investing to spend later.
   c. To manage the risk characteristics of that consumption plan.

2. But stockholders do not need the financial manager's help to achieve the best time pattern of consumption. They can do that on their own, provided they have free access to competitive financial markets. They can also choose the risk characteristics of their consumption plan by investing in more- or less-risky securities.

3. How then can the financial manager help the firm's stockholders? There is only one way: by increasing their wealth. That means increasing the market value of the firm and the current price of its shares.

Economists have proved this value-maximization principle with great rigor and generality. After you have absorbed this chapter, take a look at its Appendix, which contains a further example. The example, though simple, illustrates how the principle of value maximization follows from formal economic reasoning.

We have suggested that shareholders want to be richer rather than poorer. But sometimes you hear managers speak as if shareholders have different goals. For example, managers may say that their job is to "maximize profits." That sounds reasonable. After all, don't shareholders want their company to be profitable? But taken literally, profit maximization is not a well-defined financial objective for at least two reasons:

1. Maximize profits? Which year's profits? A corporation may be able to increase current profits by cutting back on outlays for maintenance or staff training, but those outlays may have added long-term value. Shareholders will not welcome higher short-term profits if long-term profits are damaged.
2. A company may be able to increase future profits by cutting this year's dividend and investing the freed-up cash in the firm. That is not in the shareholders' best interest if the company earns only a modest return on the money.

The Investment Trade-off

OK, let's take the objective as maximizing market value. But why do some investments increase market value, while others reduce it? The answer is given by Figure 1.2, which sets out the fundamental trade-off for corporate investment decisions. The corporation has a proposed investment project (a real asset). Suppose it has cash on hand sufficient to finance the project. The financial manager is trying to decide whether to invest in the project. If the financial manager decides not to invest, the corporation can pay out the cash to shareholders, say as an extra dividend. (The investment and dividend arrows in Figure 1.2 are arrows 2 and 4*b* in Figure 1.1.)

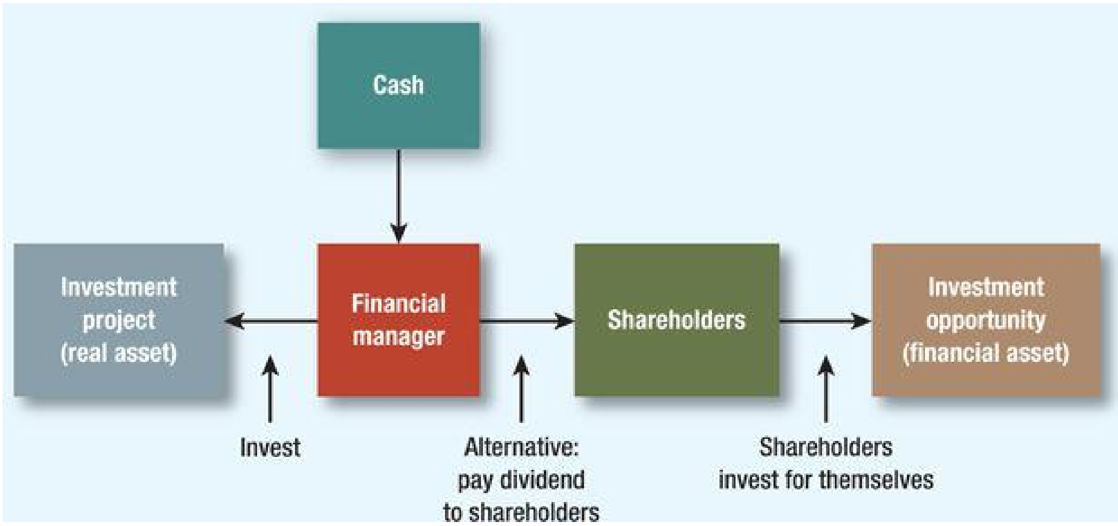

FIGURE 1.2 The firm can either keep and reinvest cash or return it to investors. (Arrows represent possible cash flows or transfers.) If cash is reinvested, the opportunity cost is the expected rate of return that shareholders could have obtained by investing in financial assets.

Assume that the financial manager is acting in the interests of the corporation's owners, its stockholders. What do these stockholders want the financial manager to do? The answer depends on the rate of return on the investment project and on the rate of return that the stockholders can earn by investing in financial markets. If the return offered by the investment project is higher than the rate of return that shareholders can get by investing on their own, then the shareholders would vote for the investment project. If the investment project offers a lower return than shareholders can achieve on their own, the shareholders would vote to cancel the project and take the cash instead.

Figure 1.2 could apply to Walmart's decisions to invest in new retail stores, for example. Suppose Walmart has cash set aside to build 100 new stores in 2014. It could go ahead with the new stores, or it could choose to cancel the investment project and instead pay the cash out to its stockholders. If it pays out the cash, the stockholders can then invest for themselves.

Suppose that Walmart's new-stores project is just about as risky as the U.S. stock market and that investment in the stock market offers a 10% expected rate of return. If the new stores offer a superior rate of return, say 20%, then Walmart's stockholders would be happy to let Walmart keep the cash and invest it in the new stores. If the new stores offer only a 5% return, then the stockholders are better off with the cash and without the new stores; in that case, the financial manager should turn down the investment project.

As long as a corporation's proposed investments offer higher rates of return than its shareholders can earn for themselves in the stock market (or in other financial markets), its shareholders will applaud the investments and its stock price will increase. But if the company earns an inferior return, shareholders boo, stock price falls, and stockholders demand their money back so that they can invest on their own.

In our example, the minimum acceptable rate of return on Walmart's new stores is 10%. This minimum rate of return is called a *hurdle rate* or *cost of capital.* It is really an **opportunity cost of capital** because it depends on the investment *opportunities* available to investors in financial markets. Whenever a corporation invests cash in a new project, its shareholders lose the opportunity to invest the cash on their own. Corporations increase value by accepting all investment projects that earn more than the opportunity cost of capital.

Notice that the opportunity cost of capital depends on the risk of the proposed investment project. Why? It's not just because shareholders are risk-averse. It's also because shareholders have to trade off risk against return when they invest on their own. The safest investments, such as U.S. government debt, offer low rates of return. Investments with higher expected rates of return—the stock market, for example—are riskier and sometimes deliver painful losses. (The U.S. stock market was down 38% in 2008, for example.) Other investments are riskier still. For example, high-tech growth stocks offer the prospect of higher rates of return but are even more volatile.

Notice too that the opportunity cost of capital is generally *not* the interest rate that the company pays on a loan from a bank. If the company is making a risky investment, the opportunity cost is the expected return that investors can achieve in financial markets at the same level of risk. The expected return on risky securities is normally well above the interest rate on a bank loan.

Managers look to the financial markets to measure the opportunity cost of capital for the firm's investment projects. They can observe the opportunity cost of capital for safe investments by looking up current interest rates on safe debt securities. For risky investments, the opportunity cost of capital has to be estimated. We start to tackle this task in Chapter 7.

Should Managers Look After the Interests of Their Shareholders?

We have described managers as the agent of shareholders, who want them to maximize their wealth. But perhaps this begs the questions: Is it *desirable* for managers to act in the selfish interests of their shareholders? Does a focus on enriching the shareholders mean that managers must act as greedy mercenaries riding roughshod over the weak and helpless?

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Ethical dilemmas

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Most of this book is devoted to financial policies that increase value. None of these policies requires gallops over the weak and helpless. In most instances, little conflict arises between doing well (maximizing value) and doing good. Profitable firms are those with satisfied customers and loyal employees; firms with dissatisfied customers and a disgruntled workforce will probably end up with declining profits and a low stock price.

Most established corporations can add value by building long-term relationships with their customers and establishing a reputation for fair dealing and financial integrity. When something happens to undermine that reputation, the costs can be enormous.

So, when we say that the objective of the firm is to maximize shareholder wealth, we do not mean that anything goes. The law deters managers from making blatantly dishonest decisions, but most managers are not simply concerned with observing the letter of the law or with keeping to written contracts. In business and finance, as in other day-to-day affairs, there are unwritten rules of behavior. These rules make routine financial transactions feasible, because each party to the transaction has to trust the other to keep to his or her side of the bargain.[7]

FINANCE IN PRACTICE

A Trojan Horse

Eurozone countries are threatened with hefty fines if their total government debt exceeds 60% of GDP or if they run a budget deficit of more than 3% of GDP. Greece has never managed to stick to this 60% debt limit, and only once in the past 20 years has it adhered to the 3% deficit ceiling.

When it comes to totting up its debts, the Greek government has shown remarkable ingenuity. But in 2001, shortly after joining the Eurozone, Greece entered into an unusually imaginative scheme that would remove about 2% of the country's debts from its accounts. It did so by entering into a number of swap transactions with the investment bank, Goldman Sachs. We explain swaps in Chapter 26, but a simple example will illustrate how the scheme worked. Suppose Greece borrows $10 billion, which it then swaps into euros. This means that Greece pays the $10 billion over to Goldman Sachs and receives euros in exchange. From then on Goldman pays Greece enough dollars to service its dollar loan, and in return Greece makes a series of euro payments to Goldman. Such an arrangement is a normal part of government financing. Europe's governments obtain funds from investors around the world by borrowing dollars or yen. But they need euros to pay their daily bills. The swap allows them to get those euros. What was unusual about the Greek government deal with Goldman Sachs was that the swap was arranged so that Greece got *more* than $10 billion worth of euros up front — but, of course, had to pay back correspondingly *more* euros in the future. Those extra euros that it received and the extra euros that it promised to pay in the future were in effect additional debt but they were buried in the swap and did not appear in the official statistics.

The deal with Goldman was a good bit more complicated than our simple example suggests. It also included a separate contract that linked Greece's payments to future rates of interest. As interest rates fell after 2001, that bet began to sour, and within 4 years the debt had almost doubled in size.

There was nothing illegal about the Greek deal. Nor was it the first Eurozone country to disguise its borrowing in this way, for Italy had used a similar subterfuge in 1995. However, when the deal was uncovered, many observers felt that the Greek government had behaved unethically. Goldman Sachs also was criticized for participating in the subterfuge and for constructing a scheme that was highly profitable to them but poorly understood by the Greek government.

Of course trust is sometimes misplaced. Charlatans and swindlers are often able to hide behind booming markets. It is only "when the tide goes out that you learn who's been swimming naked."[8] The tide went out in 2008 and a number of frauds were exposed. One notorious example was the Ponzi scheme run by the New York financier Bernard Madoff.[9] Individuals and institutions put about $65 billion in the scheme before it collapsed in 2008. (It's not clear what Madoff did with all this money, but much of it was apparently paid out to early investors in the scheme to create an impression of superior investment performance.) With hindsight, the investors should not have trusted Madoff or the financial advisers who steered money to Madoff.

Madoff's Ponzi scheme was (we hope) a once-in-a-lifetime event.[10] It was astonishingly unethical, illegal, and bound to end in tears. More complex ethical issues were raised by the banking crisis of 2007–2009. Look, for example, at the box on the previous page that describes a deal involving the investment bank Goldman Sachs. Some observers believed that Goldman's actions reflected all that is worst on Wall Street, but others argued that Goldman was simply following its client's wishes.

Agency Problems and Corporate Governance

We have emphasized the *separation of ownership and control* in public corporations. The owners (shareholders) cannot control what the managers do, except indirectly through the board of directors. This separation is necessary but also dangerous. You can see the dangers. Managers may be tempted to buy sumptuous corporate jets or to schedule business meetings at tony resorts. They may shy away from attractive but risky projects because they are worried more about the safety of their jobs than about value is a workhorse concept of corporate finance that shows up in almost every chapter.

- *How do I calculate the rate of return?* The rate of return is calculated from the cash inflows and outflows generated by the investment project. See Chapters 2 and 5.
- *What are the cash flows?* The future cash flows from an investment project should sum up all cash inflows and outflows caused by the decision to invest. Cash flows are calculated after corporate taxes are paid. They are the free cash flows that can be paid out to shareholders or reinvested on their behalf. Chapter 6 explains free cash flows in detail.
- *How does the financial manager judge whether cash-flow forecasts are realistic?* As Niels Bohr, the 1922 Nobel Laureate in Physics, observed, “Prediction is difficult, especially if it’s about the future.” But good financial managers take care to assemble relevant information and to purge forecasts of bias and thoughtless optimism. See Chapters 6 and 9 through 11.
- *How do we measure risk?* We look to the risks borne by shareholders, recognizing that investors can dilute or eliminate some risks by holding diversified portfolios (Chapters 7 and 8).
- *How does risk affect the opportunity cost of capital?* Here we need a theory of risk and return in financial markets. The most widely used theory is the Capital Asset Pricing Model (Chapters 8 and 9).
- *What determines value in financial markets?* We cover valuation of bonds and common stocks in Chapters 3 and 4. We will return to valuation principles again and again in later chapters. As you will see, corporate finance is all about valuation.
- *Where does financing come from?* Broadly speaking, from borrowing or from cash invested or reinvested by stockholders. But financing can get complicated when you get down to specifics. Chapter 14 gives an overview of financing. Chapters 23 through 25 cover sources of debt financing, including financial leases, which are debt in disguise.
    - *Debt or equity? Does it matter?* Not in a world of perfect financial markets. In the real world, the choice between debt and equity does matter, but for many possible reasons, including taxes, the risks of bankruptcy, information differences, and incentives. See Chapters 17 and 18.

maximizing shareholder value. They may work just to maximize their own bonuses, and therefore redouble their efforts to make and resell flawed subprime mortgages.

Conflicts between shareholders' and managers' objectives create *agency problems*. Agency problems arise when *agents* work for *principals*. The shareholders are the principals; the managers are their agents. **Agency costs** are incurred when (1) managers do not attempt to maximize firm value and (2) shareholders incur costs to monitor the managers and constrain their actions.

Agency problems can sometimes lead to outrageous behavior. For example, when Dennis Kozlowski, the CEO of Tyco, threw a $2 million 40th birthday bash for his wife, he charged half of the cost to the company. This of course was an extreme conflict of interest, as well as illegal. But more subtle and moderate agency problems arise whenever managers think just a little less hard about spending money when it is not their own.

Later in the book we will look at how good systems of governance ensure that shareholders' pockets are close to the managers' hearts. This means well-designed incentives for managers, standards for accounting and disclosure to investors, requirements for boards of directors, and legal sanctions for self-dealing by management. When scandals happen, we say that corporate governance has broken down. When corporations compete effectively and ethically to deliver value to shareholders, we are comforted that governance is working properly.

