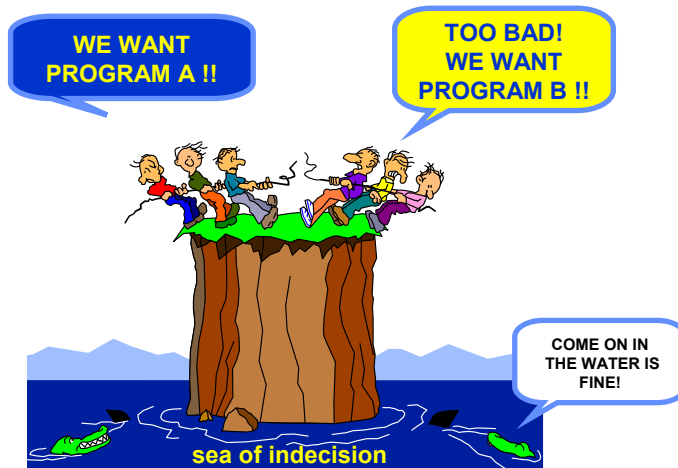


An Illustrated Guide to the
ANALYTIC HIERARCHY PROCESS

Dr. Aditya K Sahu

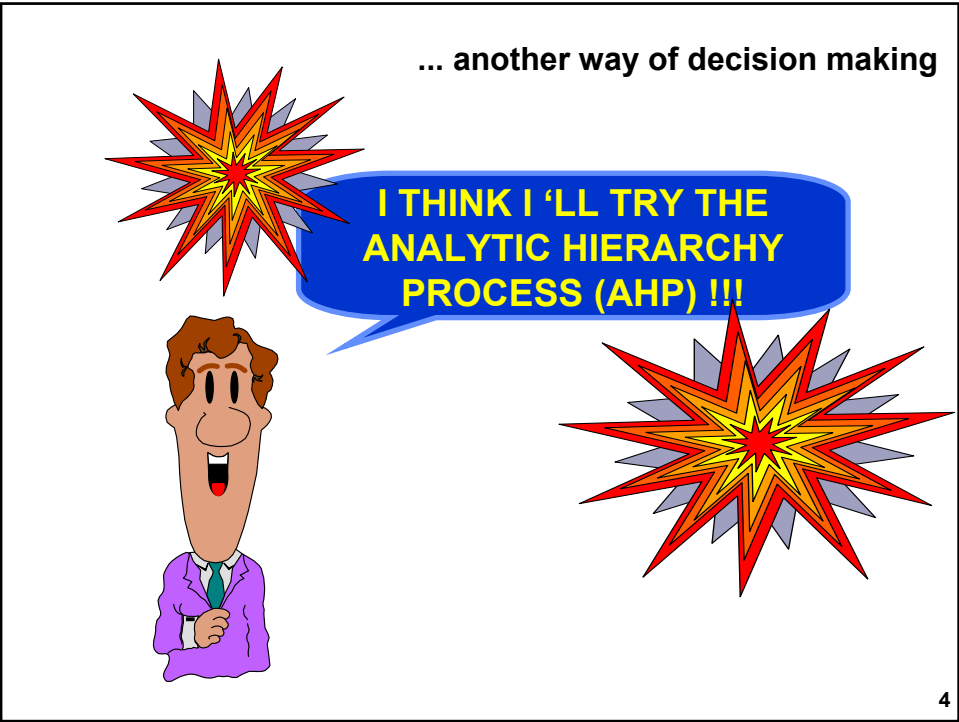
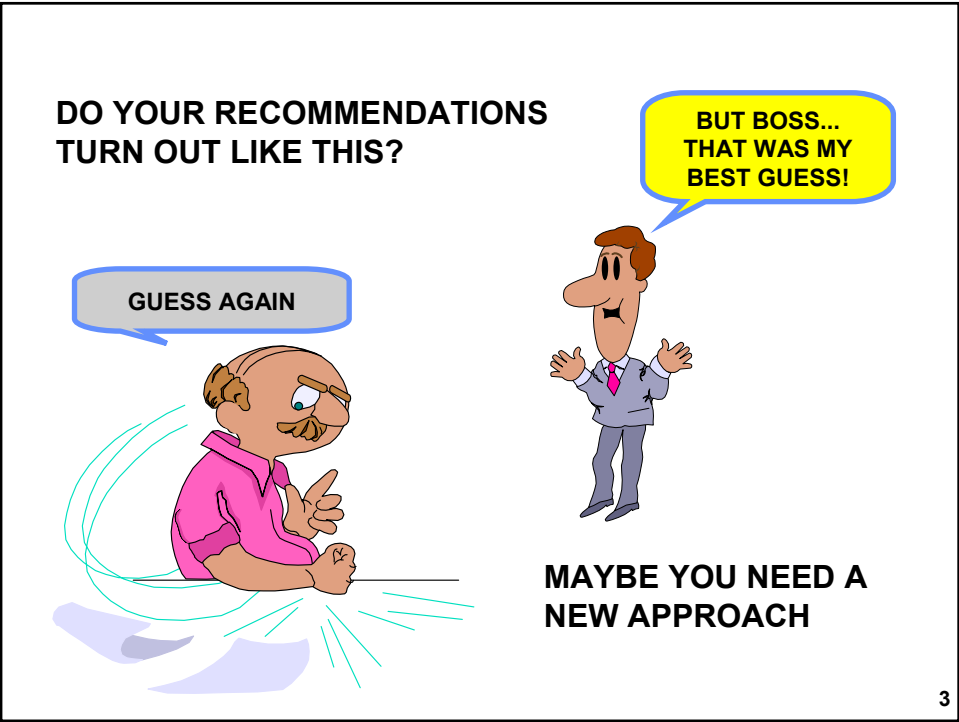
1

