



Multi-Criterion Decision Analysis (MCDA)

Aditya K. Sahu
IIM Rohtak

Example 2: Multi-Criterion Decision Analysis

- You want to go on a vacation.
- You have 3 options

Hogwarts



Hogsmeade



Azkaban



How to decide..???

Example 2: Multi-Criterion Decision Analysis

- Each option can be evaluated against certain criteria.
- Criteria for vacation destinations can be:
 - Entertainment
 - Facilities
 - Accommodation cost
 - Travel cost

Multi-Criterion Decision Analysis

Definition: Selection of the best, from a set of alternatives, each of which is evaluated against multiple criteria.

Some problem solving techniques are :

- Scoring Model
- AHP (The Analytical Hierarchy Process)
- TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution)
- DEA (Data Envelopment Analysis)

Example

A graduating college student with a double major in Finance and Accounting has received the following three job offers:

- financial analyst for an investment firm in Chicago
- accountant for a manufacturing firm in Denver
- auditor for a CPA firm in Houston



Example

- The student made the following comments:
 - “The financial analyst position provides the best opportunity for my long-run career advancement.”
 - “I would prefer living in Denver rather than in Chicago or Houston.”
 - “I like the management style and philosophy at the Houston CPA firm the best.”
- Clearly, this is a multicriteria decision.



Example



- Considering only the long-run career advancement criterion:
 - The financial analyst position in Chicago is the best decision alternative.
- Considering only the location criterion:
 - The accountant position in Denver is the best decision alternative.
- Considering only the style criterion:
 - The auditor position in Houston is the best alternative.



Scoring Model

Scoring Model

- Step 1: List the decision-making criteria.
- Step 2: Assign a weight to each criterion.
- Step 3: Rate how well each decision alternative satisfies each criterion.
- Step 4: Compute the score for each decision alternative.
- Step 5: Order the decision alternatives from highest score to lowest score. The alternative with the highest score is the recommended alternative.

Scoring Model for Job Selection

- Mathematical Model

$$S_j = \sum_i w_i r_{ij}$$

where:

r_{ij} = rating for criterion i and decision alternative j

S_j = score for decision alternative j

Scoring Model: Step 1



1) List the decision-making criteria.

- Career advancement
- Location
- Management
- Salary
- Prestige
- Job Security
- Enjoyable work

Scoring Model: Step 2



2) Assign a weight to each criterion.

Five-point scale chosen

<u>Importance</u>	<u>Weight</u>
Very unimportant	1
Somewhat unimportant	2
Average importance	3
Somewhat important	4
Very important	5

Scoring Model: Step 2



2) Assigning a Weight to Each Criterion.

<u>Criterion</u>	<u>Importance</u>	<u>Weight</u>
Career advancement	Very important	5
Location	Average importance	3
Management	Somewhat important	4
Salary	Average importance	3
Prestige	Somewhat unimportant	2
Job security	Somewhat important	4
Enjoyable work	Very important	5

Scoring Model: Step 3



3) Rate how well each decision alternative satisfies each criterion.

■ List of Alternatives

- Analyst in Chicago
- Accountant in Denver
- Auditor in Houston

Scoring Model: Step 3



3) Rate how well each decision alternative satisfies each criterion.

Nine-Point Scale Chosen

<u>Level of Satisfaction</u>	<u>Rating</u>
Extremely low	1
Very low	2
Low	3
Slightly low	4
Average	5
Slightly high	6
High	7
Very high	8
Extremely high	9

Scoring Model: Step 3



3) Rate how well each decision alternative satisfies each criterion.

<u>Criterion</u>	<u>Decision Alternative</u>		
	<u>Analyst Chicago</u>	<u>Accountant Denver</u>	<u>Auditor Houston</u>
Career advancement	8	6	4
Location	3	8	7
Management	5	6	9
Salary	6	7	5
Prestige	7	5	4
Job security	4	7	6
Enjoyable work	8	6	5

Scoring Model: Step 4



4) Compute the score for each decision alternative.