1-3 Preview of Coming Attractions

Figure 1.2 illustrates how the financial manager can add value for the firm and its shareholders. He or she searches for investments that offer rates of return higher than the opportunity cost of capital. But that search opens up a treasure chest of follow-up questions.

- *Is a higher rate of return on investment always better?* Not always, for two reasons. First, a lower-but-safer return can be better than a higher-but-riskier return. Second, an investment with a higher percentage return can generate less value than a lower-return investment that is larger or lasts longer. We show how to calculate the present value (PV) of a stream of cash flows in Chapter 2. Present

That's enough questions to start, but you can see certain themes emerging. For example, corporate finance is "all about valuation," not only for the reasons just listed, but because value maximization is the natural financial goal of the corporation. Another theme is the importance of the opportunity cost of capital, which is established in financial markets. The financial manager is an intermediary, who has to understand financial markets as well as the operations and investments of the corporation.

SUMMARY

Corporations face two principal financial decisions. First, what investments should the corporation make? Second, how should it pay for the investments? The first decision is the investment decision; the second is the financing decision.

The stockholders who own the corporation want its managers to maximize its overall value and the current price of its shares. The stockholders can all agree on the goal of value maximization, so long as financial markets give them the flexibility to manage their own savings and investment plans. Of course, the objective of wealth maximization does not justify unethical behavior. Shareholders do not want the maximum possible stock price. They want the maximum honest share price.

How can financial managers increase the value of the firm? Mostly by making good investment decisions. Financing decisions can also add value, and they can surely destroy value if you screw them up. But it's usually the profitability of corporate investments that separates value winners from the rest of the pack.

Investment decisions involve a trade-off. The firm can either invest cash or return it to shareholders, for example, as an extra dividend. When the firm invests cash rather than paying it out, shareholders forgo the opportunity to invest it for themselves in financial markets. The return that they are giving up is therefore called the opportunity cost of capital. If the firm's investments can earn a return higher than the opportunity cost of capital, stock price increases. If the firm invests at a return lower than the opportunity cost of capital, stock price falls.

Managers are not endowed with a special value-maximizing gene. They will consider their own personal interests, which creates a potential conflict of interest with outside shareholders. This conflict is called a principal–agent problem. Any loss of value that results is called an agency cost.

Investors will not entrust the firm with their savings unless they are confident that management will act ethically on their behalf. Successful firms have governance systems that help to align managers' and shareholders' interests.

Remember the following five themes, for you will see them again and again throughout this book:

1. Corporate finance is all about maximizing value.
2. The opportunity cost of capital sets the standard for investments.
3. A safe dollar is worth more than a risky dollar.
4. Smart investment decisions create more value than smart financing decisions.
5. Good governance matters.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect Finance*. Please see the preface for more information.

BASIC

1. Investment and financing decisions Read the following passage: "Companies usually buy (*a*) assets. These include both tangible assets such as (*b*) and intangible assets such as (*c*). To pay for these assets, they sell (*d*) assets such as (*e*). The decision about which assets to buy is usually termed the (*f*) or (*g*) decision. The decision about how to raise the money is usually termed the (*h*) decision." Now fit each of the following terms into the most appropriate space: *financing, real, bonds, investment, executive airplanes, financial, capital budgeting, brand names.*

2. Investment and financing decisions Which of the following are real assets, and which are financial?

a. A share of stock.

b. A personal IOU.

c. A trademark.

d. A factory.

e. Undeveloped land.

f. The balance in the firm's checking account.

g. An experienced and hardworking sales force.

h. A corporate bond.

3. Investment and financing decisions Vocabulary test. Explain the differences between:

a. Real and financial assets.

b. Capital budgeting and financing decisions.

c. Closely held and public corporations.

d. Limited and unlimited liability.

4. Corporations Which of the following statements always apply to corporations?

a. Unlimited liability.

b. Limited life.

c. Ownership can be transferred without affecting operations.

d. Managers can be fired with no effect on ownership.

INTERMEDIATE

5. Separation of ownership In most large corporations, ownership and management are separated. What are the main implications of this separation?

6. Opportunity cost of capital F&H Corp. continues to invest heavily in a declining industry. Here is an excerpt from a recent speech by F&H's CFO:

We at F&H have of course noted the complaints of a few spineless investors and uninformed security analysts about the slow growth of profits and dividends. Unlike those confirmed doubters, we have confidence in the long-run demand for mechanical encabulators, despite competing digital products. We are therefore determined to invest to maintain our share of the overall encabulator market. F&H has a rigorous CAPEX approval process, and we are confident of returns around 8% on investment. That's a far better return than F&H earns on its cash holdings.

The CFO went on to explain that F&H invested excess cash in short-term government securities, which are almost entirely risk-free but offered only a 4% rate of return.

a. Is a forecasted 8% return in the encabulator business necessarily better than a 4% safe return on short-term government securities? Why or why not?
b. Is F&H's opportunity cost of capital 4%? How in principle should the CFO determine the cost of capital?

7. **Corporate goals** We can imagine the financial manager doing several things on behalf of the firm's stockholders. For example, the manager might:
a. Make shareholders as wealthy as possible by investing in real assets.
b. Modify the firm's investment plan to help shareholders achieve a particular time pattern of consumption.
c. Choose high- or low-risk assets to match shareholders' risk preferences.
d. Help balance shareholders' checkbooks.
But in well-functioning capital markets, shareholders will vote for *only one* of these goals. Which one? Why?

8. **Maximizing shareholder value** Ms. Espinoza is retired and depends on her investments for her income. Mr. Liu is a young executive who wants to save for the future. Both are stockholders in Scaled Composites, LLC, which is building *SpaceShipOne* to take commercial passengers into space. This investment's payoff is many years away. Assume it has a positive NPV for Mr. Liu. Explain why this investment also makes sense for Ms. Espinoza.

9. **Ethical issues** The box on page 11 describes the controversial involvement of Goldman Sachs in a financing deal with Greece. The deal was perfectly legal, but many regarded it as unethical. Was the fault solely that of the Greek government or was Goldman partly to

blame? There is some indication that the government did not fully understand the bet that it was placing on future interest rates. Did Goldman have a responsibility to ensure that it did?

10. Agency issues Why might one expect managers to act in shareholders' interests? Give some reasons.

11. Agency issues Many firms have devised defenses that make it more difficult or costly for other firms to take them over. How might such defenses affect the firm's agency problems? Are managers of firms with formidable takeover defenses more or less likely to act in the shareholders' interests rather than their own? What would you expect to happen to the share price when management proposes to institute such defenses?

12. Ethical issues Most managers have no difficulty avoiding blatantly dishonest actions. But sometimes gray areas, where it is debatable whether an action is unethical and unacceptable, exist. Suggest an important ethical dilemma that companies may face. What principles should guide their decision?

APPENDIX

Why Maximizing Shareholder Value Makes Sense

We have suggested that well-functioning financial markets allow different investors to agree on the objective of maximizing value. This idea is sufficiently important that we need to pause and examine it more carefully.

BEYOND THE PAGE

Foundations of NPV

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How Financial Markets Reconcile Preferences for Current vs. Future Consumption

Suppose that there are two possible investors with entirely different preferences. Think of A as an ant, who wishes to save for the future, and of G as a grasshopper, who would prefer to spend all his wealth on some ephemeral frolic, taking no heed of tomorrow. Suppose that each has a nest egg of exactly $100,000 in cash. G chooses to spend all of it today, while A prefers to invest it in the financial market. If the interest rate is 10%, A would then have 1.10 × $100,000 = $110,000 to spend a year from now. Of course, there are many possible intermediate strategies. For example, A or G could choose to split the difference, spending $50,000 now and putting the remaining $50,000 to work at 10% to provide 1.10 × $50,000 = $55,000 next year. The entire range of possibilities is shown by the green line in Figure 1A.1.

In our example, A used the financial market to postpone consumption. But the market can also be used to bring consumption forward in time. Let's illustrate by assuming that instead of having cash on hand of $100,000, our two friends are due to receive $110,000 each at the end of the year. In this case A will be happy to wait and spend the income when it arrives. G will prefer to borrow against his future income and party it away today. With an interest rate of 10%, G can borrow and spend $110,000/1.10 = $100,000. Thus the financial market provides a kind of time machine that allows people to separate the timing of their income from that of their spending. Notice that with an interest rate of 10%, A and G are equally happy with cash on hand of $100,000 or an income of $110,000 at the end of the year. They do not care about the timing of the cash flow; they just prefer the cash flow that has the highest value today ($100,000 in our example).

Investing in Real Assets

In practice individuals are not limited to investing in financial markets; they may also acquire plant, machinery, and other real assets. For example, suppose that A and G are offered the opportunity to invest their $100,000 in a new business that a friend is founding. This will produce a one-off surefire payment of $121,000 next year. A would clearly be happy to invest in the business. It will provide her with $121,000 to spend at the end of the year, rather than the $110,000 that she gets by investing her $100,000 in the financial market. But what about G, who wants money now, not in one year's time? He too is happy to invest, as long as he can borrow against the future payoff of the investment project. At an interest rate of 10%, G can borrow $110,000 and so will have an extra $10,000 to spend today. Both A and G are better off investing in their friend's venture. The investment increases their wealth. It moves them up from the green to the maroon line in Figure 1A.1.

Why can both A and G spend more by investing $100,000 in their friend's business? Because the business provides a return of $21,000, or 21%, whereas they would earn only $10,000, or 10%, by investing their money in the capital market.

A Crucial Assumption

The key condition that allows A and G to agree to invest in the new venture is that both have access to a well-functioning, competitive capital market, in which they can borrow and lend at the same rate. Whenever the corporation's shareholders have equal access to competitive capital markets, the goal of maximizing market value makes sense.

It is easy to see how this rule would be damaged if we did *not* have such a well-functioning capital market. For example, suppose that G could not easily borrow against future income. In that case he might well prefer to spend his cash today rather than invest it in the new venture. If A and G were shareholders in the same enterprise, A would be happy for the firm to invest, while G would be clamoring for higher current dividends.

No one believes unreservedly that capital markets function perfectly. Later in this book we discuss several cases in which differences in taxation, transaction costs, and other imperfections must be taken into account in financial decision making. However, we also discuss research indicating that, in general, capital markets function fairly well. In this case maximizing shareholder value is a sensible corporate objective. But for now, having glimpsed the problems of imperfect markets, we shall, like an economist in a shipwreck, simply *assume* our life jacket and swim safely to shore.

QUESTIONS

1. Maximizing shareholder value Look back to the numerical example graphed in Figure 1A.1. Suppose the interest rate is 20%. What would the ant (A) and grasshopper (G) do if they both start with $100,000? Would they invest in their friend's business? Would they borrow or lend? How much and when would each consume?

2. Maximizing shareholder value Answer this question by drawing graphs like Figure 1A.1. Casper Milktoast has $200,000 available to support consumption in periods 0 (now) and 1 (next year). He wants to consume *exactly* the same amount in each period. The interest rate is 8%. There is no risk.

a. How much should he invest, and how much can he consume in each period?

b. Suppose Casper is given an opportunity to invest up to $200,000 at 10% risk-free. The interest rate stays at 8%. What should he do, and how much can he consume in each period?

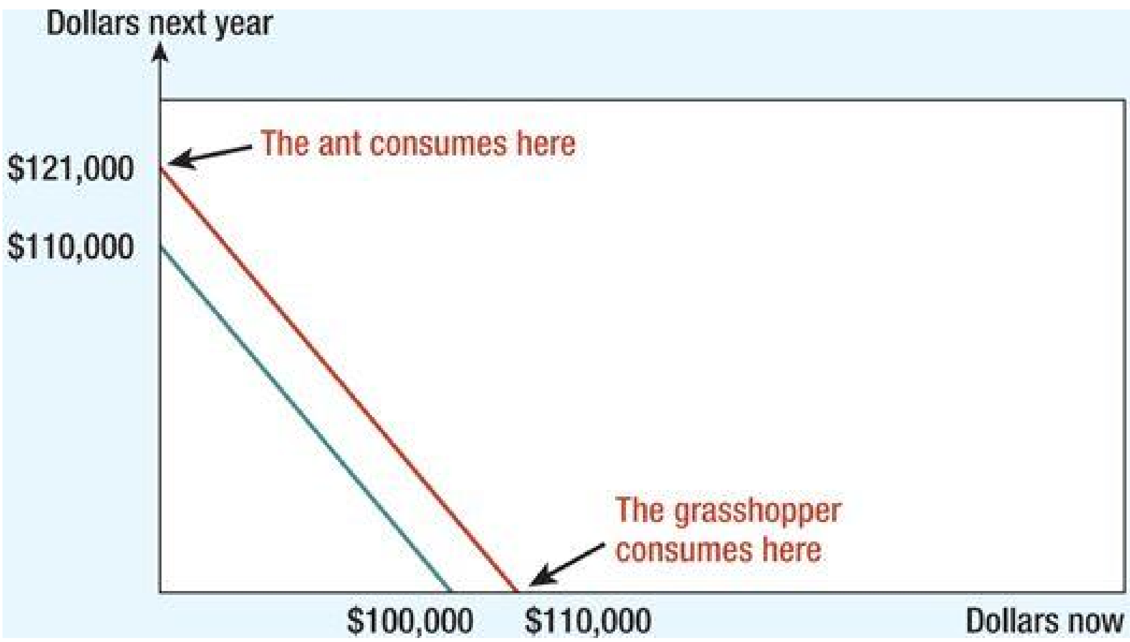


FIGURE 1A.1 The green line shows the possible spending patterns for the ant and grasshopper if they invest $100,000 in the capital market. The red line shows the possible spending patterns if they invest in their friend's business. Both are better off by investing in the business as long as the grasshopper can borrow against the future income.

[1]We have referred to the corporation's owners as "shareholders" and "stockholders." The two terms mean exactly the same thing and are used interchangeably. Corporations are also referred to casually as "companies," "firms," or "businesses." We also use these terms interchangeably.

[2]LVMH Moët Hennessy Louis Vuitton (usually abbreviated to LVMH) markets perfumes and cosmetics, wines and spirits, watches, and other fashion and luxury goods. And, yes, we know what you are thinking, but LVMH really is short for Moët Hennessy Louis Vuitton.

[3]The private investors who bought the bankrupt system concentrated on aviation, maritime, and defense markets rather than retail customers. In 2010 it arranged $1.8 billion in new financing to replace and upgrade its satellite system.

[4]In the U.S., corporations are identified by the label "Corporation," "Incorporated," or "Inc.," as in US Airways Group, Inc. The UK identifies public corporations by "plc" (short for "Public Limited Corporation"). French corporations have the suffix "SA" ("Société Anonyme"). The corresponding labels in Germany are "GmbH" ("Gesellschaft mit beschränkter Haftung") or "AG" ("Aktiengesellschaft").