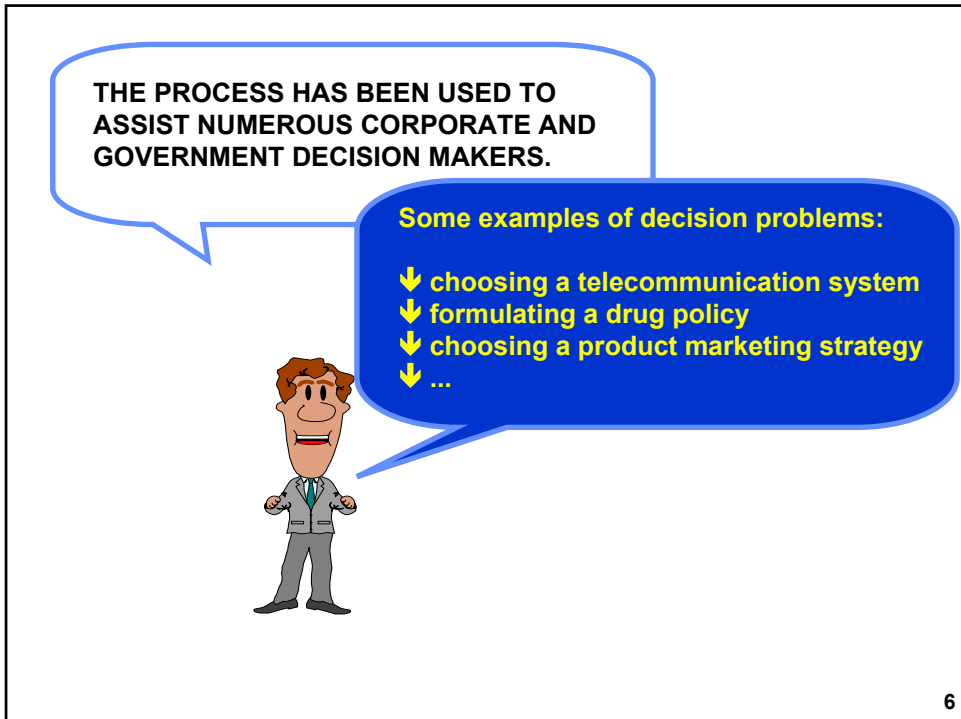
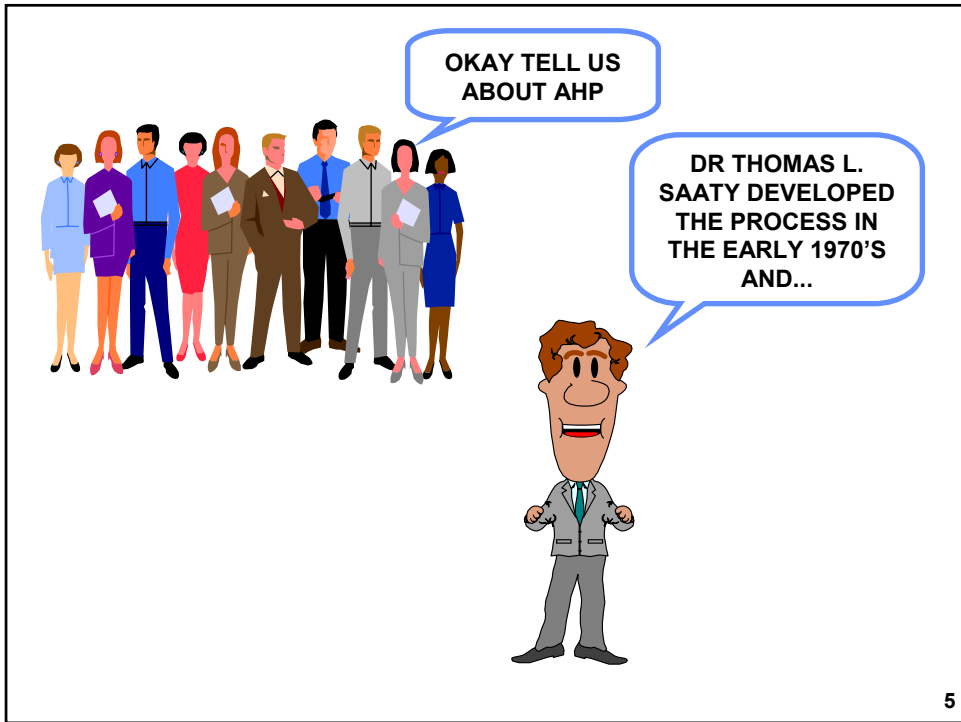
Do your decision conferences turn out like this?



