

## Discrete Probability Distributions

### DSBA

(1) What is the first big change that Indian drivers made when gas prices significantly increased? According to an Access India survey, 30% said that it was cutting recreational driving. However, 27% said that it was consolidating or reducing errands. If these figures are true for all Indian drivers, and if 20 such drivers are randomly sampled and asked what is the first big change they made due to higher gas prices,

- a. What is the probability that exactly 8 said that it was consolidating or reducing errands?
- b. What is the probability that none of them said that it was cutting recreational driving?
- c. What is the probability that more than 7 said that it was cutting recreational driving?

(2) *The Wall Street Journal* reported some interesting statistics on the job market. One statistic is that 40% of all workers say they would change jobs for “slightly higher pay.” In addition, 88% of companies say that there is a shortage of qualified job candidates. Suppose 16 workers are randomly selected and asked if they would change jobs for “slightly higher pay.”

- a. What is the probability that 9 or more say yes?
- b. What is the probability that 3, 4, 5, or 6 say yes?
- c. If 13 companies are contacted, what is the probability that exactly 10 say there is a shortage of qualified job candidates?
- d. If 13 companies are contacted, what is the probability that all of the companies say there is a shortage of qualified job candidates?
- e. If 13 companies are contacted, what is the expected number of companies that would say there is a shortage of qualified job candidates?

(3) An increasing number of consumers believe they have to look out for themselves in the marketplace. According to a survey conducted by the ORG-MARG for *Bharat Weekend* magazine, 60% of all consumers have called an 800 or 900 telephone number for information about some product. Suppose a random sample of 25 consumers is contacted and interviewed about their buying habits.

- a. What is the probability that 15 or more of these consumers have called an 800 or 900 telephone number for information about some product?

b. What is the probability that more than 20 of these consumers have called an 800 or 900 telephone number for information about some product?

c. What is the probability that fewer than 10 of these consumers have called an 800 or 900 telephone number for information about some product?

(4) Studies have shown that about half of all workers who change jobs cash out their EPF plans rather than leaving the money in the account to grow. The percentage is much higher for workers with small EPF balances. In fact, 87% of workers with EPF accounts less than \$5,000 opt to take their balance in cash rather than roll it over into individual retirement accounts when they change jobs.

a. Assuming that 50% of all workers who change jobs cash out their EPF plans, if 16 workers who have recently changed jobs that had EPF plans are randomly sampled, what is the probability that more than 10 of them cashed out their EPF plan?

b. If 10 workers who have recently changed jobs and had EPF plans with accounts less than \$5,000 are randomly sampled, what is the probability that exactly 6 of them cashed out?

(5) On Monday mornings, the First National Bank has only one teller window open for deposits and withdrawals. Experience has shown that the average number of arriving customers in a four-minute interval on Monday mornings is 2.8, and each teller can serve more than that number efficiently. These random arrivals at this bank on Monday mornings are Poisson distributed.

a. What is the probability that on a Monday morning exactly six customers will arrive in a four-minute interval?

b. What is the probability that no one will arrive at the bank to make a deposit or withdrawal during a four-minute interval?

c. Suppose the teller can serve no more than four customers in any four-minute interval at this window on a Monday morning. What is the probability that, during any given four-minute interval, the teller will be unable to meet the demand? What is the probability that the teller will be able to meet the demand? When demand cannot be met during any given interval, a second window is opened. What percentage of the time will a second window have to be opened?

d. What is the probability that exactly three people will arrive at the bank during a two-minute period on Monday mornings to make a deposit or a withdrawal? What is the probability that five or more customers will arrive during an eight-minute period?

(6) A restaurant manager is interested in taking a more statistical approach to predicting customer load. She begins the process by gathering data. One of the restaurant hosts or hostesses is assigned to count customers every 5 minutes from 7 P.M. until 8 P.M. every

Saturday night for three weeks. The data are shown here. After the data are gathered, the manager computes lambda using the data from all three weeks as one data set as a basis for probability analysis. What value of lambda did she find? Assume that these customers randomly arrive and that the arrivals are Poisson distributed. Use the value of lambda computed by the manager and help the manager calculate the following probabilities for any given 5-minute interval between 7 P.M. and 8 P.M. on Saturday night.

- a. What is the probability that no customers arrive during any given 5-minute interval? b. What is the probability that six or more customers arrive during any given 5-minute interval?
- c. What is the probability that during a 10-minute interval fewer than four customers arrive?
- d. What is the probability that between three and six (inclusive) customers arrive in any 10-minute interval?
- e. What is the probability that exactly eight customers arrive in any 15-minute interval?

(7) According to the United National Environmental Program and World Health Organization, in Mumbai, India, air pollution standards for particulate matter are exceeded an average of 5.6 days in every three-week period. Assume that the distribution of number of days exceeding the standards per three-week period is Poisson distributed.

- a. What is the probability that the standard is not exceeded on any day during a three-week period?
- b. What is the probability that the standard is exceeded exactly 6 days of a three-week period?
- c. What is the probability that the standard is exceeded 15 or more days during a three-week period? If this outcome actually occurred, what might you conclude?