[5]The corporation's bylaws set out in more detail the duties of the board of directors and how the firm should conduct its business.

[6]Here we use "financial markets" as shorthand for the financial sector of the economy. Strictly speaking, we should say "access to well-functioning financial markets and institutions." Many investors deal mostly with financial institutions, for example, banks, insurance companies, or mutual funds. The financial institutions in turn engage in financial markets, including the stock and bond markets. The institutions act as financial intermediaries on behalf of individual investors.

[7]See L. Guiso, L. Zingales, and P. Sapienza, "Trusting the Stock Market," *Journal of Finance* 63 (December 2008), pp. 2557–600. The authors show that an individual's lack of trust is a significant impediment to participation in the stock market. "Lack of trust" means a subjective fear of being cheated.

[8]The quotation is from Warren Buffett's annual letter to the shareholders of Berkshire Hathaway, March 2008.

[9]Ponzi schemes are named after Charles Ponzi who founded an investment company in 1920 that promised investors unbelievably high returns. He was soon deluged with funds from investors in New England, taking in $1 million during one three-hour period. Ponzi invested only about $30 of the money that he raised, but used part of the cash provided by later investors to pay generous dividends to the original investors. Within months the scheme collapsed and Ponzi started a five-year prison sentence.

[10]Ponzi schemes pop up frequently, but none has approached the scope and duration of Madoff's.

Part 1 Value

CHAPTER 2

How to Calculate Present Values

Companies invest in lots of things. Some are *tangible assets*—that is, assets you can kick, like factories, machinery, and offices. Others are *intangible assets*, such as patents or trademarks. In each case the company lays out some money now in the hope of receiving even more money later.

Individuals also make investments. For example, your college education may cost you $40,000 per year. That is an investment you hope will pay off in the form of a higher salary later in life. You are sowing now and expecting to reap later.

Companies pay for their investments by raising money and, in the process, assuming liabilities. For example, they may borrow money from a bank and promise to repay it with interest later. You also may have financed your investment in a college education by borrowing money that you plan to pay back out of that fat salary.

All these financial decisions require comparisons of cash payments at different dates. Will your future salary be sufficient to justify the current expenditure on college tuition? How much will you have to repay the bank if you borrow to finance your degree?

In this chapter we take the first steps toward understanding the relationship between the values of dollars today and dollars in the future. We start by looking at how funds invested at a specific interest rate will grow over time. We next ask how much you would need to invest today to produce a specified future sum of money, and we describe some shortcuts for working out the value of a series of cash payments.

The term *interest rate* sounds straightforward enough, but rates can be quoted in different ways. We, therefore, conclude the chapter by explaining the difference between the quoted rate and the true or effective interest rate.

Once you have learned how to value cash flows that occur at different points in time, we can move on in the next two chapters to look at how bonds and stocks are valued. After that we will tackle capital investment decisions at a practical level of detail.

For simplicity, every problem in this chapter is set out in dollars, but the concepts and calculations are identical in euros, yen, or any other currency.

2-1 Future Values and Present Values

Calculating Future Values

Money can be invested to earn interest. So, if you are offered the choice between \$100 today and \$100 next year, you naturally take the money now to get a year's interest. Financial managers make the same point when they say that money has a *time value* or when they quote the most basic principle of finance: *a dollar today is worth more than a dollar tomorrow.*

Suppose you invest \$100 in a bank account that pays interest of $r = 7\%$ a year. In the first year you will earn interest of $.07 \times \$100 = \$7$ and the value of your investment will grow to \$107:

$$\text{Value of investment after 1 year} = \$100 \times (1 + r) = 100 \times 1.07 = \$107$$

By investing, you give up the opportunity to spend \$100 today, but you gain the chance to spend \$107 next year.

If you leave your money in the bank for a second year, you earn interest of $.07 \times \$107 = \$7.49$ and your investment will grow to \$114.49:

$$\text{Value of investment after 2 years} = \$107 \times 1.07 = \$100 \times 1.07^2 = \$114.49$$

Notice that in the second year you earn interest on both your initial investment ($100) and the previous year's interest ($7). Thus your wealth grows at a *compound rate* and the interest that you earn is called **compound interest.**

If you invest your $100 for t years, your investment will continue to grow at a 7% compound rate to $\$100 \times (1.07)^t$. For any interest rate r, the future value of your $100 investment will be

$$\text{Future value of } \$100 = \$100 \times (1 + r)^t$$

The higher the interest rate, the faster your savings will grow. Figure 2.1 shows that a few percentage points added to the interest rate can do wonders for your future wealth. For example, by the end of 20 years $100 invested at 10% will grow to $\$100 \times (1.10)^{20} = \$672.75$. If it is invested at 5%, it will grow to only $\$100 \times (1.05)^{20} = \$265.33$.

Calculating Present Values

We have seen that $100 invested for two years at 7% will grow to a future value of $100 \times 1.07^2 = \$114.49$. Let's turn this around and ask how much you need to invest *today* to produce $114.49 at the end of the second year. In other words, what is the **present value (PV)** of the $114.49 payoff?

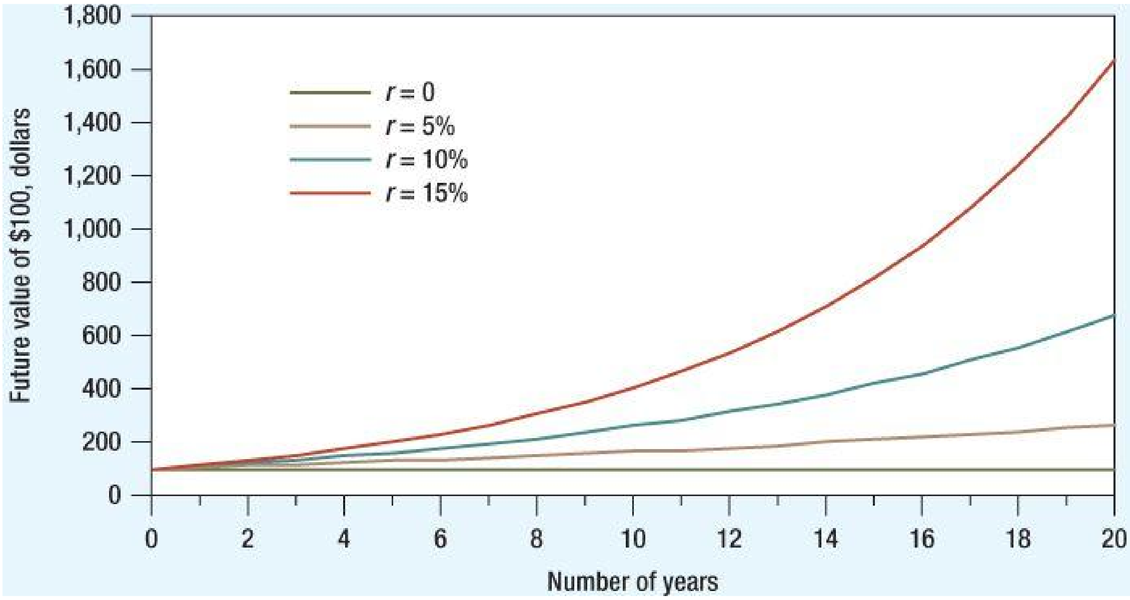
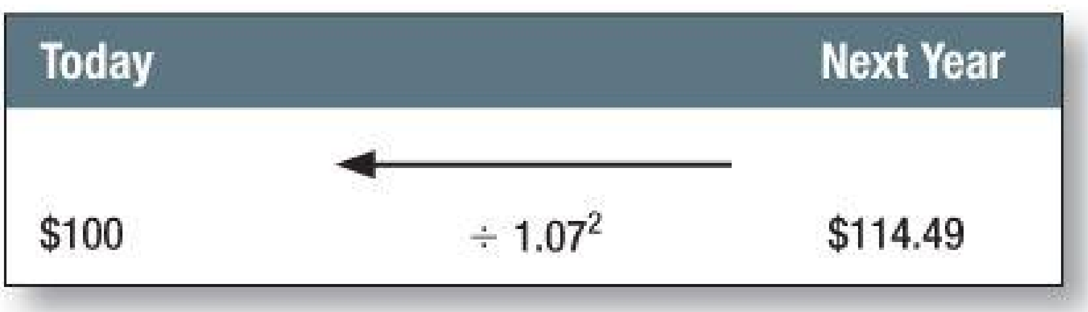


FIGURE 2.1 How an investment of $100 grows with compound interest at different interest rates.

You already know that the answer is $100. But, if you didn't know or you forgot, you can just run the future value calculation in reverse and divide the future payoff by $(1.07)^2$:

$$\text{Present value} = \text{PV} = \frac{\$114.49}{(1.07)^2} = \$100$$



In general, suppose that you will receive a cash flow of C_t dollars at the end of year t. The present value of this future payment is

$$\text{Present value} = \text{PV} = \frac{C_t}{(1 + r)^t}$$

The rate, *r,* in the formula is called the discount rate, and the present value is the discounted value of the cash flow, C_t. You sometimes see this present value formula written differently. Instead of *dividing* the future payment by $(1 + r)^t$, you can equally well *multiply* the payment by $1/(1 + r)^t$. The expression $1/(1 + r)^t$ is called the **discount factor.** It measures the present value of one dollar received in year *t.* For example, with an interest rate of 7% the two-year discount factor is

$$DF_2 = 1/(1.07)^2 = .8734$$

Investors are willing to pay \$.8734 today for delivery of \$1 at the end of two years. If each dollar received in year 2 is worth \$.8734 today, then the present value of your payment of \$114.49 in year 2 must be

$$\text{Present value} = DF_2 \times C_2 = .8734 \times 114.49 = \$100$$

The longer you have to wait for your money, the lower its present value. This is illustrated in Figure 2.2. Notice how small variations in the interest rate can have a powerful effect on the present value of distant cash flows. At an interest rate of 5%, a payment of \$100 in year 20 is worth \$37.69 today. If the interest rate increases to 10%, the value of the future payment falls by about 60% to \$14.86.

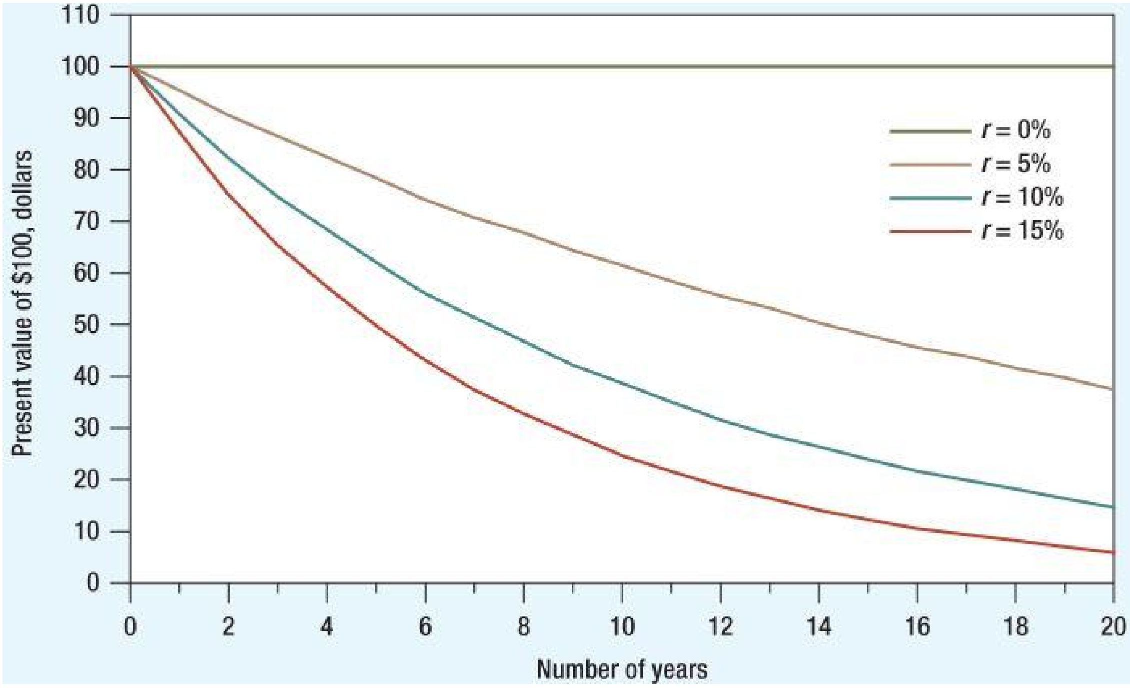


FIGURE 2.2 Present value of a future cash flow of $100. Notice that the longer you have to wait for your money, the less it is worth today.

Valuing an Investment Opportunity

How do you decide whether an investment opportunity is worth undertaking? Suppose you own a small company that is contemplating construction of a suburban office block. The cost of buying the land and constructing the building is $700,000. Your company has cash in the bank to finance construction. Your real-estate adviser forecasts a shortage of office space and predicts that you will be able to sell next year for $800,000. For simplicity, we will assume initially that this $800,000 is a sure thing.

The rate of return on this one-period project is easy to calculate. Divide the expected profit ($800,000 – 700,000 = $100,000) by the required investment ($700,000). The result is 100,000/700,000 = .143, or 14.3%.

Figure 2.3 summarizes your choices. (Note the resemblance to Figure 1.2 in the last chapter.) You can invest in the project, or pay cash out to shareholders, who can invest on their own. We assume that they can earn a 7% profit by investing for one year in safe assets (U.S. Treasury debt securities, for example). Or they can invest in the stock market, which is risky but offers an average return of 12%.