or does this happen?

2





Let's show how it works

PROBLEMS ARE DECOMPOSED INTO A HIERARCHY OF CRITERIA AND ALTERNATIVES

```
graph TD; Problem[Problem] --> C1[Criterion 1]; Problem --> C2[Criterion 2]; Problem --> Cn[Criterion n]; C1 --> C11[Criterion 1.1]; C1 --> Dots1[...]; C2 --> Dots2[...]; Cn --> Dotsn[...]; C11 --> A1[Alternative 1]; C11 --> A2[Alternative 2]; C11 --> Dots3[...]; C2 --> A2; Cn --> Dots4[...]; A1 --> Dots5[...]; A2 --> Dots5; Dots3 --> Dots5; Dots4 --> Dots5; Dots5 --> An[Alternative n];
```

7

OKAY, HERE'S A DECISION PROBLEM WE FACE IN OUR PERSONAL LIVES

8



AN IMPORTANT PART OF THE PROCESS IS TO ACCOMPLISH THESE THREE STEPS



- **STATE THE OBJECTIVE:**
 - SELECT A NEW CAR
- **DEFINE THE CRITERIA:**
 - STYLE, RELIABILITY, FUEL ECONOMY
- **PICK THE ALTERNATIVES:**
 - CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO

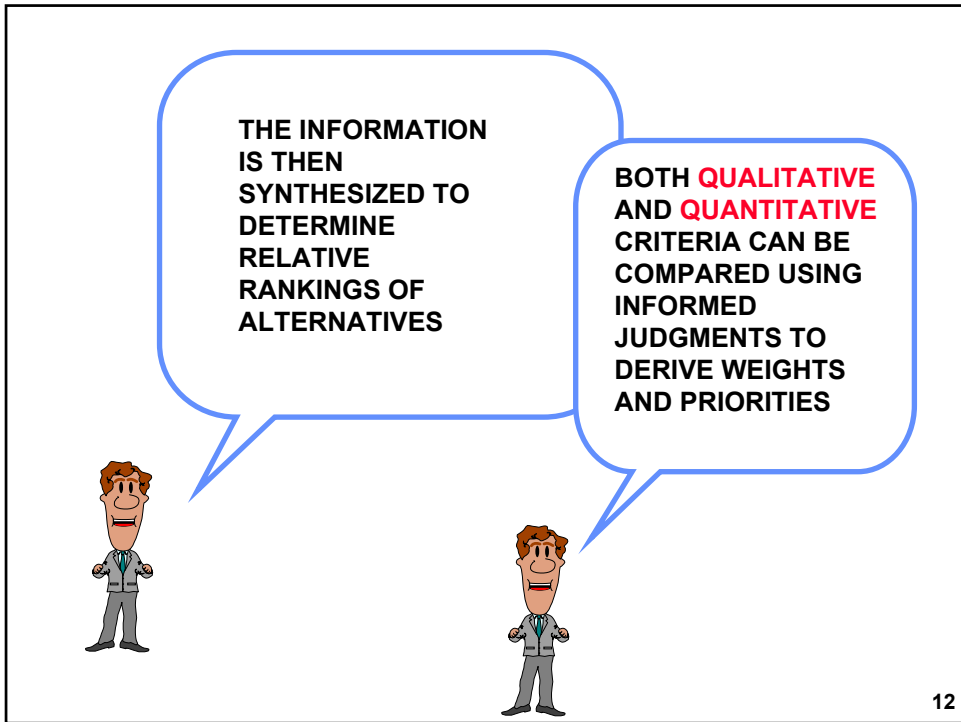
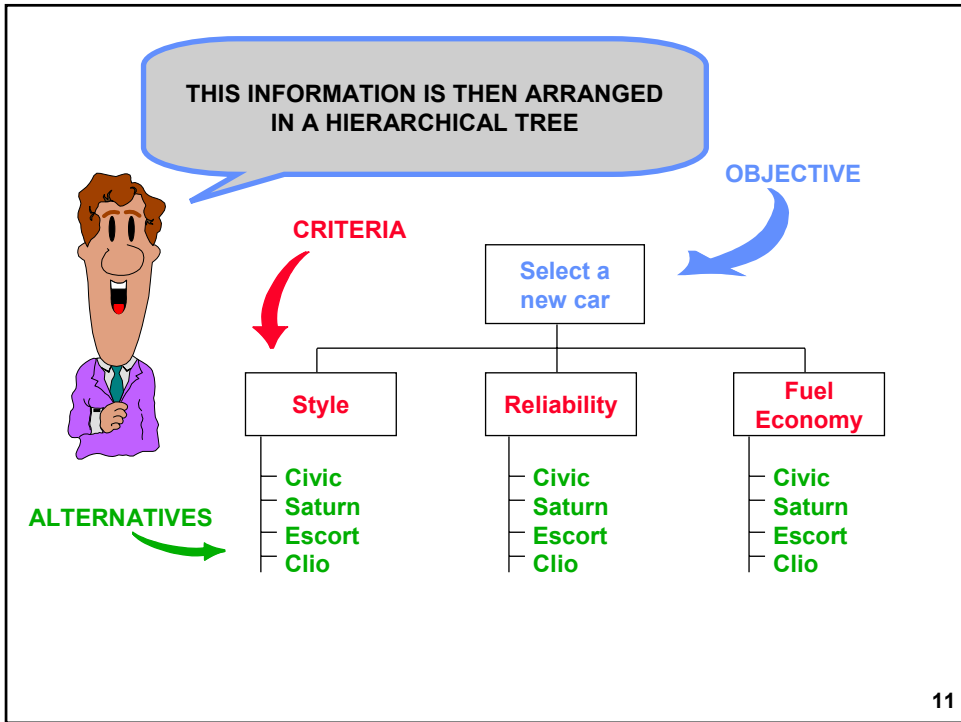
WHAT ABOUT COST?