Decision Alternative 1 - Analyst in Chicago

<u>Criterion</u>	<u>Weight (w_i)</u>		<u>Rating (r_{i1})</u>		<u>$w_i r_{i1}$</u>
Career advancement	5	x	8	=	40
Location	3		3		9
Management	4		5		20
Salary	3		6		18
Prestige	2		7		14
Job security	4		4		16
Enjoyable work	5		8		40
Score					<u>157</u>

Scoring Model: Step 4



4) Compute the score for each decision alternative.

$$s_j = \sum_i w_i r_{ij}$$

$$S_1 = 5(8) + 3(3) + 4(5) + 3(6) + 2(7) + 4(4) + 5(8) = 157$$

$$S_2 = 5(6) + 3(8) + 4(6) + 3(7) + 2(5) + 4(7) + 5(6) = 167$$

$$S_3 = 5(4) + 3(7) + 4(9) + 3(5) + 2(4) + 4(6) + 5(5) = 149$$

Scoring Model: Step 4



4) Compute the score for each decision alternative.

<u>Criterion</u>	<u>Decision Alternative</u>		
	<u>Analyst Chicago</u>	<u>Accountant Denver</u>	<u>Auditor Houston</u>
Career advancement	40	30	20
Location	9	24	21
Management	20	24	36
Salary	18	21	15
Prestige	14	10	8
Job security	16	28	24
Enjoyable work	40	30	25
Score	<hr/> 157	<hr/> 167	<hr/> 149

Scoring Model: Step 5



- Order the decision alternatives from highest score to lowest score. The alternative with the highest score is the recommended alternative.
 - The accountant position in Denver has the highest score and is the recommended decision alternative.
 - Note that the analyst position in Chicago ranks first in 4 of 7 criteria compared to only 2 of 7 for the accountant position in Denver.
 - But when the weights of the criteria are considered, the Denver position is superior to the Chicago job.



Analytic Hierarchy Process

Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP), is a procedure designed to quantify managerial judgments of the relative importance of each of several conflicting criteria used in the decision making process.

Analytic Hierarchy Process

- Step 1: List the Overall Goal, Criteria, and Decision Alternatives

----- For each criterion, perform steps 2 through 5 -----

- Step 2: Develop a Pair-wise Comparison Matrix

Rate the relative importance between each pair of decision alternatives. The matrix lists the alternatives horizontally and vertically and has the numerical ratings comparing the horizontal (first) alternative with the vertical (second) alternative.

Ratings are given as follows:

. . . continued

Analytic Hierarchy Process

- Step 2: Pair-wise Comparison Matrix (continued)

<u>Compared to the second alternative, the first alternative is:</u>	<u>Numerical rating</u>
extremely preferred	9
very strongly preferred	7
strongly preferred	5
moderately preferred	3
equally preferred	1

Analytic Hierarchy Process

- Step 2: Pair-wise Comparison Matrix (continued)

Intermediate numeric ratings of 8, 6, 4, 2 can be assigned. A reciprocal rating (i.e. $1/9$, $1/8$, etc.) is assigned when the second alternative is preferred to the first. The value of 1 is always assigned when comparing an alternative with itself.

Analytic Hierarchy Process

■ Step 3: Develop a Normalized Matrix

Divide each number in a column of the pair-wise comparison matrix by its column sum.

- $\alpha_{ij} = S_{ij} / \sum(S_{ij})$

- $\alpha_{ij} = S_{ij} / \max(S_{ij})$

- $\alpha_{ij} = (S_{ij} - \min S_{ij}) / (\max S_{ij} - \min S_{ij})$

- $\alpha_{ij} = S_{ij} / \sqrt{\sum(S_{ij})^2}$

■ Step 4: Develop the Priority Vector

Average each row of the normalized matrix. These row averages form the priority vector of alternative preferences with respect to the particular criterion. The values in this vector sum to 1.

Analytic Hierarchy Process

- Step 5: Calculate a Consistency Ratio

The consistency of the subjective input in the pair-wise comparison matrix can be measured by calculating a consistency ratio. A consistency ratio of less than 0.1 is good. For ratios which are greater than 0.1, the subjective input should be re-evaluated.

----- For each criterion, perform steps 2 through 5 -----

Checking Consistency

- A key step in AHP is the making of several pairwise comparisons.
- An important consideration in this process is the consistency of the pairwise judgments provided by the decision maker.
- For example, if criterion A compared to criterion B has a numerical rating of 3 and if criterion B compared to criterion C has a numerical rating of 2, perfect consistency of criterion A compared to criterion C would have a numerical rating of $(3 \times 2) = 6$.
- If the A to C numerical rating assigned by the decision maker was 4 or 5, some inconsistency would exist among the pairwise comparison.