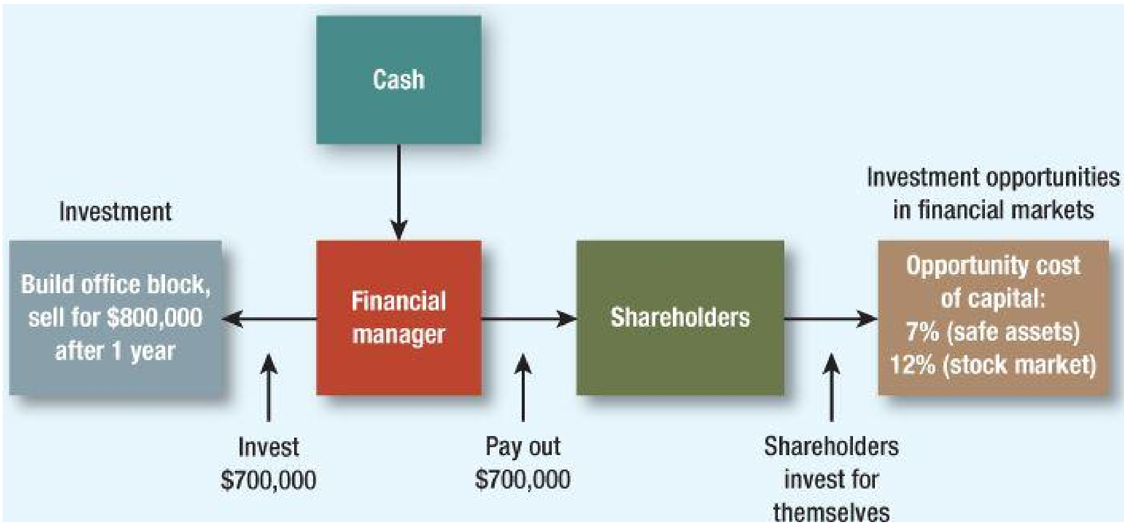


FIGURE 2.3 Your company can either invest $700,000 in an office block and sell it after 1 year for $800,000, or it can return the $700,000 to shareholders to invest in the financial markets.

What is the opportunity cost of capital, 7% or 12%? The answer is 7%: That's the rate of return that your company's shareholders could get by investing on their own at the same level of risk as the proposed project. Here the level of risk is zero. (Remember, we are assuming for now that the future value of the office block is known with certainty.) Your shareholders would vote unanimously for the investment project, because the project offers a safe return of 14% versus a safe return of only 7% in financial markets.

The office-block project is therefore a "go," but how much is it worth and how much will the investment add to your wealth? The project produces a cash flow at the end of one year. To find its present value we discount that cash flow by the opportunity cost of capital:

$$\text{Present value} = \text{PV} = \frac{C_1}{1 + r} = \frac{800{,}000}{1.07} = \$747{,}664$$

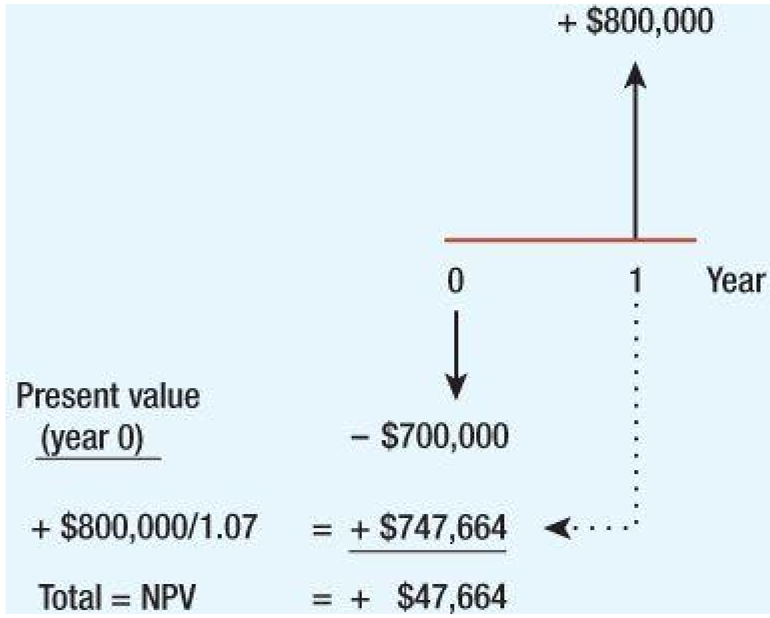


FIGURE 2.4 Calculation showing the NPV of the office development.

Risk and Present Value

We made one unrealistic assumption in our discussion of the office development: Your real estate adviser cannot be certain about the profitability of an office building. Those future cash flows represent the best forecast, but they are not a sure thing.

If the cash flows are uncertain, your calculation of NPV is wrong. Investors could achieve those cash flows with certainty by buying $747,664 worth of U.S. government securities, so they would not buy your building for that amount. You would have to cut your asking price to attract investors' interest.

Here we can invoke a second basic financial principle: *A safe dollar is worth more than a risky dollar.* Most investors dislike risky ventures and won't invest in them unless they see the prospect of a higher return. However, the concepts of present value and the opportunity cost of capital still make sense for risky investments. It is still proper to discount the payoff by the rate of return offered by a risk-equivalent investment in financial markets. But we have to think of *expected* payoffs and the *expected* rates of return on other investments.[2]

Suppose that as soon as you have bought the land and paid for the construction, you decide to sell your project. How much could you sell it for? That is an easy question. If the venture will return a surefire \$800,000, then your property ought to be worth its PV of \$747,664 today. That is what investors in the financial markets would need to pay to get the same future payoff. If you tried to sell it for more than \$747,664, there would be no takers, because the property would then offer an expected rate of return lower than the 7% available on government securities. Of course, you could always sell your property for less, but why sell for less than the market will bear? The \$747,664 present value is the only feasible price that satisfies both buyer and seller. Therefore, the present value of the property is also its market price.

Net Present Value

The office building is worth \$747,664 today, but that does not mean you are \$747,664 better off. You invested \$700,000, so the **net present value (NPV)** is \$47,664. Net present value equals present value minus the required investment:

$$\text{NPV} = \text{PV} - \text{investment} = 747{,}664 - 700{,}000 = \$47{,}664$$

In other words, your office development is worth more than it costs. It makes a *net* contribution to value and increases your wealth. The formula for calculating the NPV of your project can be written as:

$$\text{NPV} = C_0 + C_1/(1 + r)$$

Remember that C_0, the cash flow at time 0 (that is, today) is usually a negative number. In other words, C_0 is an investment and therefore a cash outflow. In our example, $C_0 = -\$700{,}000$.

When cash flows occur at different points in time, it is often helpful to draw a time line showing the date and value of each cash flow. Figure 2.4 shows a time line for your office development. It sets out the net present value calculation assuming that the discount rate r is 7%.[1]

Not all investments are equally risky. The office development is more risky than a government security but less risky than a start-up biotech venture. Suppose you believe the project is as risky as investment in the stock market and that stocks are expected to provide a 12% return. Then 12% is the opportunity cost of capital for your project. That is what you are giving up by investing in the office building and *not* investing in equally risky securities.

Now recompute NPV with $r = .12$:

$$\text{PV} = \frac{800{,}000}{1.12} = \$714{,}286$$

$$\text{NPV} = \text{PV} - 700{,}000 = \$14{,}286$$

The office building still makes a net contribution to value, but the increase in your wealth is smaller than in our first calculation, which assumed that the cash flows from the project were risk-free.

The value of the office building depends, therefore, on the timing of the cash flows and their risk. The $800,000 payoff would be worth just that if you could get it today. If the office building is as risk-free as government securities, the delay in the cash flow reduces value by $52,336 to $747,664. If the building is as risky as investment in the stock market, then the risk further reduces value by $33,378 to $714,286.

Unfortunately, adjusting asset values for both time and risk is often more complicated than our example suggests. Therefore, we take the two effects separately. For the most part, we dodge the problem of risk in Chapters 2 through 6, either by treating all cash flows as if they were known with certainty or by talking about expected cash flows and expected rates of return without worrying how risk is defined or measured. Then in Chapter 7 we turn to the problem of understanding how financial markets cope with risk.

Present Values and Rates of Return

We have decided that constructing the office building is a smart thing to do, since it is worth more than it costs. To discover how much it is worth, we asked how much you would need to invest directly in securities to achieve the same payoff. That is why we discounted the project's future payoff by the rate of return offered by these equivalent-risk securities—the overall stock market in our example.

We can state our decision rule in another way: your real estate venture is worth undertaking because its rate of return exceeds the opportunity cost of capital. The rate of return is simply the profit as a proportion of the initial outlay:

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{800{,}000 - 700{,}000}{700{,}000} = .143, \text{ or } 14.3\%$$

The cost of capital is once again the return foregone by *not* investing in financial markets. If the office building is as risky as investing in the stock market, the return foregone is 12%. Since the 14.3% return on the office building exceeds the 12% opportunity cost, you should go ahead with the project.

Building the office block is a smart thing to do, even if the payoff is just as risky as the stock market. We can justify the investment by either one of the following two rules:[3]

- *Net present value rule.* Accept investments that have positive net present values.
- *Rate of return rule.* Accept investments that offer rates of return in excess of their opportunity costs of capital.

Both rules give the same answer, although we will encounter some cases in Chapter 5 where the rate of return rule is unreliable. In those cases, you should use the net present value rule.

Calculating Present Values When There Are Multiple Cash Flows

One of the nice things about present values is that they are all expressed in current dollars—so you can add them up. In other words, the present value of cash flow (A + B) is equal to the present value of cash flow A plus the present value of cash flow B.

Suppose that you wish to value a stream of cash flows extending over a number of years. Our rule for adding present values tells us that the *total* present value is:

$$PV = \frac{C_1}{(1 + r)} + \frac{C_2}{(1 + r)^2} + \frac{C_3}{(1 + r)^3} + \cdots + \frac{C_T}{(1 + r)^T}$$

This is called the **discounted cash flow** (or **DCF**) formula. A shorthand way to write it is

$$PV = \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t}$$

where ∑ refers to the sum of the series. To find the *net* present value (NPV) we add the (usually negative) initial cash flow:

$$NPV = C_0 + PV = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t}$$

EXAMPLE 2.1 • Present Values with Multiple Cash Flows

Your real estate adviser has come back with some revised forecasts. He suggests that you rent out the building for two years at $30,000 a year, and predicts that at the end of that time you will be able to sell the building for $840,000. Thus there are now two future cash flows—a cash flow of C_1 = $30,000 at the end of one year and a further cash flow of C_2 = (30,000 + 840,000) = $870,000 at the end of the second year.

The present value of your property development is equal to the present value of C_1 plus the present value of C_2. Figure 2.5 shows that the value of the first year's cash flow is $C_1/(1 + r) = 30{,}000/1.12 = \$26{,}786$ and the value of the second year's flow is $C_2/(1 + r)^2 = 870{,}000/1.12^2 = \$693{,}559$. Therefore our rule for adding present values tells us that the *total* present value of your investment is:

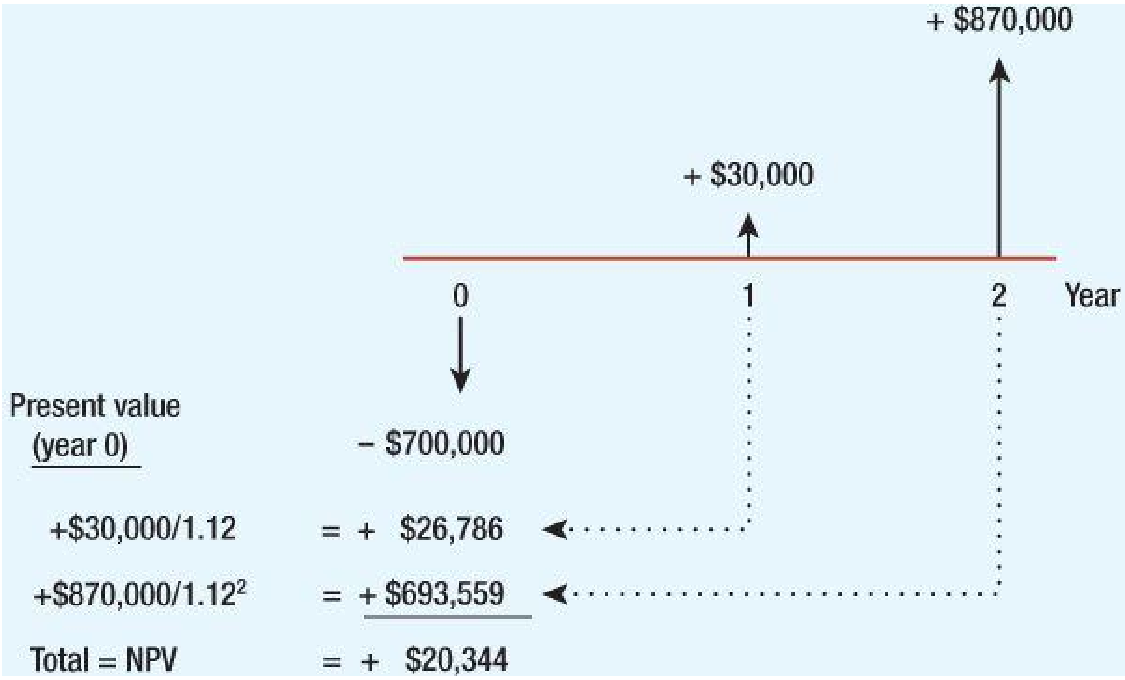


FIGURE 2.5 Calculation showing the NPV of the revised office project.

$$PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} = \frac{30,000}{1.12} + \frac{870,000}{1.12^2} = 26,786 + 693,559 = \$720,344$$

It looks as if you should take your adviser's suggestion. NPV is higher than if you sell in year 1:

$$NPV = \$720,344 - \$700,000 = \$20,344$$

Your two-period calculations in Example 2.1 required just a few keystrokes on a calculator. Real problems can be much more complicated, so financial managers usually turn to financial calculators especially programmed for present value calculations or to computer spreadsheet programs. A box near the end of the chapter introduces you to some useful Excel functions that can be used to solve discounting problems.

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Introduction to financial calculators

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The Opportunity Cost of Capital

By investing in the office building you are giving up the opportunity to earn an expected return of 12% in the stock market. The opportunity cost of capital is therefore 12%. When you discount the expected cash flows by the opportunity cost of capital, you are asking how much investors in the financial markets are prepared to pay for a security that produces a similar stream of future cash flows. Your calculations showed that these investors would need to pay $720,344 for an investment that produces cash flows of $30,000 at year 1 and $870,000 at year 2. Therefore, they won't pay any more than that for your office building.

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Introduction to Excel

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Confusion sometimes sneaks into discussions of the cost of capital. Suppose a banker approaches. "Your company is a fine and safe business with few debts," she says. "My bank will lend you the $700,000 that you need for the office block at 8%." Does this mean that the cost of capital is 8%? If so, the project would be even more worthwhile. At an 8% cost of capital, PV would be $30{,}000/1.08 + 870{,}000/1.08^2 = \$773{,}663$ and NPV = $773,663 – $700,000 = + $73,663.