(BE QUIET, WE'LL TALK ABOUT THAT LATER)

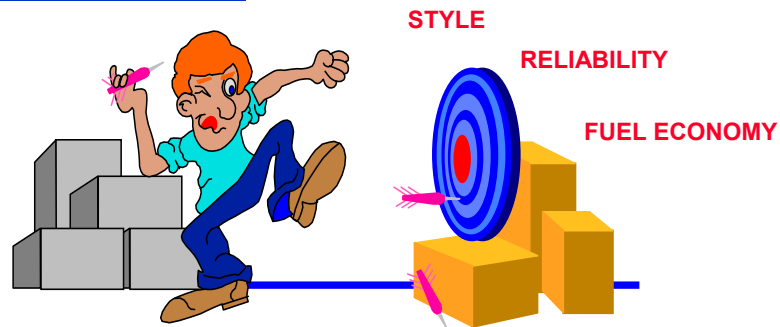
SKEPTIC-GATOR

10



HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here's one way !

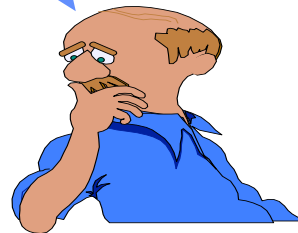


13

HERE'S ANOTHER WAY

Hmm, I think reliability is the most important followed by style and fuel economy is least important so I will make the following judgements

USING **JUDGMENTS** TO DETERMINE THE RANKING OF THE CRITERIA



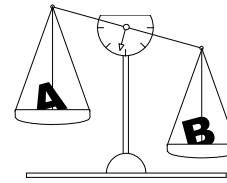
1. **RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE**
2. **STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY**
3. **RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY**

he's not very consistent here ... that's o.k.

14



Pairwise Comparisons

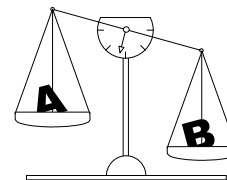


USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

15



Pairwise Comparisons



USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

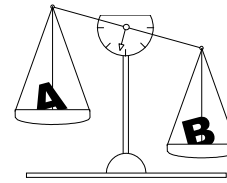
1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY		1/1	4/1
FUEL ECONOMY			1/1

16



Pairwise Comparisons



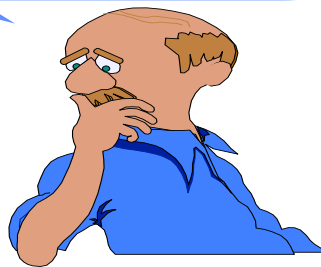
USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

17

How do you turn this MATRIX into ranking of criteria?



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

18

HOW DO YOU GET A RANKING OF PRIORITIES FROM A PAIRWISE MATRIX?

AND THE SURVEY SAYS



ACTUALLY...

DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.

REFERENCE : THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY

19

HERE'S HOW TO SOLVE FOR THE EIGENVECTOR:

1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.
2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.
3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.

SAY WHAT!

SHOW ME AN EXAMPLE



20

IT'S MATRIX ALGEBRA TIME !!!



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

FOR NOW, LET'S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

21

STEP 1: SQUARING THE MATRIX

THIS TIMES

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

THIS

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

I.E. $(1.0000 * 1.0000) + (0.5000 * 2.0000) + (3.0000 * 0.3333) = 3.0000$

RESULTS IN THIS

3.0000	1.7500	8.0000
5.3332	3.0000	14.0000
1.1666	0.6667	3.0000

22

**STEP 2 : NOW, LET'S COMPUTE OUR FIRST EIGENVECTOR
(TO FOUR DECIMAL PLACES)**

FIRST, WE SUM THE ROWS

$$\begin{array}{r}
 \boxed{\begin{array}{r} 3.0000 + 1.7500 + 8.0000 \\ 5.3332 + 3.0000 + 14.0000 \\ 1.1666 + 0.6667 + 3.0000 \end{array}} = \begin{array}{r} 12.7500 \\ 22.3332 \\ 4.8333 \end{array} \quad \begin{array}{r} 0.3194 \\ 0.5595 \\ 0.1211 \end{array}
 \end{array}$$

SECOND, WE SUM THE ROW TOTALS \rightarrow $\begin{array}{r} 39.9165 \\ 1.0000 \end{array}$

FINALLY, WE **NORMALIZE** BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

THE RESULT IS OUR EIGENVECTOR
(A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)

$$\boxed{\begin{array}{r} 0.3194 \\ 0.5595 \\ 0.1211 \end{array}}$$

23

THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION (REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)