Checking Consistency

- With numerous pairwise comparisons, perfect consistency is difficult to achieve.
- In fact, some degree of inconsistency can be expected to exist in almost any set of pairwise comparisons.
- To handle the consistency issue, AHP provides a method for measuring the degree of consistency among the pairwise comparisons provided by the decision maker.
- If the degree of consistency is unacceptable, the decision maker should review and revise the pairwise comparisons before proceeding with the AHP analysis.

Checking Consistency

- AHP provides a measure of the consistency for the pairwise comparisons by computing a consistency ratio.
- This ratio is designed in such a way that a value greater than 0.10 indicates an inconsistency in the pairwise judgments.
- Thus, if the consistency ratio is 0.10 or less, the consistency of the pairwise comparisons is considered reasonable, and the AHP process can continue with the synthesization computations.

Determining the Consistency Ratio

- Step i:

For each row of the pair-wise comparison matrix, determine a weighted sum by summing the multiples of the entries by the priority of its corresponding (column) alternative.

- Step ii:

For each row, divide its weighted sum by the priority of its corresponding (row) alternative.

- Step iii:

Determine the average, λ_{\max} , of the results of step ii.

- Step iv:

Compute the consistency index, CI, of the n alternatives by: $CI = (\lambda_{\max} - n) / (n - 1)$.

Determining the Consistency Ratio

- Step v:

Determine the random index, RI, as follows:

<u>Number of Alternative (n)</u>	<u>Random Index (RI)</u>	<u>Number of Alternative (n)</u>	<u>Random Index (RI)</u>
3	0.58	6	1.24
4	0.90	7	1.32
5	1.12	8	1.41

- Step vi:

Compute the consistency ratio: $CR = CI/RI$.

Note: RI: Consistency index of a randomly generated pair-wise comparison matrix. The value of RI depends on the number of items being compared.

Analytic Hierarchy Process

- Step 6: Develop a Priority Matrix

After steps 2 through 5 has been performed for all criteria, the results of step 4 are summarized in a priority matrix by listing the decision alternatives horizontally and the criteria vertically. The column entries are the priority vectors for each criterion.

Analytic Hierarchy Process

- **Step 7: Develop a Priority Vector for Criteria**

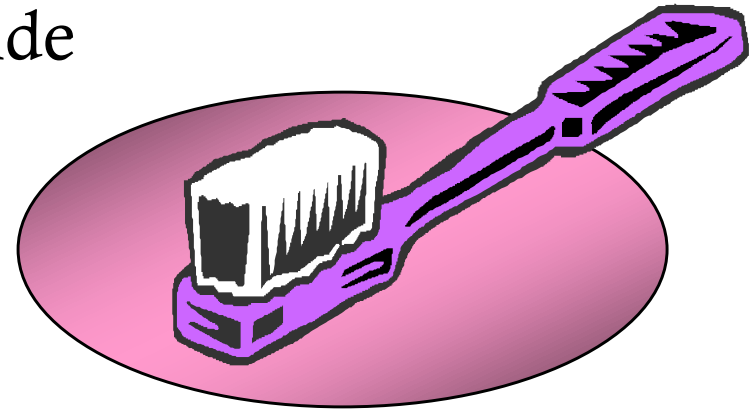
This is done in the same manner as that used to construct alternative pair-wise comparison matrices by using subjective ratings (step 2). Similarly, normalize the matrix (step 3) and develop a criteria priority vector (step 4).

- **Step 8: Develop an Overall Priority Vector**

Multiply the criteria priority vector (from step 7) by the priority matrix (from step 6).

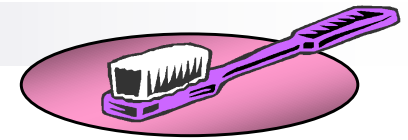
Example: Gill Glass

Designer Gill Glass must decide which of three manufacturers will develop his "signature" toothbrushes. Three factors are important to Gill: (1) his costs; (2) reliability of the product; and, (3) delivery time of the orders.



The three manufacturers are Cornell Industries, Brush Pik, and Picobuy. Cornell Industries will sell toothbrushes to Gill Glass for \$100 per gross, Brush Pik for \$80 per gross, and Picobuy for \$144 per gross.