But that can't be right. First, the interest rate on the loan has nothing to do with the risk of the project: it reflects the good health of your existing business. Second, whether you take the loan or not, you still face the choice between the office building and an equally risky investment in the stock market. The stock market investment could generate the same expected payoff as your office building at a lower cost. A financial manager who borrows $700,000 at 8% and invests in an office building is not smart, but stupid, if the company or its shareholders can borrow at 8% and invest the money at an even higher return. That is why the 12% expected return on the stock market is the opportunity cost of capital for your project.

2-2 Looking for Shortcuts—Perpetuities and Annuities

How to Value Perpetuities

Sometimes there are shortcuts that make it easy to calculate present values. Let us look at some examples.

On occasion, the British and the French have been known to disagree and sometimes even to fight wars. At the end of some of these wars the British consolidated the debt they had issued during the war. The securities issued in such cases were called consols. Consols are **perpetuities.** These are bonds that the government is under no obligation to repay but that offer a fixed income for each year to perpetuity. The British government is still paying interest on consols issued all those years ago. The annual rate of return on a perpetuity is equal to the promised annual payment divided by the present value:[4]

$$\text{Return} = \frac{\text{cash flow}}{\text{present value}}$$

$$r = \frac{C}{\text{PV}}$$

We can obviously twist this around and find the present value of a perpetuity given the discount rate *r* and the cash payment *C:*

$$\text{PV} = \frac{C}{r}$$

The year is 2030. You have been fabulously successful and are now a billionaire many times over. It was fortunate indeed that you took that finance course all those years ago. You have decided to follow in the footsteps of two of your heroes, Bill Gates and Warren Buffet. Malaria is still a scourge and you want to help eradicate it and other infectious diseases by endowing a foundation to combat these diseases. You aim to provide \$1 billion a year in perpetuity, starting next year. So, if the interest rate is 10%, you are going to have to write a check today for

$$\text{Present value of perpetuity} = \frac{C}{r} = \frac{\$1 \text{ billion}}{.1} = \$10 \text{ billion}$$

Two warnings about the perpetuity formula. First, at a quick glance you can easily confuse the formula with the present value of a single payment. A payment of \$1 at the end of one year has a present value of $1/(1 + r)$. The perpetuity has a value of $1/r$. These are quite different.

Second, the perpetuity formula tells us the value of a regular stream of payments starting one period from now. Thus your \$10 billion endowment would provide the foundation with its first payment in one year's time. If you also want to provide an up-front sum, you will need to lay out an extra \$1 billion.

Sometimes you may need to calculate the value of a perpetuity that does not start to make payments for several years. For example, suppose that you decide to provide \$1 billion a year with the first payment four years from now. Figure 2.6 provides a timeline of these payments. Think first about how much they will be worth in year 3. At that point the endowment will be an ordinary perpetuity with the first payment due at the end of the year. So our perpetuity formula tells us that in year 3 the endowment will be worth $\$1/r = \$1/.1 = \$10$ billion. But it is not worth that much now. To find *today's* value we need to multiply by the three-year discount factor $1/(1 + r)^3 = 1/(1.1)^3 = .751$. Thus, the "delayed" perpetuity is worth \$10 billion × .751 = \$7.51 billion. The full calculation is:

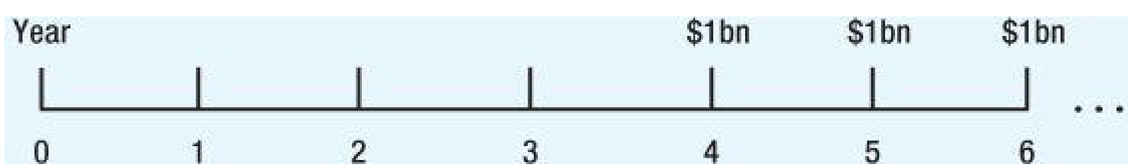

FIGURE 2.6 This perpetuity makes a series of payments of \$1 billion a year starting in year 4.

$$PV = \$1 \text{ billion} \times \frac{1}{r} \times \frac{1}{(1+r)^3} = \$1 \text{ billion} \times \frac{1}{.10} \times \frac{1}{(1.10)^3} = \$7.51 \text{ billion}$$

How to Value Annuities

An **annuity** is an asset that pays a fixed sum each year for a specified number of years. The equal-payment house mortgage or installment credit agreement are common examples of annuities. So are interest payments on most bonds, as we see in the next chapter.

You can always value an annuity by calculating the value of each cash flow and finding the total. However, it is often quicker to use a simple formula that states that if the interest rate is *r,* then the present value of an annuity that pays $\$C$ a period for each of *t* periods is:

$$\text{Present value of } t\text{-year annuity} = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

The expression in brackets shows the present value of \$1 a year for each of *t* years. It is generally known as the *t*-year **annuity factor.**

If you are wondering where this formula comes from, look at Figure 2.7. It shows the payments and values of three investments.

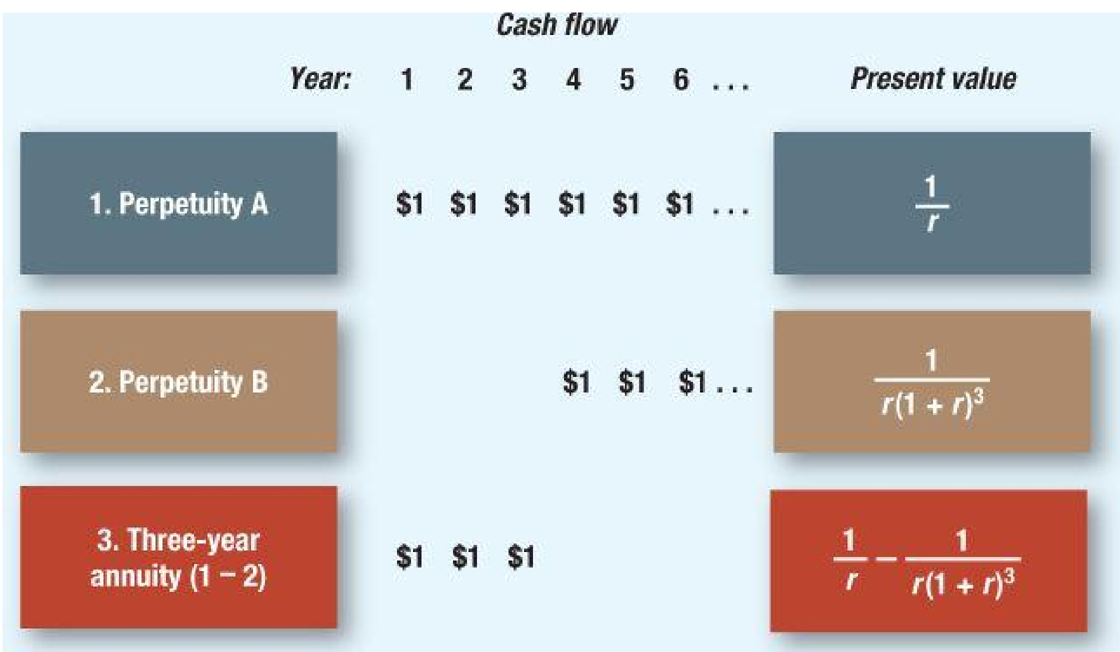

FIGURE 2.7 An annuity that makes payments in each of years 1 through 3 is equal to the difference between two perpetuities.

Row 1 The investment in the first row provides a perpetual stream of $1 starting at the end of the first year. We have already seen that this perpetuity has a present value of $1/r$.

Row 2 Now look at the investment shown in the second row of Figure 2.7. It also provides a perpetual stream of $1 payments, but these payments don't start until year 4. This stream of payments is identical to the payments in row 1, except that they are delayed for an additional three years. In year 3, the investment will be an ordinary perpetuity with payments starting in one year and will therefore be worth $1/r$ in year 3. To find the value *today,* we simply multiply this figure by the three-year discount factor. Thus

$$PV = \frac{1}{r} \times \frac{1}{(1 + r)^3}$$

Row 3 Finally, look at the investment shown in the third row of Figure 2.7. This provides a level payment of $1 a year for each of three years. In other words, it is a three-year annuity. You can also see that, taken together, the investments in rows 2 and 3 provide exactly the same cash payments as the investment in row 1. Thus the value of our annuity (row 3) must be equal to the value of the row 1 perpetuity less the value of the delayed row 2 perpetuity:

$$\text{Present value of a 3-year annuity of \$1 a year} = \frac{1}{r} - \frac{1}{r(1 + r)^3}$$

Remembering formulas is about as difficult as remembering other people's birthdays. But as long as you bear in mind that an annuity is equivalent to the difference between an immediate and a delayed perpetuity, you shouldn't have any difficulty.[5]

EXAMPLE 2.2 • Costing an Installment Plan

Most installment plans call for level streams of payments. Suppose that Tiburon Autos offers an “easy payment” scheme on a new Toyota of $5,000 a year, paid at the end of each of the next five years, with no cash down. What is the car really costing you?

First let us do the calculations the slow way, to show that, if the interest rate is 7%, the present value of these payments is $20,501. The time line in Figure 2.8 shows the value of each cash flow and the total present value. The annuity formula, however, is generally quicker; you simply need to multiply the $5,000 cash flow by the annuity factor:

$$\text{PV} = 5{,}000\left[\frac{1}{.07} - \frac{1}{.07(1.07)^5}\right] = 5{,}000 \times 4.100 = \$20{,}501$$

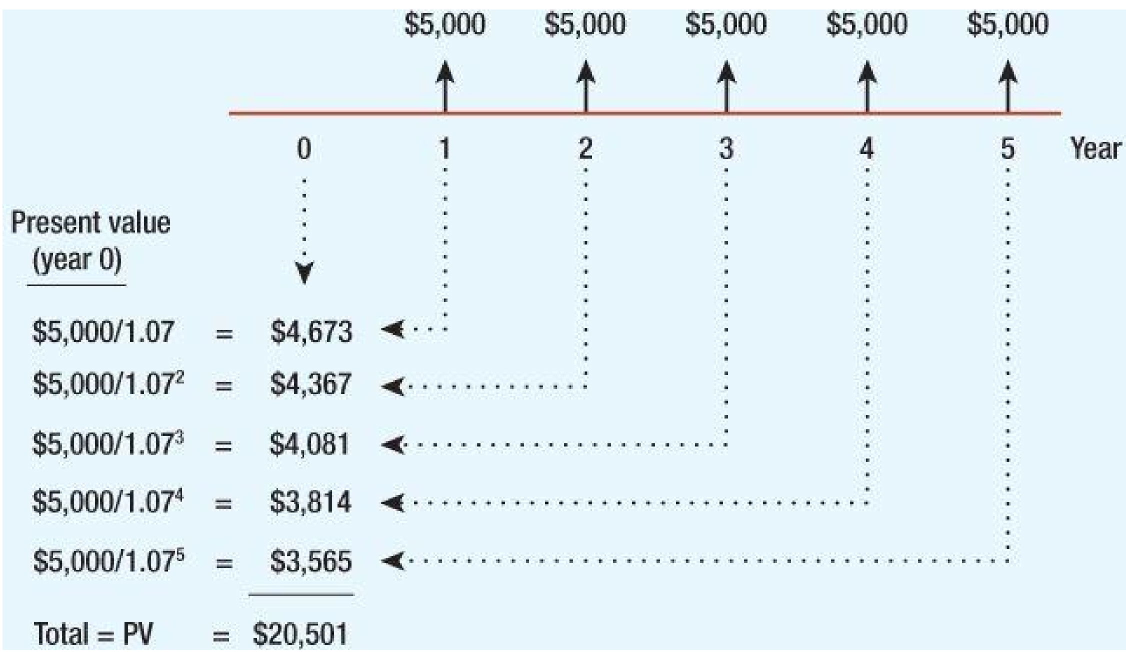

FIGURE 2.8 Calculations showing the year-by-year present value of the installment payments.

EXAMPLE 2.3 • Winning Big at the Lottery

In August 2006 eight lucky meatpackers from Nebraska pooled their money to buy Powerball lottery tickets and won a record $365 million. We suspect that the winners received unsolicited congratulations, good wishes, and requests for money from dozens of more or less worthy charities, relations, and newly devoted friends. In response, they could fairly point out that the prize wasn't really worth \$365 million. That sum was to be paid in 30 equal annual installments of \$12.167 million each. Assuming that the first payment occurred at the end of one year, what was the present value of the prize? The interest rate at the time was 6.0%.

These payments constitute a 30-year annuity. To value this annuity we simply multiply \$12.167 million by the 30-year annuity factor:

$$\text{PV} = 12.167 \times \text{30-year annuity factor}$$

$$= 12.167 \times \left[\frac{1}{r} - \frac{1}{r(1+r)^{30}}\right]$$

At an interest rate of 6.0%, the annuity factor is

$$\left[\frac{1}{.060} - \frac{1}{.060(1.060)^{30}}\right] = 13.765$$

The present value of the cash payments is \$12.167 × 13.765 = \$167.5 million, much below the well-trumpeted prize, but still not a bad day's haul.

Lottery operators generally make arrangements for winners with big spending plans to take an equivalent lump sum. In our example the winners could either take the \$365 million spread over 30 years or receive \$167.5 million up front. Both arrangements had the same present value.

Valuing Annuities Due

When we used the annuity formula to value the Powerball lottery prize in Example 2.3, we presupposed that the first payment was made at the end of one year. In fact, the first of the 30 yearly payments was made immediately. How does this change the value of the prize?

If we discount each cash flow by one less year, the present value is increased by the multiple $(1 + r)$. In the case of the lottery prize the value becomes $167.5 \times (1 + r) = 167.5 \times 1.060 = \$177.5$ million.

A level stream of payments starting immediately is called an **annuity due.** An annuity due is worth $(1 + r)$ times the value of an ordinary annuity.

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Try It! More on annuities

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Calculating Annual Payments

Annuity problems can be confusing on first acquaintance, but you will find that with practice they are generally straightforward. For example, here is a case where you need to use the annuity formula to find the amount of the payment *given* the present value.

EXAMPLE 2.4 • Paying Off a Bank Loan

Bank loans are paid off in equal installments. Suppose that you take out a four-year loan of $1,000. The bank requires you to repay the loan evenly over the four years. It must therefore set the four annual payments so that they have a present value of $1,000. Thus,

$$\text{PV} = \text{annual loan payment} \times \text{4-year annuity factor} = \$1{,}000$$

$$\text{Annual loan payment} = \$1{,}000/\text{4-year annuity factor}$$

Suppose that the interest rate is 10% a year. Then

$$\text{4-year annuity factor} = \left[\frac{1}{.10} - \frac{1}{.10(1.10)^4}\right] = 3.17$$

and

$$\text{Annual loan payment} = 1{,}000/3.17 = \$315.47$$

Let's check that this annual payment is sufficient to repay the loan. Table 2.1 provides the calculations. At the end of the first year, the interest charge is 10% of $1,000, or $100. So $100 of the first payment is absorbed by interest, and the remaining $215.47 is used to reduce the loan balance to $784.53.