CONTINUING OUR EXAMPLE,
AGAIN, STEP 1: WE SQUARE THIS MATRIX

$$\boxed{\begin{array}{r} 3.0000 \quad 1.7500 \quad 8.0000 \\ 5.3332 \quad 3.0000 \quad 14.0000 \\ 1.1666 \quad 0.6667 \quad 3.0000 \end{array}}$$

WITH THIS RESULT \rightarrow

$$\boxed{\begin{array}{r} 27.6653 \quad 15.8330 \quad 72.4984 \\ 48.3311 \quad 27.6662 \quad 126.6642 \\ 10.5547 \quad 6.0414 \quad 27.6653 \end{array}}$$

24

AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)

27.6653	+	15.8330	+	72.4984	=	115.9967	0.3196
48.3311	+	27.6662	+	126.6642	=	202.6615	0.5584
10.5547	+	6.0414	+	27.6653	=	44.2614	0.1220
						TOTALS	362.9196 1.0000

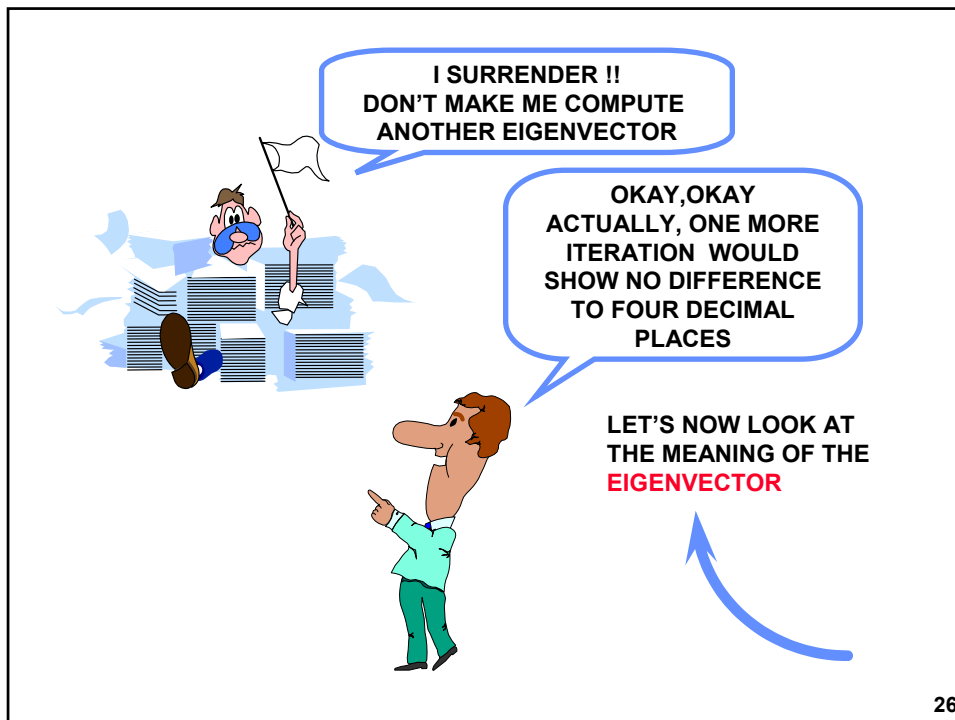
COMPUTE THE DIFFERENCE OF THE PREVIOUS COMPUTED EIGENVECTOR TO THIS ONE:

0.3194	-	0.3196	=	- 0.0002
0.5595	-	0.5584	=	0.0011
0.1211	-	0.1220	=	- 0.0009

TO FOUR DECIMAL PLACES THERE'S NOT MUCH DIFFERENCE
HOW ABOUT ONE MORE ITERATION?




25



26

HERE'S OUR PAIRWISE MATRIX WITH THE NAMES



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1


AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

STYLE	0.3196	← THE SECOND MOST IMPORTANT CRITERION
RELIABILITY	0.5584	← THE MOST IMPORTANT CRITERION
FUEL ECONOMY	0.1220	← THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIEARCHICAL TREE...