Example: Gill Glass



■ Hierarchy for the Manufacturer Selection Problem

Overall Goal

Select the Best Toothbrush Manufacturer

Criteria

Cost

Reliability

Deliv. Time

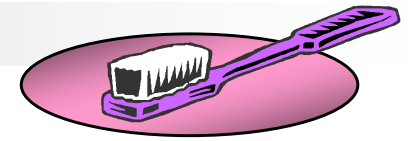
Decision
Alternatives

Cornell
Brush Pik
Picobuy

Cornell
Brush Pik
Picobuy

Cornell
Brush Pik
Picobuy

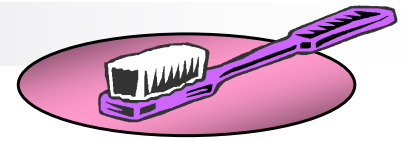
Pairwise Comparison Matrix: Cost



Gill has decided that in terms of price, Brush Pik is moderately preferred to Cornell and very strongly preferred to Picobuy. In turn Cornell is strongly to very strongly preferred to Picobuy.

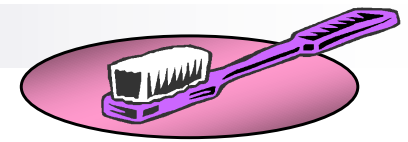
	Cornell	Brush Pik	Picobuy
Cornell	1	1/3	6
Brush Pik	3	1	7
Picobuy	1/6	1/7	1

Pairwise Comparison Matrix: Cost



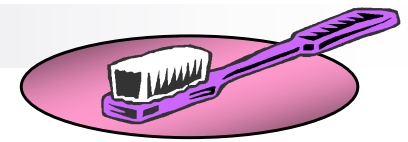
- Since Brush Pik is moderately preferred to Cornell, Cornell's entry in the Brush Pik row is 3 and Brush Pik's entry in the Cornell row is $1/3$.
- Since Brush Pik is very strongly preferred to Picobuy, Picobuy's entry in the Brush Pik row is 7 and Brush Pik's entry in the Picobuy row is $1/7$.
- Since Cornell is strongly to very strongly preferred to Picobuy, Picobuy's entry in the Cornell row is 6 and Cornell's entry in the Picobuy row is $1/6$.

Pairwise Comparison Matrix: Cost



	Cornell	Brush Pik	Picobuy
Cornell	1	1/3	6
Brush Pik	3	1	7
Picobuy	1/6	1/7	1

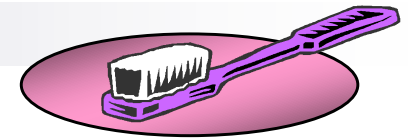
Normalized Matrix: Cost



Divide each entry in the pair-wise comparison matrix by its corresponding column sum. For example, for Cornell the column sum = $1 + 3 + 1/6 = 25/6$. This gives:

	Cornell	Brush Pik	Picobuy
Cornell	$6/25$	$7/31$	$6/14$
Brush Pik	$18/25$	$21/31$	$7/14$
Picobuy	$1/25$	$3/31$	$1/14$

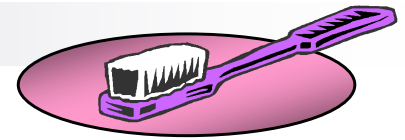
Priority Vector: Cost



The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

$$\begin{array}{l} \text{Cornell:} \quad (6/25 + 7/31 + 6/14)/3 = \left[.298 \right] \\ \text{Brush Pik:} \quad (18/25 + 21/31 + 7/14)/3 = \left[.632 \right] \\ \text{Picobuy:} \quad (1/25 + 3/31 + 1/14)/3 = \left[.069 \right] \end{array}$$

Checking Consistency



- **Step i:** Multiply each column of the pair-wise comparison matrix by its priority:

$$.298 \begin{bmatrix} 1 \\ 3 \\ 1/6 \end{bmatrix} + .632 \begin{bmatrix} 1/3 \\ 1 \\ 1/7 \end{bmatrix} + .069 \begin{bmatrix} 6 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} .923 \\ 2.009 \\ .209 \end{bmatrix}$$

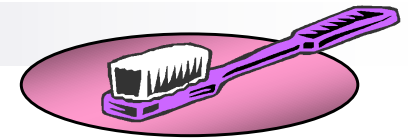
- **Step ii:** Divide these number by their priorities:

$$.923 / .298 = 3.097$$

$$2.009 / .632 = 3.179$$

$$.209 / .069 = 3.029$$

Checking Consistency



- **Step iii:** Compute the average of the values found in step ii. This average is denoted by λ_{\max} .

$$\lambda_{\max} = (3.097 + 3.179 + 3.029)/3 = 3.102$$

- **Step iv:** Compute the consistence index, CI.

$$CI = (\lambda_{\max} - n)/(n - 1) = (3.102 - 3)/2 = .051$$

- **Step v:** Compute the consistency ratio, CR, by CI/RI , where $RI = .58$ for 3 factors:

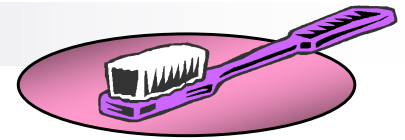
$$CR = CI/RI = .051/.58 = .088$$

Since the consistency ratio, CR, is less than 0.1, this is well within the acceptable range for consistency.