Year	Beginning-of-Year Balance	Year-end Interest on Balance	Total Year-end Payment	Amortization of Loan	End-of-Year Balance
1	$1,000.00	$100.00	$315.47	$215.47	$784.53
2	784.53	78.45	315.47	237.02	547.51
3	547.51	54.75	315.47	260.72	286.79
4	286.79	28.68	315.47	286.79	0

TABLE 2.1 An example of an amortizing loan. If you borrow $1,000 at an interest rate of 10%, you would need to make an annual payment of $315.47 over four years to repay that loan with interest.

Next year, the outstanding balance is lower, so the interest charge is only $78.45. Therefore $315.47 – $78.45 = $237.02 can be applied to paying off the loan. Because the loan is progressively paid off, the fraction of each payment devoted to interest steadily falls over time, while the fraction used to reduce the loan increases. By the end of year 4, the amortization is just enough to reduce the balance of the loan to zero.

Loans that involve a series of level payments are known as *amortizing loans.* "Amortizing" means that part of the regular payment is used to pay interest on the loan and part is used to reduce the amount of the loan.

EXAMPLE 2.5 • Calculating Mortgage Payments

Most mortgages are amortizing loans. For example, suppose that you take out a $250,000 house mortgage from your local savings bank when the interest rate is 12%. The bank requires you to repay the mortgage in equal annual installments over the next 30 years.

Thus,

$$\text{Annual mortgage payment} = \$250{,}000/\text{30-year annuity factor}$$

$$\text{30-year annuity factor} = \left[\frac{1}{.12} - \frac{1}{.12(1.12)^{30}}\right] = 8.055$$

and

$$\text{Annual mortgage payment} = 250{,}000/8.055 = \$31{,}036$$

Figure 2.9 shows that in the early years, almost all of the mortgage payment is eaten up by interest and only a small fraction is used to reduce the amount of the loan. Even after 15 years, the bulk of the annual payment goes to pay the interest on the loan. From then on, the amount of the loan begins to decline rapidly.

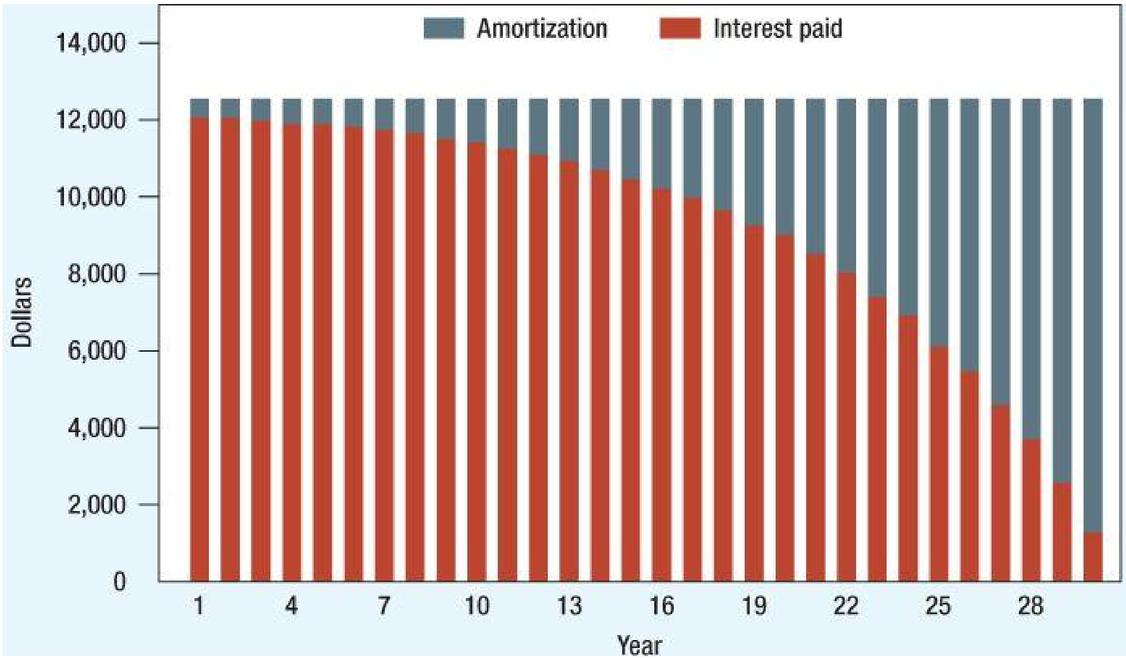

FIGURE 2.9 Mortgage amortization. This figure shows the breakdown of mortgage payments between interest and amortization.

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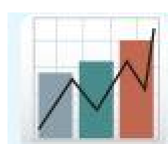
Try It! Figure 2.9: The amortzation schedule



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Future Value of an Annuity

Sometimes you need to calculate the *future* value of a level stream of payments.

EXAMPLE 2.6 • Saving to Buy a Sailboat

Perhaps your ambition is to buy a sailboat; something like a 40-foot Beneteau would fit the bill very well. But that means some serious saving. You estimate that, once you start work, you could save \$20,000 a year out of your income and earn a return of 8% on these savings. How much will you be able to spend after five years?

We are looking here at a level stream of cash flows—an annuity. We have seen that there is a shortcut formula to calculate the *present* value of an annuity. So there ought to be a similar formula for calculating the *future* value of a level stream of cash flows.

Think first how much your savings are worth today. You will set aside \$20,000 in each of the next five years. The present value of this five-year annuity is therefore equal to

$$\text{PV} = \$20{,}000 \times \text{5-year annuity factor}$$

$$= \$20{,}000 \times \left[\frac{1}{.08} - \frac{1}{.08(1.08)^5}\right] = \$79{,}854$$

Once you know today's value of the stream of cash flows, it is easy to work out its value in the future. Just multiply by $(1.08)^5$:

$$\text{Value at end of year 5} = \$79{,}854 \times 1.08^5 = \$117{,}332$$

You should be able to buy yourself a nice boat for \$117,000.

In Example 2.6 we calculate the future value of an annuity by first calculating its present value and then multiplying by $(1 + r)^t$. The general formula for the future value of a level stream of cash flows of \$1 a year for t years is, therefore,

$$\text{Future value of annuity} = \text{present value of annuity of \$1 a year} \times (1 + r)^t$$

$$= \left[\frac{1}{r} - \frac{1}{r(1 + r)^t}\right] \times (1 + r)^t = \frac{(1 + r)^t - 1}{r}$$

There is a general point here. If you can find the present value of *any* series of cash flows, you can always calculate future value by multiplying by $(1 + r)^t$:

$$\text{Future value at the end of year } t = \text{present value} \times (1 + r)^t$$

2-3 More Shortcuts—Growing Perpetuities and Annuities

Growing Perpetuities

You now know how to value level streams of cash flows, but you often need to value a stream of cash flows that grows at a constant rate. For example, think back to your plans to donate \$10 billion to fight malaria and other infectious diseases. Unfortunately, you made no allowance for the growth in salaries and other costs, which will probably average about 4% a year starting in year 1. Therefore, instead of providing \$1 billion a year in perpetuity, you must provide \$1 billion in year 1, 1.04 × \$1 billion in year 2, and so on. If we call the growth rate in costs g, we can write down the present value of this stream of cash flows as follows:

$$PV = \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} + \frac{C_3}{(1 + r)^3} + \cdots$$

$$= \frac{C_1}{1 + r} + \frac{C_1(1 + g)}{(1 + r)^2} + \frac{C_1(1 + g)^2}{(1 + r)^3} + \cdots$$

Fortunately, there is a simple formula for the sum of this geometric series.[6] If we assume that r is greater than g, our clumsy-looking calculation simplifies to

$$\text{Present value of growing perpetuity} = \frac{C_1}{r - g}$$

Therefore, if you want to provide a perpetual stream of income that keeps pace with the growth rate in costs, the amount that you must set aside today is

$$\text{PV} = \frac{C_1}{r - g} = \frac{\$1 \text{ billion}}{.10 - .04} = \$16.667 \text{ billion}$$

You will meet this perpetual-growth formula again in Chapter 4, where we use it to value the stock of mature, slowly growing companies.

Growing Annuities

You are contemplating membership in the St. Swithin's and Ancient Golf Club. The annual membership fee for the coming year is $5,000, but you can make a single payment today of $12,750, which will provide you with membership for the next three years. Which is the better deal? The answer depends on how rapidly membership fees are likely to increase over the three-year period. For example, suppose that the annual fee is payable at the end of each year and is expected to increase by 6% per annum. The discount rate is 10%.

The problem is to calculate the present value of the three-year stream of growing payments. The first payment occurs at the end of year 1 and is C = $5,000. Thereafter, the payments grow at the rate of $g = .06$ each year. Thus in year 2 the expected payment is $5,000 × 1.06, and in year 3 it is $5,000 × 1.06^2. Of course, you could calculate these cash flows and discount them at 10%. The alternative is to use the following formula for the present value of a growing annuity:[7]

Cash Flow, $							
Year:	0	1	2 . . .	. . . t − 1	t	t + 1 . . .	Present Value
Perpetuity		1	1 . . .	1	1	1 . . .	$\frac{1}{r}$
t-period annuity		1	1 . . .	1	1		$\frac{1}{r} - \frac{1}{r(1+r)^t}$
t-period annuity due	1	1	1 . . .	1			$(1+r)\left(\frac{1}{r} - \frac{1}{r(1+r)^t}\right)$
Growing perpetuity		1	$1 \times (1+g)$ . . .	$1 \times (1+g)^{t-2}$	$1 \times (1+g)^{t-1}$	$1 \times (1+g)^{t}$ . . .	$\frac{1}{r-g}$
t-period growing annuity		1	$1 \times (1+g)$ . . .	$1 \times (1+g)^{t-2}$	$1 \times (1+g)^{t-1}$		$\frac{1}{r-g}\left[1 - \frac{(1+g)^t}{(1+r)^t}\right]$

TABLE 2.2 Some useful shortcut formulas.

$$\text{PV of growing annuity} = C \times \frac{1}{r-g}\left[1 - \frac{(1+g)^t}{(1+r)^t}\right]$$

In our golf club example, the present value of the membership fees for the next three years is

$$\text{PV} = \$5{,}000 \times \frac{.1}{.10 - .06}\left[1 - \frac{(1.06)^3}{(1.10)^3}\right] = \$5{,}000 \times 2.629 = \$13{,}147$$

If you can find the cash, you would be better off paying now for a three-year membership.

Too many formulas are bad for the digestion. So we will stop at this point and spare you any more of them. The formulas discussed so far appear in Table 2.2.

2-4 How Interest Is Paid and Quoted

In our examples we have assumed that cash flows occur only at the end of each year. This is sometimes the case. For example, in France and Germany the government pays interest on its bonds annually. However, in the United States and Britain government bonds pay interest semiannually. So if the interest rate on a U.S. government bond is quoted as 10%, the investor in practice receives interest of 5% every six months.

If the first interest payment is made at the end of six months, you can earn an additional six months' interest on this payment. For example, if you invest \$100 in a bond that pays interest of 10% compounded semiannually, your wealth will grow to $1.05 \times \$100 = \$105$ by the end of six months and to $1.05 \times \$105 = \$110.25$ by the end of the year. In other words, an interest rate of 10% compounded semiannually is equivalent to 10.25% compounded annually. The *effective annual interest rate* on the bond is 10.25%.

Let's take another example. Suppose a bank offers you an automobile loan at an **annual percentage rate,** or **APR,** of 12% with interest to be paid monthly. This means that each month you need to pay one-twelfth of the annual rate, that is, 12/12 = 1% a month. Thus the bank is *quoting* a rate of 12%, but the effective annual interest rate on your loan is $1.01^{12} - 1 = .1268$, or 12.68%.[8]

Our examples illustrate that you need to distinguish between the *quoted* annual interest rate and the *effective* annual rate. The quoted annual rate is usually calculated as the total annual payment divided by the number of payments in the year. When interest is paid once a year, the quoted and effective rates are the same. When interest is paid more frequently, the effective interest rate is higher than the quoted rate.

In general, if you invest \$1 at a rate of r per year compounded m times a year, your investment at the end of the year will be worth $[1 + (r/m)]^m$ and the effective interest rate is $[1 + (r/m)]^m - 1$. In our automobile loan example $r = .12$ and $m = 12$. So the effective annual interest rate was $[1 + .12/12]^{12} - 1 = .1268$, or 12.68%.

Continuous Compounding

Instead of compounding interest monthly or semiannually, the rate could be compounded weekly ($m = 52$) or daily ($m = 365$). In fact there is no limit to how frequently interest could be paid. One can imagine a situation where the payments are spread evenly and continuously throughout the year, so the interest rate is continuously compounded.[9] In this case m is infinite.

It turns out that there are many occasions in finance when continuous compounding is useful. For example, one important application is in option pricing models, such as the Black–Scholes model that we introduce in Chapter 21. These are continuous time models. So you will find that most pay more for the continuous cash payments because the cash starts to flow in immediately.

Example 3 After you have retired, you plan to spend $200,000 a year for 20 years. The annually compounded interest rate is 10%. How much must you save by the time you retire to support this spending plan?

Let us first do the calculations assuming that you spend the cash at the end of each year. In this case we can use the simple annuity formula that we derived earlier:

$$\text{PV} = C\left(\frac{1}{r} - \frac{1}{r} \times \frac{1}{(1+r)^t}\right)$$

$$= \$200{,}000\left(\frac{1}{.10} - \frac{1}{.10} \times \frac{1}{(1.10)^{20}}\right) = \$200{,}000 \times 8.514 = \$1{,}702{,}800$$

Thus, you will need to have saved nearly $1¾ million by the time you retire.

Instead of waiting until the end of each year before you spend any cash, it is more reasonable to assume that your expenditure will be spread evenly over the year. In this case, instead of using the annually compounded rate of 10%, we must use the continuously compounded rate of $r = 9.53\%$ ($e^{.0953} = 1.10$). Therefore, to cover a steady stream of expenditure, you need to set aside the following sum:[10]

USEFUL SPREADSHEET FUNCTIONS

Discounting Cash Flows

Spreadsheet programs such as Excel provide built-in functions to solve discounted-cash-flow (DCF) problems. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel asks you for the inputs that it needs. At the bottom left of the function box there is a Help facility with an example of how the function is used.

computer programs for calculating option values ask for the continuously compounded interest rate.