27

HERE'S THE TREE WITH THE CRITERIA WEIGHTS



OBJECTIVE


CRITERIA

- Select a new car 1.00
 - Style .3196
 - Civic
 - Saturn
 - Escort
 - Clio
 - Reliability .5584
 - Civic
 - Saturn
 - Escort
 - Clio
 - Fuel Economy .1220
 - Civic
 - Saturn
 - Escort
 - Clio

ALTERNATIVES

WHAT ABOUT THE ALTERNATIVES?

I'M GLAD YOU ASKED...



28

IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



STYLE

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	1/4	4/1	1/6
SATURN	4/1	1/1	4/1	1/4
ESCORT	1/4	1/4	1/1	1/5
CLIO	6/1	4/1	5/1	1/1

AND...

29

IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



RELIABILITY

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	2/1	5/1	1/1
SATURN	1/2	1/1	3/1	2/1
ESCORT	1/5	1/3	1/1	1/4
CLIO	1/1	1/2	4/1	1/1

ITS MATRIX ALGEBRA TIME!!!

30

COMPUTING THE EIGENVECTOR
DETERMINES THE RELATIVE
RANKING OF ALTERNATIVES
UNDER EACH CRITERION



RANKING	STYLE		RANKING	RELIABILITY	
3	CIVIC	.1160	1	CIVIC	.3790
2	SATURN	.2470	2	SATURN	.2900
4	ESCORT	.0600	4	ESCORT	.0740
1	CLIO	.5770	3	CLIO	.2570

WHAT ABOUT FUEL ECONOMY?



SKEPTIC-GATOR

ANOTHER GOOD QUESTION...

31

AS STATED EARLIER,
AHP CAN COMBINE BOTH QUALITATIVE
AND QUANTITATIVE INFORMATION



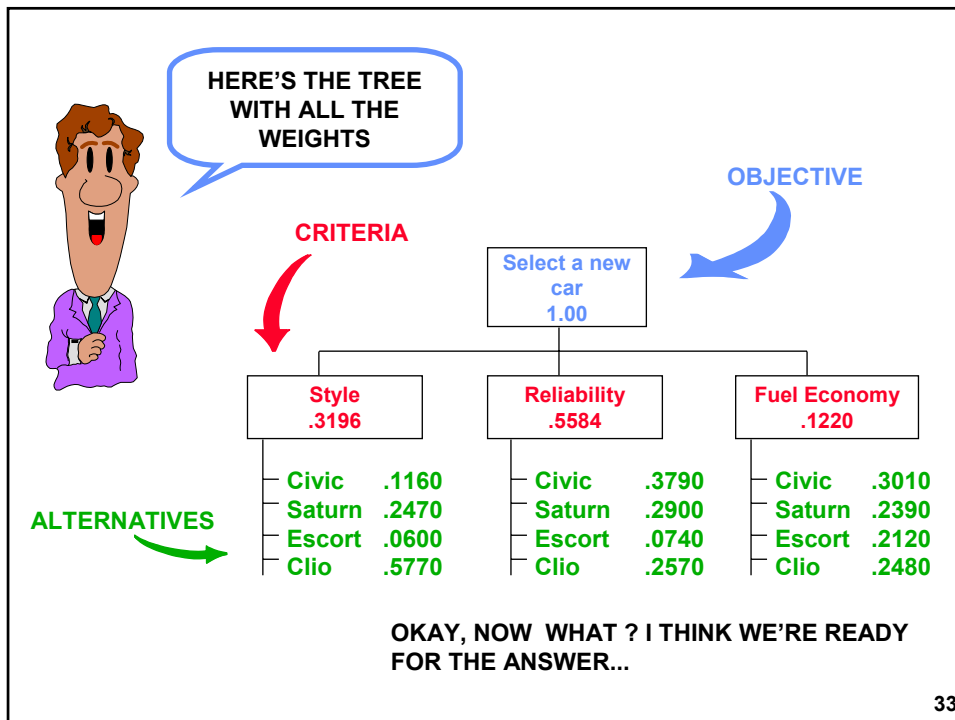
FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH
ALTERNATIVE:

	FUEL ECONOMY (MILES/GALLON)		
CIVIC	34	$34 / 113 =$.3010
SATURN	27	$27 / 113 =$.2390
ESCORT	24	$24 / 113 =$.2120
CLIO	28	$28 / 113 =$.2480
	<u>113</u>		<u>1.0000</u>