NOTE: *RI is the consistency index of a randomly generated pairwise comparison matrix. The value of RI depends on the number of items being compared and is given as follows:*

n	3	4	5	6	7	8
RI	0.58	0.90	1.12	1.24	1.32	1.41

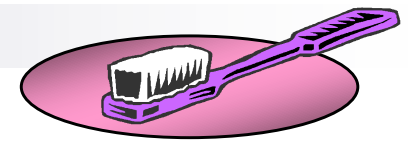
Pairwise Comparison Matrix: Reliability



Cornell is equally to moderately preferred to Brushpik.

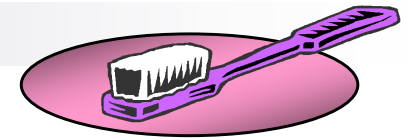
Cornell is very strongly preferred to Picobuy. Brush Pik is strongly preferred to Picobuy.

Pairwise Comparison Matrix: Reliability



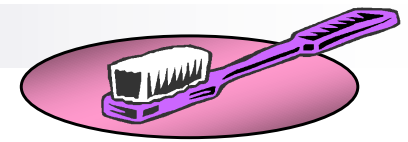
	Cornell	Brush Pik	Picobuy
Cornell	1	2	7
Brush Pik	1/2	1	5
Picobuy	1/7	1/5	1

Normalized Matrix: Reliability



Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

Reliability	Cornel	Brush Pik	Picobuy
Cornel	0.609	0.625	0.538
Brush Pik	0.304	0.313	0.385
Picobuy	0.087	0.063	0.077



Priority Vector: Reliability

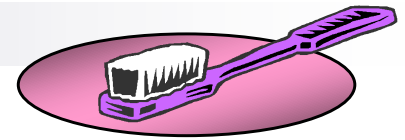
The priority vector is determined by averaging the row entries in the normalized matrix.

Reliability	Priority Vector
Cornel	0.591
Brush Pik	0.334
Picobuy	0.075

- **Checking Consistency**

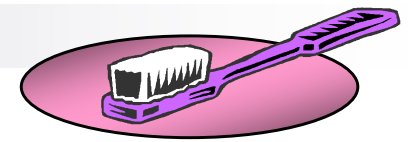
Gill Glass' responses to reliability could be checked for consistency in the same manner as was cost.

Pairwise Comparison Matrix: Delivery Time



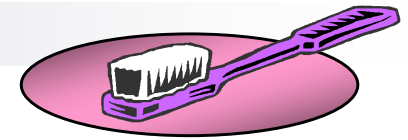
Gill Glass has determined that for delivery time, Cornell is equally preferable to Picobuy. Both Cornell and Picobuy are very strongly to extremely preferable to Brush Pik.

Pairwise Comparison Matrix: Delivery Time



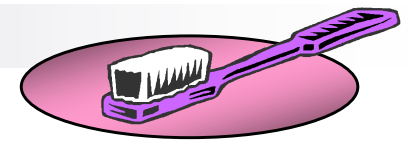
	Cornell	Brush Pik	Picobuy
Cornell	1	8	1
Brush Pik	1/8	1	1/8
Picobuy	1	8	1

Normalized Matrix: Delivery Time



Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cornell	Brush Pik	Picobuy
Cornell	$8/17$	$8/17$	$8/17$
Brush Pik	$1/17$	$1/17$	$1/17$
Picobuy	$8/17$	$8/17$	$8/17$



Priority Vector: Delivery Time

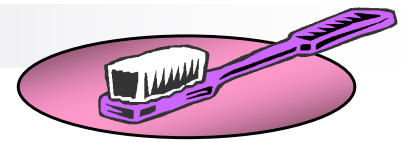
The priority vector is determined by averaging the row entries in the normalized matrix. Converting to decimals we get:

$$\begin{array}{l} \text{Cornell:} \quad (8/17 + 8/17 + 8/17)/3 = \left[\begin{array}{c} .471 \\ .059 \\ .471 \end{array} \right] \\ \text{Brush Pik:} \quad (1/17 + 1/17 + 1/17)/3 = \\ \text{Picobuy:} \quad (8/17 + 8/17 + 8/17)/3 = \end{array}$$

■ Checking Consistency

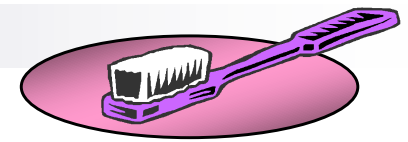
Gill Glass' responses to delivery time could be checked for consistency in the same manner as was cost.