It may seem that a lot of calculations would be needed to find a continuously compounded interest rate. However, think back to your high school algebra. You may recall that as m approaches infinity $[1 + (r/m)]^m$ approaches $(2.718)^r$. The figure 2.718—or e, as it is called—is the base for natural logarithms. Therefore, $1 invested at a continuously compounded rate of r will grow to $e^r = (2.718)^r$ by the end of the first year. By the end of t years it will grow to $e^{rt} = (2.718)^{rt}$.

Example 1 Suppose you invest $1 at a continuously compounded rate of 11% ($r = .11$) for one year ($t = 1$). The end-year value is $e^{.11}$, or $1.116. In other words, investing at 11% a year *continuously* compounded is exactly the same as investing at 11.6% a year *annually* compounded.

Example 2 Suppose you invest $1 at a continuously compounded rate of 11% ($r = .11$) for two years ($t = 2$). The final value of the investment is $e^{rt} = e^{.22}$, or $1.246.

Sometimes it may be more reasonable to assume that the cash flows from a project are spread evenly over the year rather than occurring at the year's end. It is easy to adapt our previous formulas to handle this. For example, suppose that we wish to compute the present value of a perpetuity of C dollars a year. We already know that if the payment is made at the end of the year, we divide the payment by the *annually* compounded rate of r:

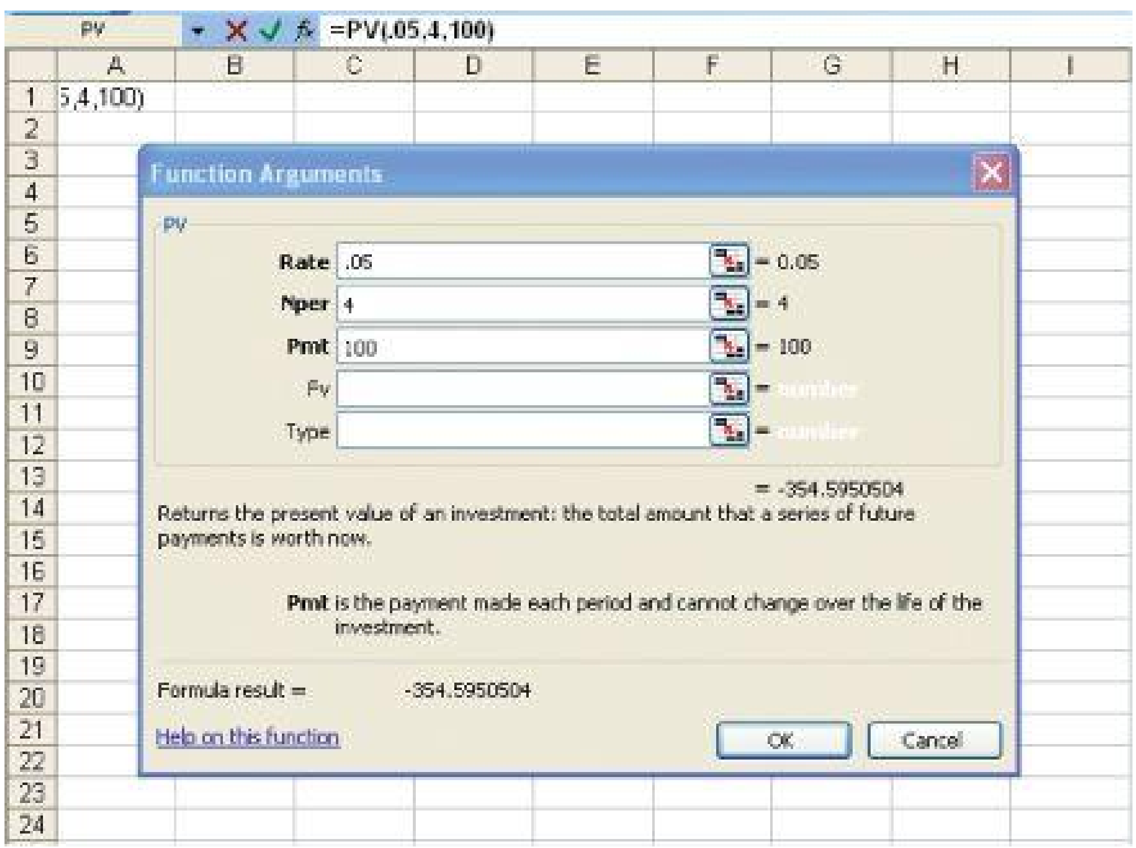
$$\text{PV} = \frac{C}{r}$$

If the same total payment is made in an even stream throughout the year, we use the same formula but substitute the *continuously* compounded rate.

Suppose the annually compounded rate is 18.5%. The present value of a $100 perpetuity, with each cash flow received at the end of the year, is 100/.185 = $540.54. If the cash flow is received continuously, we must divide $100 by 17%, because 17% continuously compounded is equivalent to 18.5% annually compounded ($e^{.17} = 1.185$). The present value of the continuous cash flow stream is 100/.17 = $588.24. Investors are prepared to

Here is a list of useful functions for DCF problems and some points to remember when entering data:

- **FV:** Future value of single investment or annuity.
- **PV:** Present value of single future cash flow or annuity.
- **RATE:** Interest rate (or rate of return) needed to produce given future value or annuity.
- **NPER:** Number of periods (e.g., years) that it takes an investment to reach a given future value or series of future cash flows.
- **PMT:** Amount of annuity payment with a given present or future value.
- **NPV:** Calculates the value of a stream of negative and positive cash flows. (When using this function, note the warning below.)
- **XNPV:** Calculates the net present value at the date of the first cash flow of a series of cash flows occurring at uneven intervals.
- **EFFECT:** The effective annual interest rate, given the quoted rate (APR) and number of interest payments in a year.



• **NOMINAL:** The quoted interest rate (APR) given the effective annual interest rate.

All the inputs in these functions can be entered directly as numbers or as the addresses of cells that contain the numbers.

Three warnings:

1. PV is the amount that needs to be invested today to produce a given future value. It should therefore be entered as a negative number. Entering both PV and FV with the same sign when solving for RATE results in an error message.

2. Always enter the interest or discount rate as a decimal value (e.g., .05 rather than 5%).

3. Use the NPV function with care. Better still, don't use it at all. It gives the value of the cash flows one period *before* the first cash flow and not the value at the date of the first cash flow.

Spreadsheet Questions

The following questions provide opportunities to practice each of the Excel functions.

1. (FV) In 1880 five aboriginal trackers were each promised the equivalent of 100 Australian dollars for helping to capture the notorious outlaw Ned Kelly. One hundred and thirteen years later the granddaughters of two of the trackers claimed that this reward had not been paid. If the interest rate over this period averaged about 4.5%, how much would the A$100 have accumulated to?

2. (PV) Your company can lease a truck for $10,000 a year (paid at the end of the year) for six years, or it can buy the truck today for $50,000. At the end of the six years the truck will be worthless. If the interest rate is 6%, what is the present value of the lease payments? Is the lease worthwhile?

3. (RATE) Ford Motor stock was one of the victims of the 2008 credit crisis. In June 2007, Ford stock price stood at $9.42. Eighteen months later it was $2.72. What was the annual rate of return over this period to an investor in Ford stock?

4. (NPER) An investment adviser has promised to double your money. If the interest rate is 7% a year, how many years will she take to do so?

5. (PMT) You need to take out a home mortgage for $200,000. If payments are made annually over 30 years and the interest rate is 8%, what is the amount of the annual payment?

6. (XNPV) Your office building requires an initial cash outlay of $370,000. Suppose that you plan to rent it out for three years at $20,000 a year and then sell it for $400,000. If the cost of capital is 12%, what is its net present value?

7. (EFFECT) Banque Nationale Supérieure pays 6.2% interest compounded annually. Banque Nationale Principale pays 6% interest compounded monthly. Which bank offers the higher effective annual interest rate?

8. (NOMINAL) What monthly compounded interest rate would Banque Nationale Principale need to pay on savings deposits to provide an effective rate of 6.2%?

$$\text{PV} = C\left(\frac{1}{r} - \frac{1}{r} \times \frac{1}{e^{rt}}\right)$$

$$= \$200{,}000\left(\frac{1}{.0953} - \frac{1}{.0953} \times \frac{1}{6.727}\right) = \$200{,}000 \times 8.932 = \$1{,}786{,}400$$

To support a steady stream of outgoings, you must save an additional $83,600.

Often in finance we need only a ballpark estimate of present value. An error of 5% in a present value calculation may be perfectly acceptable. In such cases it doesn't usually matter whether we assume that cash flows occur at the end of the year or in a continuous stream. At other times precision matters, and we do need to worry about the exact frequency of the cash flows.

SUMMARY

Firms can best help their shareholders by accepting all projects that are worth more than they cost. In other words, they need to seek out projects with positive net present values. To find net present value we first calculate present value. Just discount future cash flows by an appropriate rate r, usually called the *discount rate, hurdle rate, or opportunity cost of capital:*

$$\text{Present value(PV)} = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \cdots$$

Net present value is present value plus any immediate cash flow:

$$\text{Net present value(NPV)} = C_0 + \text{PV}$$

Remember that C_0 is negative if the immediate cash flow is an investment, that is, if it is a cash outflow.

The discount rate r is determined by rates of return prevailing in capital markets. If the future cash flow is absolutely safe, then the discount rate is the interest rate on safe securities such as U.S. government debt. If the future cash flow is uncertain, then the expected cash flow should be discounted at the expected rate of return offered by equivalent-risk securities. (We talk more about risk and the cost of capital in Chapters 7 to 9.)

Cash flows are discounted for two simple reasons: because (1) a dollar today is worth more than a dollar tomorrow and (2) a safe dollar is worth more than a risky one. Formulas for PV and NPV are numerical expressions of these ideas.

Financial markets, including the bond and stock markets, are the markets where safe and risky future cash flows are traded and valued. That is why we look to rates of return prevailing in the financial markets to determine how much to discount for time and risk. By calculating the present value of an asset, we are estimating how much people will pay for it if they have the alternative of investing in the capital markets.

You can always work out any present value using the basic formula, but shortcut formulas can reduce the tedium. We showed how to value an investment that makes a level stream of cash flows forever (a *perpetuity*) and one that produces a level stream for a limited period (an *annuity*). We also showed how to value investments that produce growing streams of cash flows.

When someone offers to lend you a dollar at a quoted interest rate, you should always check how frequently the interest is to be paid. For example, suppose that a $100 loan requires six-month payments of $3. The total yearly interest payment is $6 and the interest will be quoted as a rate of 6% compounded semiannually. The equivalent *annually compounded rate* is $(1.03)^2 - 1 = .061$, or 6.1%. Sometimes it is convenient to assume that interest is paid evenly over the year, so that interest is quoted as a continuously compounded rate.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect Finance*. Please see the preface for more information.

BASIC

1. Future values At an interest rate of 12%, the six-year discount factor is .507. How many dollars is $.507 worth in six years if invested at 12%?

2. Discount factors If the PV of $139 is $125, what is the discount factor?

3. Present values If the cost of capital is 9%, what is the PV of $374 paid in year 9?

4. Present values A project produces a cash flow of $432 in year 1, $137 in year 2, and $797 in year 3. If the cost of capital is 15%, what is the project's PV?

5. Futures values If you invest $100 at an interest rate of 15%, how much will you have at the end of eight years?

6. Perpetuities An investment costs $1,548 and pays $138 in perpetuity. If the interest rate is 9%, what is the NPV?

7. Growing perpetuities A common stock will pay a cash dividend of $4 next year. After that, the dividends are expected to increase indefinitely at 4% per year. If the discount rate is 14%, what is the PV of the stream of dividend payments?

8. Perpetuities and annuities The interest rate is 10%.

a. What is the PV of an asset that pays $1 a year in perpetuity?

b. The value of an asset that appreciates at 10% per annum approximately doubles in seven years. What is the approximate PV of an asset that pays $1 a year in perpetuity beginning in year 8?

c. What is the approximate PV of an asset that pays $1 a year for each of the next seven years?

d. A piece of land produces an income that grows by 5% per annum. If the first year's income is $10,000, what is the value of the land?

9. Future values and annuities

a. The cost of a new automobile is $10,000. If the interest rate is 5%, how much would you have to set aside now to provide this sum in five years?

b. You have to pay $12,000 a year in school fees at the end of each of the next six years. If the interest rate is 8%, how much do you need to set aside today to cover these bills?

c. You have invested $60,476 at 8%. After paying the above school fees, how much would remain at the end of the six years?

10. Continuous compounding The continuously compounded interest rate is 12%.

a. You invest $1,000 at this rate. What is the investment worth after five years?

b. What is the PV of $5 million to be received in eight years?

c. What is the PV of a continuous stream of cash flows, amounting to $2,000 per year, starting immediately and continuing for 15 years?

11. Compounding intervals You are quoted an interest rate of 6% on an investment of $10 million. What is the value of your investment after four years if interest is compounded:

a. Annually?

b. Monthly? or

c. Continuously?

INTERMEDIATE

12. Present values What is the PV of $100 received in:

a. Year 10 (at a discount rate of 1%)?

b. Year 10 (at a discount rate of 13%)?

c. Year 15 (at a discount rate of 25%)?

d. Each of years 1 through 3 (at a discount rate of 12%)?

13. Discount factors and present values

a. If the one-year discount factor is .905, what is the one-year interest rate?

b. If the two-year interest rate is 10.5%, what is the two-year discount factor?

c. Given these one- and two-year discount factors, calculate the two-year annuity factor.

d. If the PV of $10 a year for three years is $24.65, what is the three-year annuity factor?

e. From your answers to (c) and (d), calculate the three-year discount factor.

14. Present values A factory costs $800,000. You reckon that it will produce an inflow after operating costs of $170,000 a year for 10 years. If the opportunity cost of capital is 14%, what is the net present value of the factory? What will the factory be worth at the end of five years?

15. Present values A machine costs $380,000 and is expected to produce the following cash flows:

Year	1	2	3	4	5	6	7	8	9	10
Cash flow ($000s)	50	57	75	80	85	92	92	80	68	50

If the cost of capital is 12%, what is the machine's NPV?

16. Growing annuities ***Yasuo Obuchi*** is 30 years of age and his salary next year will be ***¥4,000,000***. ***He*** forecasts that his salary will increase at a steady rate of 5% per annum until his retirement at age 60.

a. If the discount rate is 8%, what is the PV of these future salary payments?

b. If Mr Obuchi saves 5% of his salary each year and invests these savings at an interest rate of 8%, how much will he have saved by age 60?

c. If he plans to spend these savings in even amounts over the subsequent 20 years, how much can he spend each year?