NORMALIZING THE FUEL ECONOMY INFO
ALLOWS US TO USE IT WITH OTHER RANKINGS



32



A LITTLE MORE MATRIX ALGEBRA GIVES US THE SOLUTION:

	STYLE	RELI- ABILITY	FUEL ECONOMY	CRITERIA RANKING	
CIVIC	.1160	.3790	.3010	0.3196	STYLE
SATURN	.2470	.2900	.2390	0.5584	RELIABILITY
ESCORT	.0600	.0740	.2120	0.1220	FUEL ECONOMY
CLIO	.5770	.2570	.2480		

I.E. FOR THE CIVIC $(.1160 * .3196) + (.3790 * .5584) + (.3010 * .1220) = .3060$

Civic	.3060
Saturn	.2720
Escort	.0940
Clio	.3280

AND THE WINNER IS !!!
THE CLIO IS THE HIGHEST RANKED CAR

34

IN SUMMARY, THE ANALYTIC HIERARCHY PROCESS PROVIDES A LOGICAL FRAMEWORK TO DETERMINE THE BENEFITS OF EACH ALTERNATIVE



- | | |
|-----------|-------|
| 1. Clio | .3280 |
| 2. Civic | .3060 |
| 3. Saturn | .2720 |
| 4. Escort | .0940 |

WHAT ABOUT COSTS?



SKEPTIC-GATOR

WELL, I'LL TELL YOU...

35

ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, **COSTS** SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED



OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON'T CARE ABOUT ITS BENEFITS



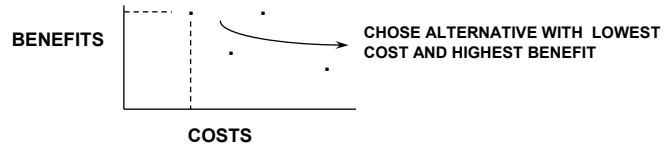
DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES

36

WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:



1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE



2. BENEFIT TO COST RATIOS

3. LINEAR PROGRAMMING

4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS

IN OUR EXAMPLE...

37

LET'S USE BENEFIT TO COST RATIOS



	COST \$	NORMALIZED COSTS	BENEFIT - COST RATIOS
1. CLIO	18,000	.3333	.3280 / .3333 = .9840
2. CIVIC	12,000	.2222	.3060 / .2222 = 1.3771
3. SATURN	15,000	.2778	.2720 / .2778 = .9791
4. ESCORT	9,000	.1667	.0940 / .1667 = .5639
	<u>54,000</u>	<u>1.0000</u>	

(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

AND...

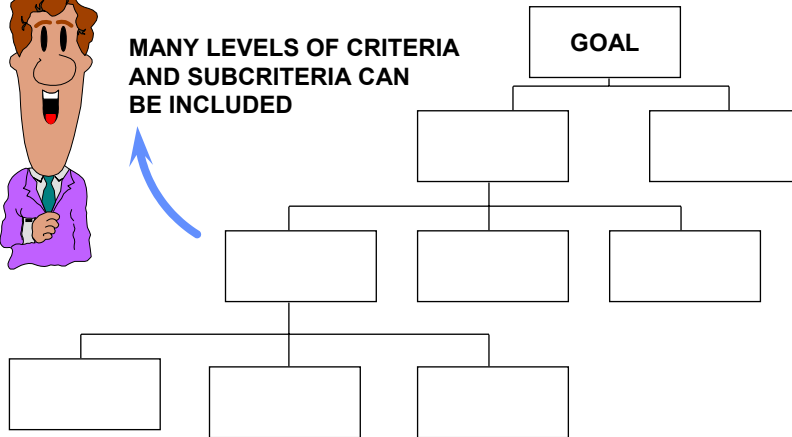
THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO

38

AHP CAN BE USED FOR VERY COMPLEX DECISIONS



MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED



HERE'S SOME EXAMPLES

39

AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS



- STRATEGIC PLANNING**
- RESOURCE ALLOCATION**
- SOURCE SELECTION**
- BUSINESS/PUBLIC POLICY**
- PROGRAM SELECTION**
- AND MUCH MUCH MORE...**

40