Pairwise Comparison Matrix: Criteria



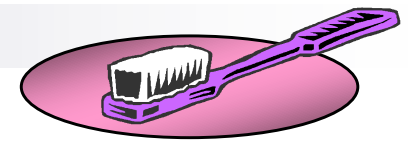
The accounting department has determined that in terms of criteria, cost is extremely preferable to delivery time and very strongly preferable to reliability, and that reliability is very strongly preferable to delivery time.

Pairwise Comparison Matrix: Criteria



	Cost	Reliability	Delivery
Cost	1	3	9
Reliability	1/3	1	7
Delivery	1/9	1/7	1

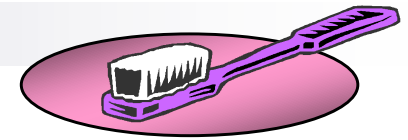
Normalized Matrix: Criteria



Divide each entry in the pair-wise comparison matrix by its corresponding column sum.

	Cost	Reliability	Delivery
Cost	0.692	0.724	0.529
Reliability	0.231	0.241	0.412
Delivery	0.077	0.034	0.059

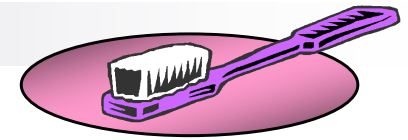
Priority Vector: Criteria



The priority vector is determined by averaging the row entries in the normalized matrix.

	Priority Vector
Cost	0.649
Reliability	0.295
Delivery	0.057

Overall Priority Vector



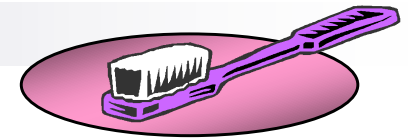
The overall priorities are determined by multiplying the priority vector of the criteria by the priorities for each decision alternative for each objective.

Priority Vector

for Criteria [.649 .295 .057]

	Cost	Reliability	Delivery
Cornell	.298	.591	.471
Brush Pik	.632	.334	.059
Picobuy	.069	.075	.471

Overall Priority Vector



Thus, the overall priority vector is:

Cornel	0.394
Brush Pik	0.512
Picobuy	0.094

Brush Pik appears to be the overall recommendation.

Car Selection Problem

Let us consider a car purchasing decision problem faced by Diane Payne. After a preliminary analysis of the makes and models of several used cars, Diane narrowed her list of decision alternatives to three cars: a [Honda Accord](#), a [Saturn](#), and a [Chevrolet Cavalier](#). Below Table summarizes the information Diane collected about these cars.

Decision Alternative			
Characteristics	Accord	Saturn	Cavalier
Price	\$13,100	\$11,200	\$9500
Color	Black	Red	Blue
Miles per gallon	19	23	28
Interior	Deluxe	Above Average	Standard
Body type	4-door midsize	2-door sport	2-door compact
Sound system	AM/ FM, tape, CD	AM/ FM	AM/ FM

Car Selection Problem

Diane decided that the following criteria were relevant for her car selection decision process:

- Price
- Miles per gallon (MPG)
- Comfort
- Style

Decision Alternative			
Characteristics	Accord	Saturn	Cavalier
Price	\$13,100	\$11,200	\$9500
Color	Black	Red	Blue
Miles per gallon	19	23	28
Interior	Deluxe	Above Average	Standard
Body type	4-door midsize	2-door sport	2-door compact
Sound system	AM/ FM, tape, CD	AM/ FM	AM/ FM

Car Selection Problem

- ❑ Data regarding the Price and MPG are provided in the Table.
- ❑ However, measures of Comfort and Style cannot be specified so directly.
 - ❑ Diane will need to consider factors such as the car's interior, type of audio system, ease of entry, seat adjustments, and driver visibility in order to determine the comfort level of each car.
 - ❑ The style criterion will have to be based on Diane's subjective evaluation of the color and the general appearance of each car.
- ❑ Using AHP, find out the best alternative for Diane.



Thank You !!!

Contact:
aditya.sahu@iimrohtak.ac.in