17. Present values A factory costs $400,000. It will produce an inflow after operating costs of $100,000 in year 1, $200,000 in year 2, and \$300,000 in year 3. The opportunity cost of capital is 12%. Calculate the NPV.

18. Present values Halcyon Lines is considering the purchase of a new bulk carrier for \$8 million. The forecasted revenues are \$5 million a year and operating costs are \$4 million. A major refit costing \$2 million will be required after both the fifth and tenth years. After 15 years, the ship is expected to be sold for scrap at \$1.5 million. If the discount rate is 8%, what is the ship's NPV?

19. Present values As winner of a breakfast cereal competition, you can choose one of the following prizes:

a. \$100,000 now.

b. \$180,000 at the end of five years.

c. \$11,400 a year forever.

d. \$19,000 for each of 10 years.

e. \$6,500 next year and increasing thereafter by 5% a year forever.

If the interest rate is 12%, which is the most valuable prize?

20. Annuities Siegfried Basset is 65 years of age and has a life expectancy of 12 more years. He wishes to invest \$20,000 in an annuity that will make a level payment at the end of each year until his death. If the interest rate is 8%, what income can Mr. Basset expect to receive each year?

21. Annuities David and Helen Zhang are saving to buy a boat at the end of five years. If the boat costs \$20,000 and they can earn 10% a year on their savings, how much do they need to put aside at the end of years 1 through 5?

22. Annuities Kangaroo Autos is offering free credit on a new \$10,000 car. You pay \$1,000 down and then \$300 a month for the next 30 months. Turtle Motors next door does not offer free credit but will give you \$1,000 off the list price. If the rate of interest is 10% a year (about .83% a month), which company is offering the better deal?

23. Present values Recalculate the NPV of the office building venture in Example 2.1 at interest rates of 5, 10, and 15%. Plot the points on a graph with NPV on the vertical axis and the discount rates on the horizontal axis. At what discount rate (approximately) would the project have zero NPV? Check your answer.

24. Perpetuities and continuous compounding If the interest rate is 7%, what is the value of the following three investments?

a. An investment that offers you $100 a year in perpetuity with the payment at the *end* of each year.

b. A similar investment with the payment at the *beginning* of each year.

c. A similar investment with the payment spread evenly over each year.

25. Perpetuities and annuities Refer back to Sections 2-3 through 2-4. If the rate of interest is 8% rather than 10%, how much would you need to set aside to provide each of the following?

a. $1 billion at the end of each year in perpetuity.

b. A perpetuity that pays $1 billion at the end of the first year and that grows at 4% a year.

c. $1 billion at the end of each year for 20 years.

d. $1 billion a year spread evenly over 20 years.

26. Continuous compounding How much will you have at the end of 20 years if you invest $100 today at 15% *annually* compounded? How much will you have if you invest at 15% *continuously* compounded?

27. Perpetuities You have just read an advertisement stating, "Pay us $100 a year for 10 years and we will pay you $100 a year thereafter in perpetuity." If this is a fair deal, what is the rate of interest?

28. Compounding intervals Which would you prefer?

a. An investment paying interest of 12% compounded annually.

b. An investment paying interest of 11.7% compounded semiannually.

c. An investment paying 11.5% compounded continuously.

Work out the value of each of these investments after 1, 5, and 20 years.

29. Compounding intervals A leasing contract calls for an immediate payment of $100,000 and nine subsequent $100,000 semiannual payments at six-month intervals. What is the PV of these payments if the *annual* discount rate is 8%?

30. Annuities Several years ago *The Wall Street Journal* reported that the winner of the Massachusetts State Lottery prize had the misfortune to be both bankrupt and in prison for fraud. The prize was $9,420,713, to be paid in 19 equal annual installments. (There were 20 installments, but the winner had already received the first payment.) The bankruptcy court judge ruled that the prize should be sold off to the highest bidder and the proceeds used to pay off the creditors.

a. If the interest rate was 8%, how much would you have been prepared to bid for the prize?

b. Enhance Reinsurance Company was reported to have offered $4.2 million. Use Excel to find the return that the company was looking for.

31. Amortizing loans A mortgage requires you to pay $70,000 at the end of each of the next eight years. The interest rate is 8%.

a. What is the present value of these payments?

b. Calculate for each year the loan balance that remains outstanding, the interest payment on the loan, and the reduction in the loan balance.

32. Growing annuities You estimate that by the time you retire in 35 years, you will have accumulated savings of $2 million. If the interest rate is 8% and you live 15 years after retirement, what annual level of expenditure will those savings support?

Unfortunately, inflation will eat into the value of your retirement income. Assume a 4% inflation rate and work out a spending program for your retirement that will allow you to increase your expenditure in line with inflation.

33. Annuities The *annually* compounded discount rate is 5.5%. You are asked to calculate the present value of a 12-year annuity with payments of $50,000 per year. Calculate PV for each of the following cases.

a. The annuity payments arrive at one-year intervals. The first payment arrives one year from now.

b. The first payment arrives in six months. Following payments arrive at one-year intervals (i.e., at 18 months, 30 months, etc.).

34. Annuities Dear Financial Adviser,

My spouse and I are each 62 and hope to retire in three years. After retirement we will receive $7,500 per month after taxes from our employers' pension plans and $1,500 per month after taxes from Social Security. Unfortunately our monthly living expenses are $15,000. Our social obligations preclude further economies.

We have $1,000,000 invested in a high-grade, tax-free municipal-bond mutual fund. The return on the fund is 3.5% per year. We plan to make annual withdrawals from the mutual fund to cover the difference between our pension and Social Security income and our living expenses. How many years before we run out of money?

Sincerely,

Luxury Challenged

Marblehead, MA

You can assume that the withdrawals (one per year) will sit in a checking account (no interest). The couple will use the account to cover the monthly shortfalls.

35. Present values Your firm's geologists have discovered a small oil field in central Hong Kong. The field is forecasted to produce a cash flow of C_1 = $HK20 million in the first year. You estimate that you could earn an expected return of r = 12% from investing in stocks with a similar degree of risk to your oil field. Therefore, 12% is the opportunity cost of capital.

What is the present value? The answer, of course, depends on what happens to the cash flows after the first year. Calculate present value for the following cases:

a. The cash flows are forecasted to continue forever, with no expected growth or decline.

b. The cash flows are forecasted to continue for 20 years only, with no expected growth or decline during that period.

c. The cash flows are forecasted to continue forever, increasing by 3% per year because of inflation.

d. The cash flows are forecasted to continue for 20 years only, increasing by 3% per year because of inflation.

36. Amortizing loans Suppose that you take out a $200,000, 20-year mortgage loan to buy a condo. The interest rate on the loan is 6%, and payments on the loan are made annually at the end of each year.

a. What is your annual payment on the loan?

b. Construct a mortgage amortization table in Excel similar to Table 2.1, showing the interest payment, the amortization of the loan, and the loan balance for each year.

c. What fraction of your initial loan payment is interest? What about the last payment? What fraction of the loan has been paid off after 10 years? Why is the fraction less than half?

37. PV of annuities at high discount rates Interest rates in developed countries are mostly in single digits, but double-digit rates are common in developing countries. In October 2012, for example, interest rates on 5-year government bonds were about 14.5% in Egypt and 16% in Nigeria. If you skip ahead to Fig. 27-4, you will see that high inflation pushed the short-term interest rate in the Ukraine up to 16%. Thus it's useful to see how much PVs fall when interest rates climb from, say, 5% to 15%. Calculate the PVs of the following cash-flow streams at these two interest rates.

a. A 10-year annuity of $100 per year.

b. A 10-year annuity due of $100 per year (first cash flow arrives immediately). How does the switch from a standard annuity to an annuity due increase PV if the interest rate is 15% instead of 5%?

c. A 20-year annuity starting at $100 per year but growing at 5% per year.

d. A 20-year annuity starting at $100 per year but declining at 5% per year.

CHALLENGE

38. Future values and continuous compounding Here are two useful rules of thumb. The "Rule of 72" says that with discrete compounding the time it takes for an investment to double in value is roughly 72/interest rate (in percent). The "Rule of 69" says that with continuous compounding the time that it takes to double is exactly 69.3/interest rate (in percent).

a. If the annually compounded interest rate is 12%, use the Rule of 72 to calculate roughly how long it takes before your money doubles. Now work it out exactly.

b. Can you prove the Rule of 69?

39. Annuities Use Excel to construct your own set of annuity tables showing the annuity factor for a selection of interest rates and years.

40. Declining perpetuities and annuities You own an oil pipeline that will generate a $2 million cash return over the coming year. The pipeline's operating costs are negligible, and it is expected to last for a very long time. Unfortunately, the volume of oil shipped is declining, and cash flows are expected to decline by 4% per year. The discount rate is 10%.

a. What is the PV of the pipeline's cash flows if its cash flows are assumed to last forever?

b. What is the PV of the cash flows if the pipeline is scrapped after 20 years?

FINANCE ON THE WEB

Finance.yahoo.com is a marvelous source of stock price data. You should get used to using it.

1. Go to **finance.yahoo.com** and look up the most recent stock prices and the prices five years ago for Amazon (AMZN), Microsoft (MSFT), Google (GOOG), and Apple (AAPL). What was the compound rate of growth in the price of each stock over the five-year period? If each price continues to grow at the same rate for the next five years, what will be the price at the end of that period?

2. You need to have accumulated savings of $2 million by the time that you retire in 20 years. You currently have savings of $200,000. How much do you need to save each year to meet your goal? Find the savings calculator on **finance.yahoo.com** to check your answer.

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[1]You sometimes hear lay people refer to "net present value" when they mean "present value," and vice versa. Just remember, *present value* is the value of the investment today; *net present value* is the addition that the investment makes to your wealth.

[2]We define "expected" more carefully in Chapter 9. For now think of expected payoff as a realistic forecast, neither optimistic nor pessimistic. Forecasts of expected payoffs are correct on average.

[3]You might check for yourself that these are equivalent rules. In other words, if the return of $100,000/$700,000 is greater than r, then the net present value – $700,000 + [$800,000/(1 + r)] *must* be greater than 0.

[4]You can check this by writing down the present value formula

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots$$

Now let $C/(1 + r) = a$ and $1/(1 + r) = x$. Then we have (1) $PV = a(1 + x + x^2 + \ldots)$. Multiplying both sides by x, we have (2) $PV_x = a(x + x^2 + \ldots)$.

Subtracting (2) from (1) gives us PV$(1 - x) = a$. Therefore, substituting for a and x,

$$\text{PV}\left(1 - \frac{1}{1 + r}\right) = \frac{C}{1 + r}$$

Multiplying both sides by $(1 + r)$ and rearranging gives

$$\text{PV} = \frac{C}{r}$$

[5]Some people find the following equivalent formula more intuitive:

$$\text{Present value of annuity} = \frac{1}{r} \times \left[1 - \frac{1}{(1 + r)^t}\right]$$

perpetuity formula — $1 starting next year — minus $1 starting at $t + 1$

[6]We need to calculate the sum of an infinite geometric series PV $= a(1 + x + x^2 + \cdot\cdot\cdot)$ where $a = C_1/(1 + r)$ and $x = (1 + g)/(1 + r)$. In footnote 4 we showed that the sum of such a series is $a/(1 - x)$. Substituting for a and x in this formula,

$$\text{PV} = \frac{C_1}{r - g}$$

[7]We can derive the formula for a growing perpetuity by taking advantage of our earlier trick of finding the difference between the values of two perpetuities. Imagine three investments (A, B, and C) that make the following dollar payments:

Year	1	2	3	4	5	6	. . .
A	$1	$(1 + g)$	$(1 + g)^2$	$(1 + g)^3$	$(1 + g)^4$	$(1 + g)^5$	etc.
B				$(1 + g)^3$	$(1 + g)^4$	$(1 + g)^5$	etc.
C	$1	$(1 + g)$	$(1 + g)^2$				

Investments A and B are growing perpetuities; A makes its first payment of \$1 in year 1, while B makes its first payment of $\$(1 + g)^3$ in year 4. C is a three-year growing annuity; its cash flows are equal to the difference between the cash flows of A and B. You know how to value growing perpetuities such as A and B. So you should be able to derive the formula for the value of growing annuities such as C:

$$\text{PV(A)} = \frac{1}{(r-g)}$$

$$\text{PV(B)} = \frac{(1+g)^3}{(r-g)} \times \frac{1}{(1+r)^3}$$

So

$$\text{PV(C)} = \text{PV(A)} - \text{PV(B)} = \frac{1}{(r-g)} - \frac{(1+g)^3}{(r-g)} \times \frac{1}{(1+r)^3} = \frac{1}{r-g}\left[1 - \frac{(1+g)^3}{(1+r)^3}\right]$$

If $r = g$, then the formula blows up. In that case, the cash flows grow at the same rate as the amount by which they are discounted. Therefore, each cash flow has a present value of $C/(1 + r)$ and the total present value of the annuity equals $t \times C/(1 + r)$. If $r < g$, then this particular formula remains valid.

[8]In the U.S., truth-in-lending laws oblige the company to quote an APR that is calculated by multiplying the payment each period by the number of payments in the year. APRs are calculated differently in other countries. For example, in the European Union APRs must be expressed as annually compounded rates, so consumers know the effective interest rate that they are paying.

[9]When we talk about *continuous* payments, we are pretending that money can be dispensed in a continuous stream like water out of a faucet. One can never quite do this. For example, instead of paying out \$1 billion every year to combat malaria, you could pay out about \$1 million every 8¾ hours or \$10,000 every 5¼ minutes or \$10 every 31/6 seconds but you could not pay it out *continuously.* Financial managers *pretend* that payments are continuous rather than hourly, daily, or weekly because (1) it simplifies the calculations and (2) it gives a very close approximation to the NPV of frequent payments.

[10]Remember that an annuity is simply the difference between a perpetuity received today and a perpetuity received in year *t.* A continuous stream of *C* dollars a year in perpetuity is worth *C*/*r,* where *r* is the continuously compounded rate. Our annuity, then, is worth

$$\text{PV} = \frac{C}{r} - \text{present value of } \frac{C}{r} \text{ received in year } t$$

Since *r* is the continuously compounded rate, *C*/*r* received in year *t* is worth $(C/r) \times (1/e^{rt})$ today. Our annuity formula is therefore

$$\text{PV} = \frac{C}{r} - \frac{C}{r} \times \frac{1}{e^{rt}}$$

sometimes written as

$$\frac{C}{r}(1 - e^{-rt})$$